Nuclear Symmetry Energy and Its Components at Zero and Finite Temperatures

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II. Study of the temperature dependence of the symmetry energy coefficient in finite nuclei and other properties (nucleon densities, rms radii, neutron skins)

- **A.N. Antonov, M.K. Gaidarov, P. Sarriguren, E. Moya de Guerra**, "Volume and Surface Contributions to the Nuclear Symmetry Energy within the Coherent Density Fluctuation Model", Phys. Rev. C **94**, 014319 (2016)

- **A.N. Antonov, D.N. Kadrev, M.K. Gaidarov, P. Sarriguren, E. Moya de Guerra**, "Temperature Dependence of the Symmetry Energy and Neutron Skins in Ni, Sn, and Pb Isotopic Chains", Phys. Rev. C **95**, 024314 (2017)

- **A.N. Antonov, D.N. Kadrev, M.K. Gaidarov, P. Sarriguren, E. Moya de Guerra**, "Temperature Dependence of the Volume and Surface Contributions to the Nuclear Symmetry Energy within the Coherent Density Fluctuation Model", Phys. Rev. C **98**, 054315 (2018)

Theoretical Formalism

In the liquid drop model [Myers,Swiatecki (1969); Steiner *et al.* (2005)]:

$$E(A,Z) = -BA + E_S A^{2/3} + S^V A \frac{(1-2Z/A)^2}{1+S^S A^{-1/3}/S^V} + E_C \frac{Z^2}{A^{1/3}} + E_{dif}\frac{Z^2}{A} + E_{ex}\frac{Z^{4/3}}{A^{1/3}} + a\Delta A^{-1/2} \quad (5)$$

The third term in (5) is

$$S(T)\frac{(N-Z)^2}{A} \quad (6)$$

where

$$S(T) = \frac{S^V(T)}{1+\frac{S^S(T)}{S^V(T)}A^{-1/3}} = \frac{S^V(T)}{1+A^{-1/3}/\kappa(T)} \quad (7)$$

with

$$\kappa(T) \equiv \frac{S^V(T)}{S^S(T)}. \quad (8)$$

Theoretical Formalism

In nuclear matter ($A \to \infty$ and $S^S/S^V \to 0$), we have $S(T) = S^V(T)$. Also at large A Eq.(7) becomes:

$$S(T) = \frac{S^V(T)}{1 + \dfrac{S^S(T)}{S^V(T)} A^{-1/3}} \simeq c_3 - \frac{c_4}{A^{1/3}}, \tag{9}$$

where $c_3 = S^V$ and $c_4 = S^S$.

From Eq.(7):

$$S^V(T) = S(T)\left(1 + \frac{1}{\kappa(T) A^{1/3}}\right) \tag{10}$$

$$S^S(T) = \frac{S(T)}{\kappa(T)}\left(1 + \frac{1}{\kappa(T) A^{1/3}}\right) \tag{11}$$

Theoretical Formalism

CDFM (Sofia group 1979 - till now)

$$\rho(\mathbf{r},\mathbf{r}') = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_x(\mathbf{r},\mathbf{r}') \tag{12}$$

$$\rho_x(\mathbf{r},\mathbf{r}') = 3\rho_0(x)\frac{j_1(k_F(x)|\mathbf{r}-\mathbf{r}'|)}{(k_F(x)|\mathbf{r}-\mathbf{r}'|)}\Theta\left(x-\frac{|\mathbf{r}+\mathbf{r}'|}{2}\right) \tag{13}$$

$$\rho_0(x) = \frac{3A}{4\pi x^3} \tag{14}$$

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\beta}{x} \qquad \beta = \left(\frac{9\pi A}{8}\right)^{1/3} \simeq 1.52A^{1/3} \tag{15}$$

$$\rho(\mathbf{r}) = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_0(x)\Theta(x-|\mathbf{r}|) \tag{16}$$

$$\text{At } \frac{d\rho}{dr} \le 0: \qquad |\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)}\left.\frac{d\rho(r)}{dr}\right|_{r=x} \tag{17}$$

Theoretical Formalism

In PRC **84**, 034316 (2011) and other:

$$s = \int_0^\infty dx |\mathcal{F}(x)|^2 S(\rho(x)) \tag{18}$$

The T-dependent $S(T)$:

$$S(T) = \int_0^\infty dx |\mathcal{F}(x,T)|^2 S(\rho(x,T)). \tag{19}$$

$$|\mathcal{F}(x,T)|^2 = -\frac{1}{\rho_0(x)} \left. \frac{d\rho_{total}(r,T)}{dr} \right|_{r=x} \tag{20}$$

$$\rho_{total}(r,T) = \rho_p(r,T) + \rho_n(r,T) \tag{21}$$

$\rho_p(r,T)$ and $\rho_n(r,T)$ calculated using the HFBTHO code

$$\kappa(T) = \frac{3}{R\rho_0} \int_0^\infty dx |\mathcal{F}(x,T)|^2 x\rho_0(x) \left\{ \frac{S(\rho_0)}{S[\rho(x,T)]} - 1 \right\} \tag{22}$$

$R = r_0 A^{1/3}$ [Dieperink, Van Isacker (2009)]

Theoretical Formalism

$\Rightarrow$Danielewicz, Dieperink, Van Isacker:

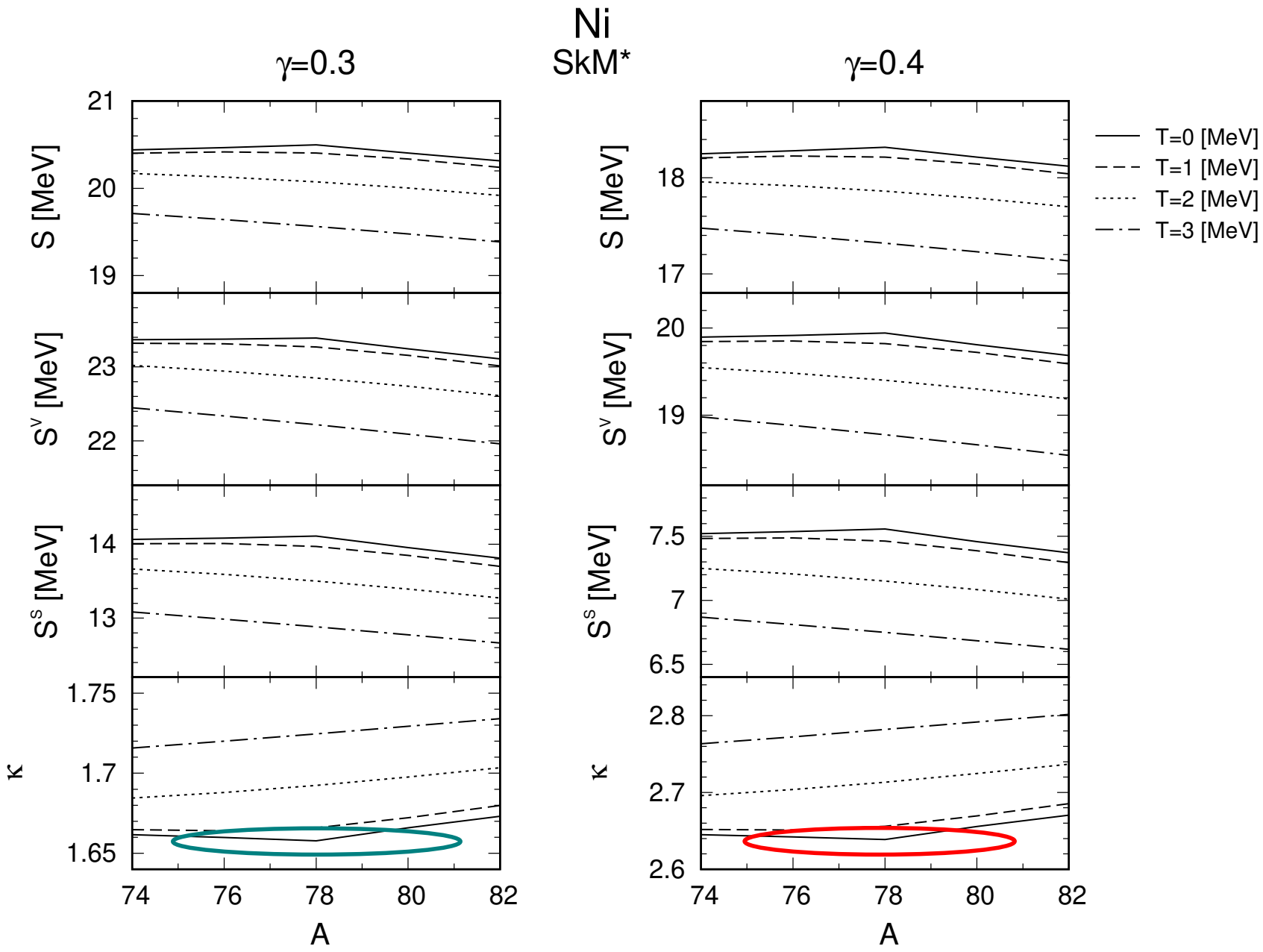
$$S(\rho(x,T)) = S^V(T)\left(\frac{\rho(x,T)}{\rho_0}\right)^\gamma. \tag{23}$$

Various estimations for γ:
$\gamma = 0.5 \pm 0.1$
$0.54 \leq \gamma \leq 0.77$
$\gamma = 0.60 \pm 0.05; \quad \gamma = 0.55 \pm 0.03$
$\gamma = 0.55; \quad \gamma = 0.79; \quad \gamma = 0.72 \pm 0.19$

$$S(T) = S(\rho_0)\int_0^\infty dx|\mathcal{F}(x,T)|^2\left(\frac{\rho(x,T)}{\rho_0}\right)^\gamma \tag{24}$$

$$\kappa(T) \equiv \frac{S^V(T)}{S^S(T)} = \frac{3}{R\rho_0}\int_0^\infty dx|\mathcal{F}(x,T)|^2 x\rho_0(x)\left\{\left(\frac{\rho_0}{\rho(x,T)}\right)^\gamma - 1\right\} \tag{25}$$

Results and Discussion



κ with Brueckner EDF

- Our results:

$$2.10 \leq \kappa \leq 2.90 \tag{1}$$

- Comparison with data (Dieperink, Van Isacker, 2007):

- from IAS and skins (Danielewicz, 2004):

$$2.6 \leq \kappa \leq 3.0 \tag{2}$$

- from masses and skins (Danielewicz, 2003):

$$2.0 \leq \kappa \leq 2.8 \tag{3}$$

- from masses and skins (Dieperink, Van Isacker, 2007):

$$1.6 \leq \kappa \leq 2.0 \tag{4}$$

- ”kinks” at ^{78}Ni and ^{132}Sn (double-magic nuclei)

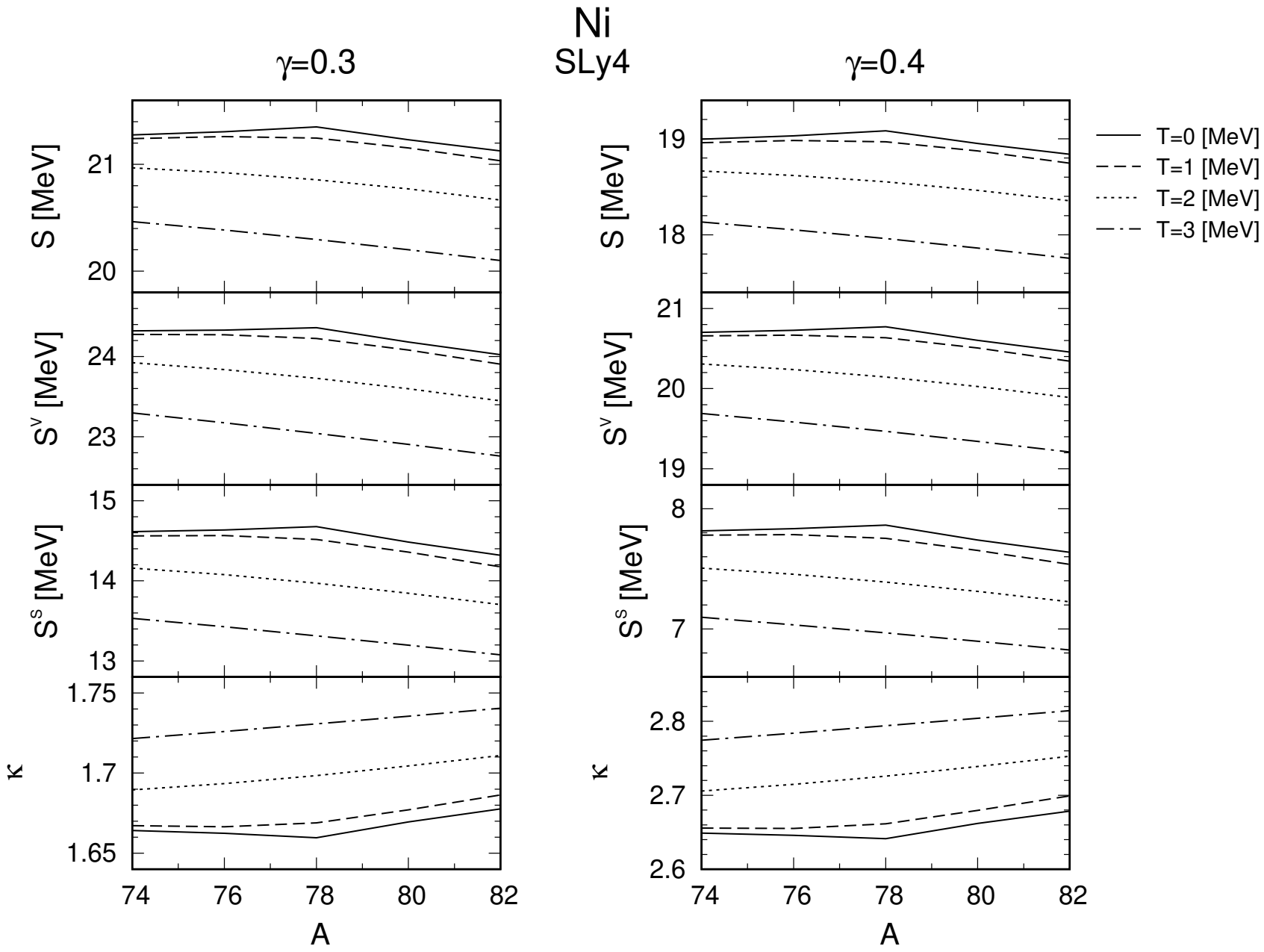
Ranges of changes of κ with Skyrme EDF are:

- Ni isotopic chain
$1.5 \leq \kappa \leq 1.7$ (SLy4 and SGII)
$0.88 \leq \kappa \leq 1.05$ (Sk3)

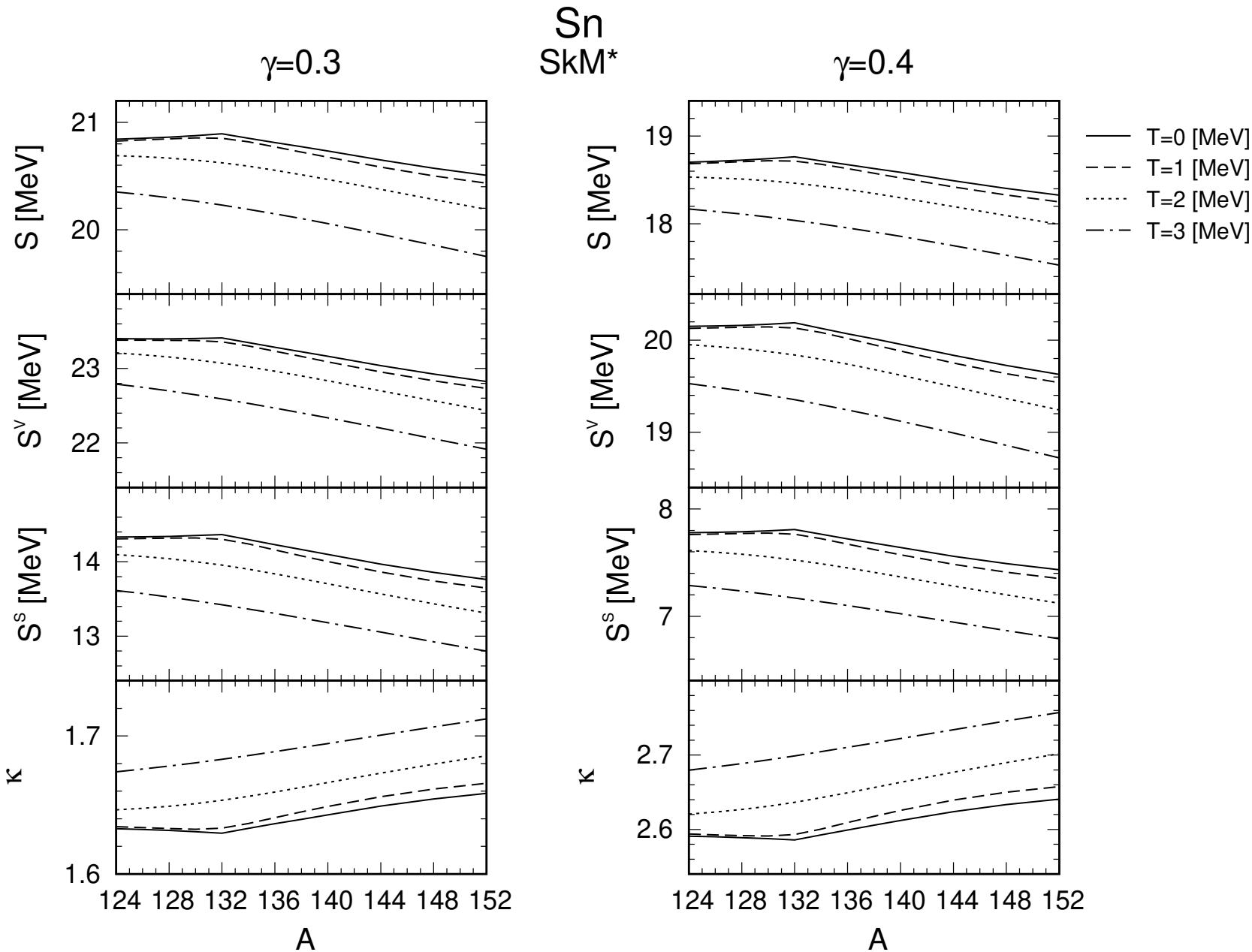
- Sn isotopic chain
$1.52 \leq \kappa \leq 2.1$ (SLy4 and SGII)
$0.82 \leq \kappa \leq 1.14$ (Sk3)

- Pb isotopic chain
$1.65 \leq \kappa \leq 1.75$ (SLy4 and SGII)
$0.84 \leq \kappa \leq 0.88$ (Sk3)

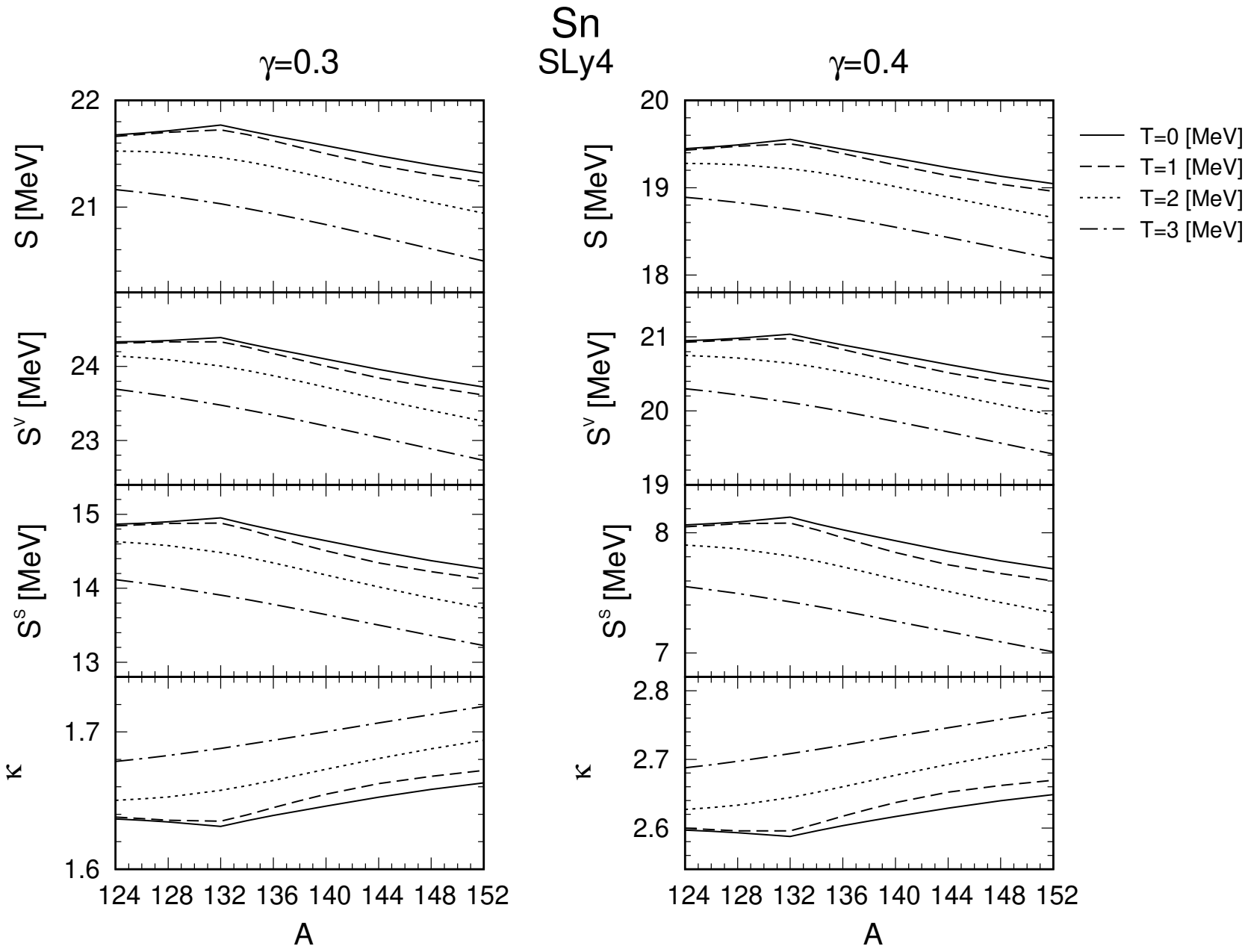
Results and Discussion



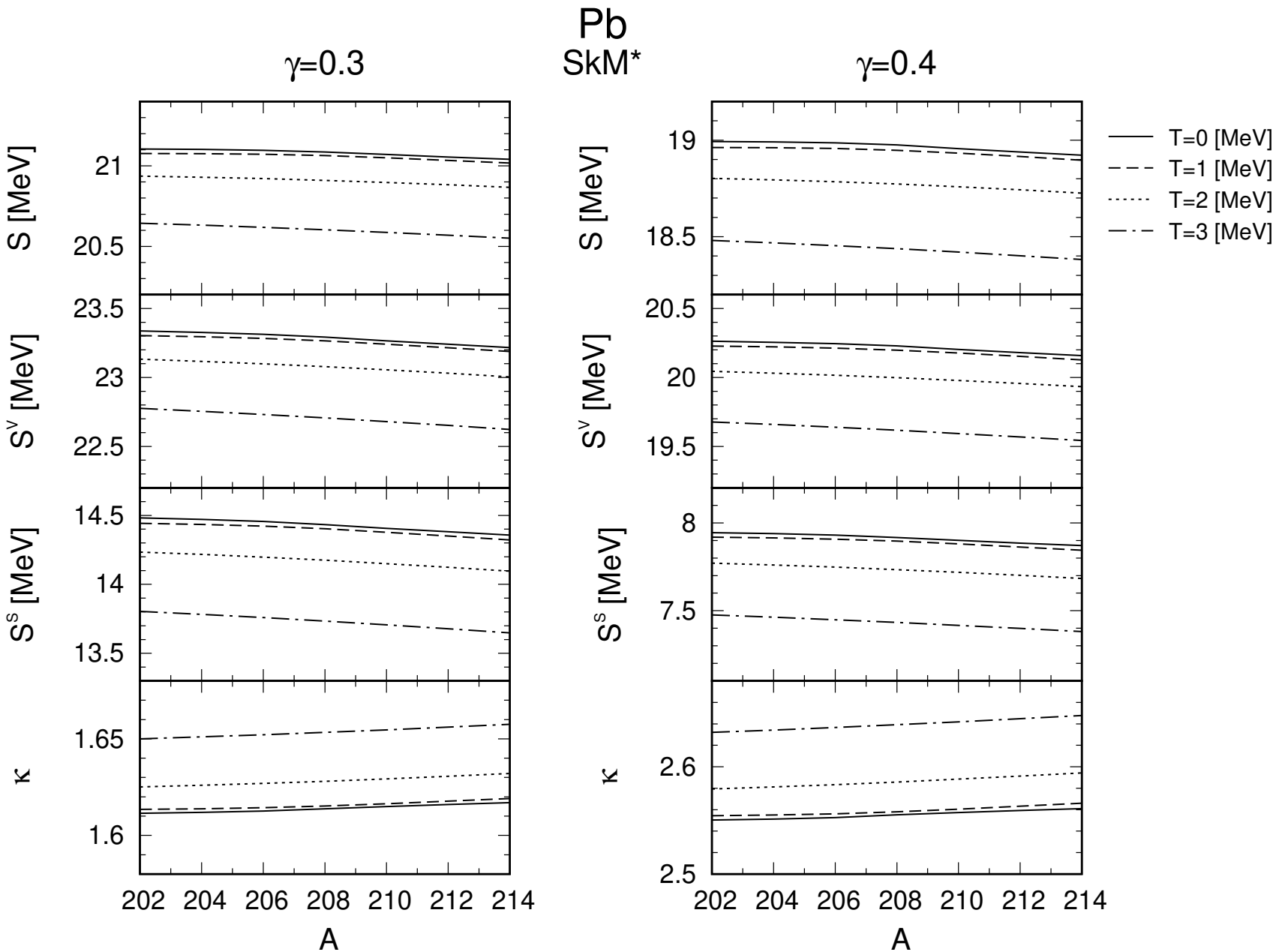
Results and Discussion



Results and Discussion



Results and Discussion



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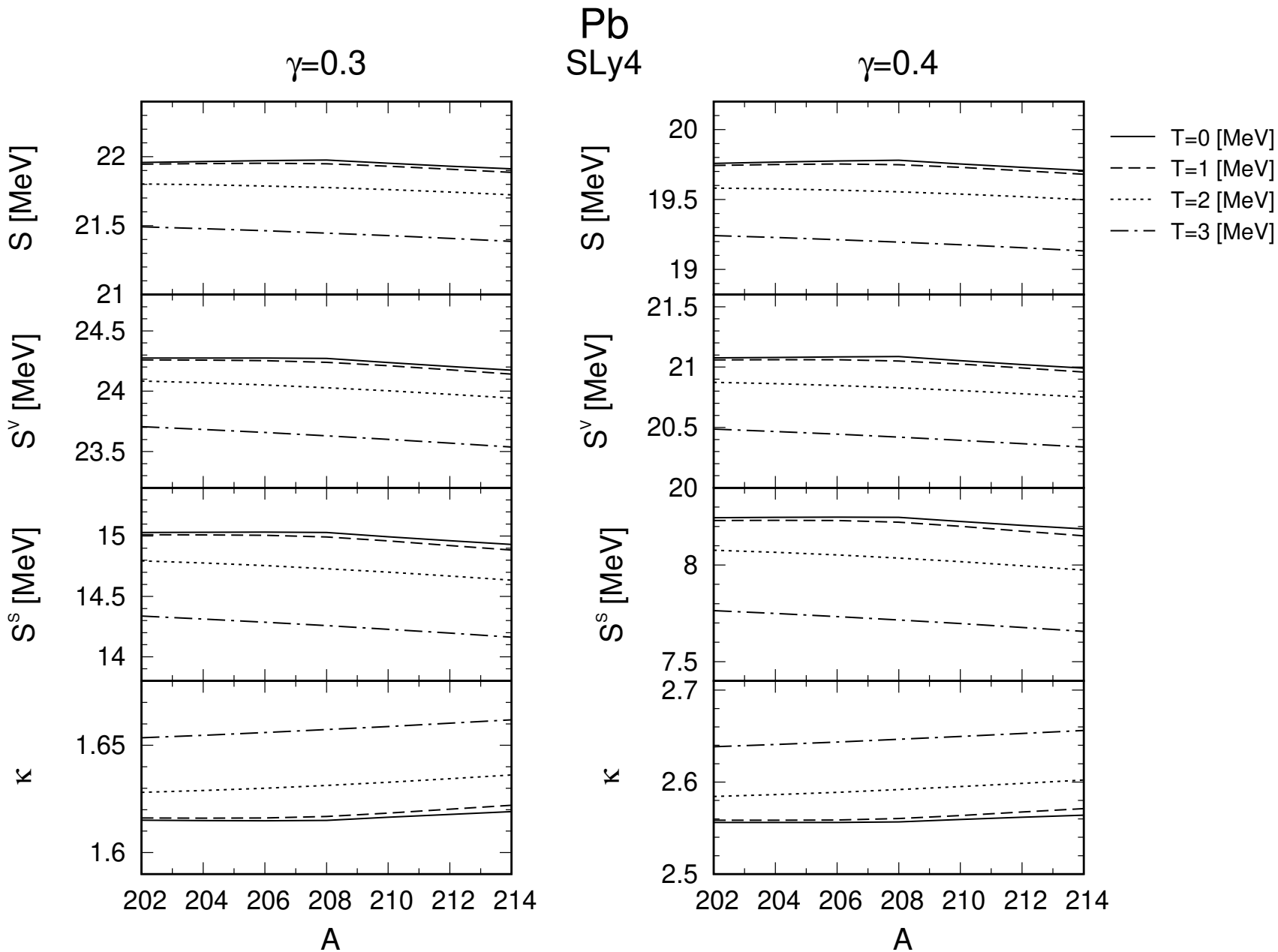
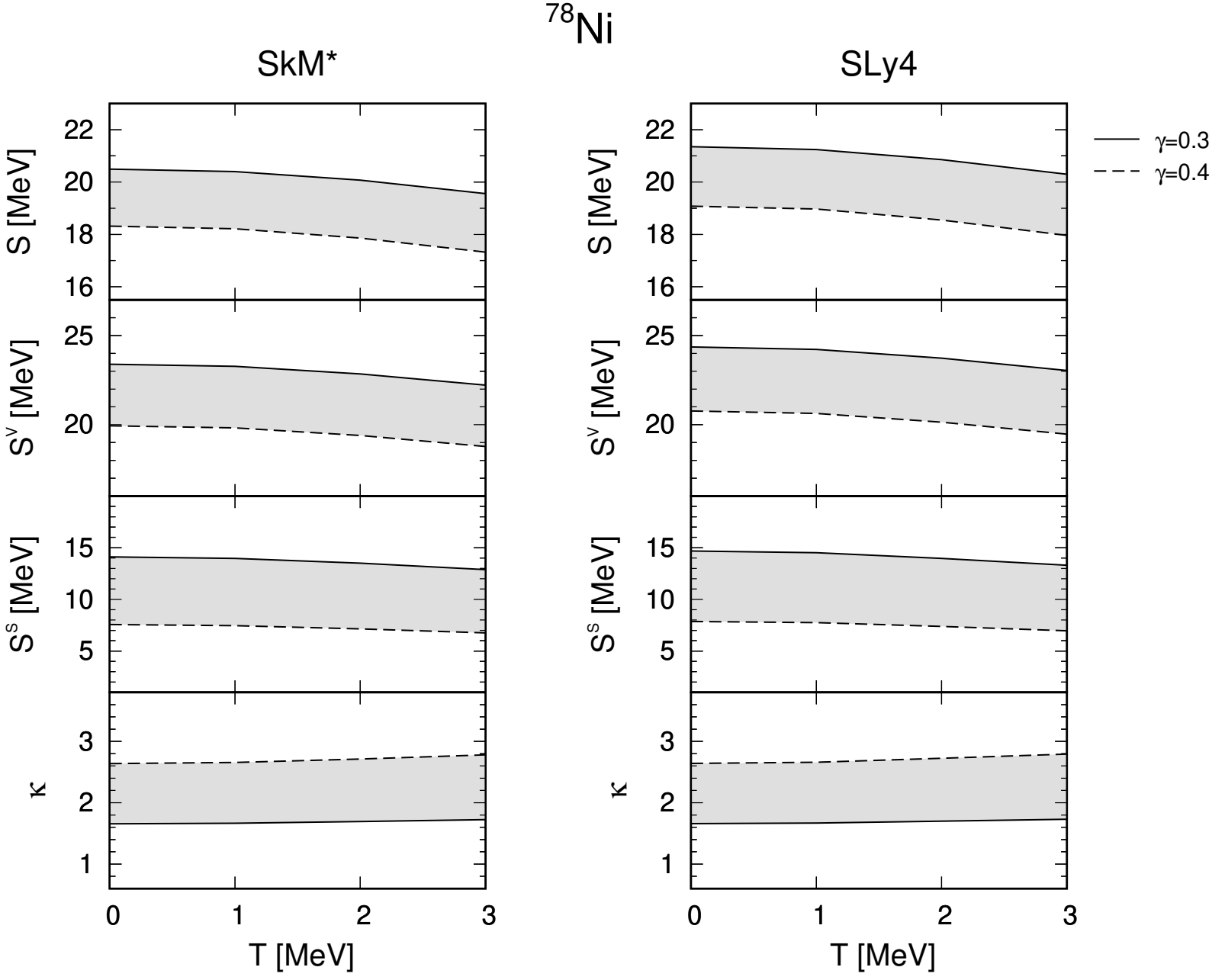
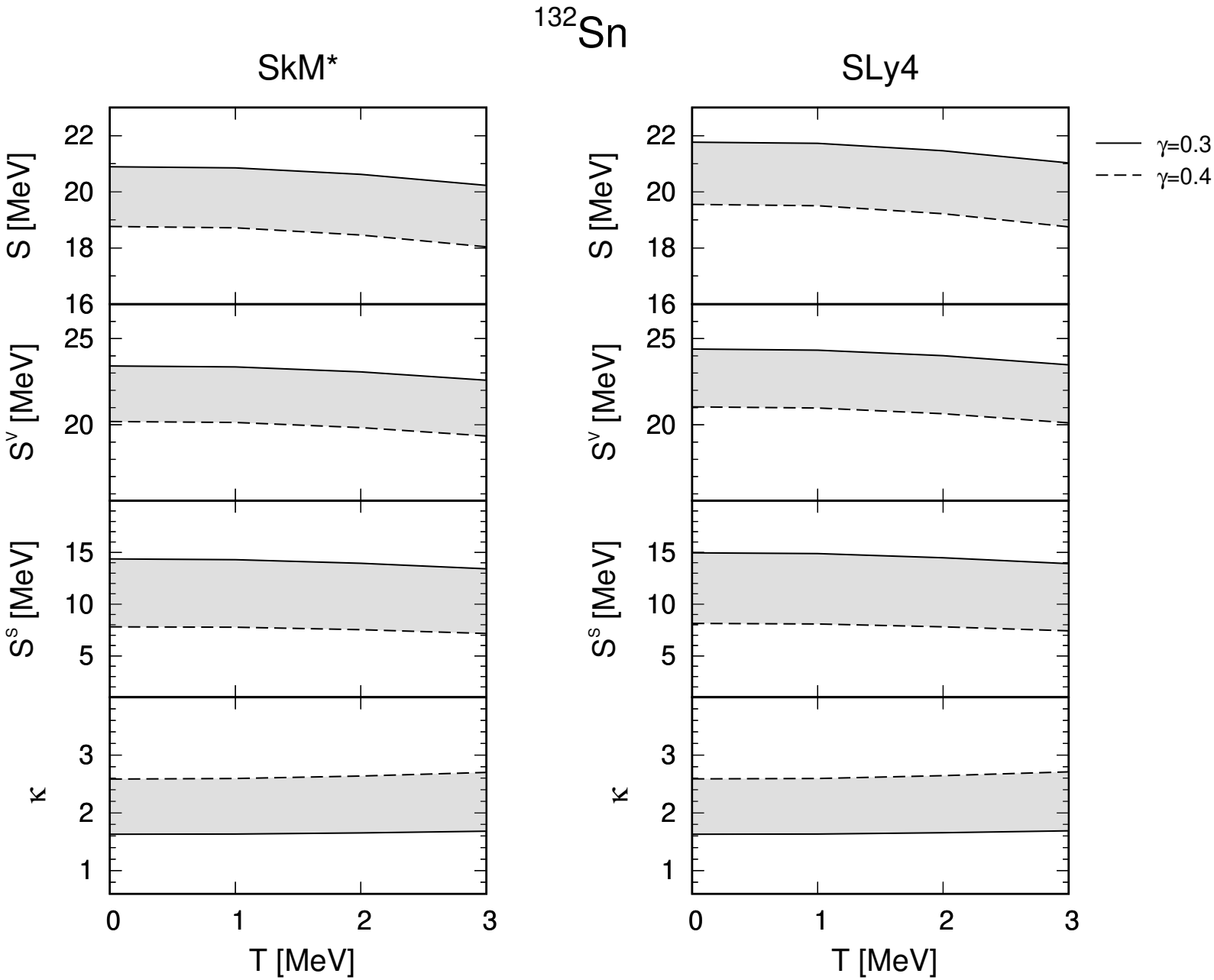


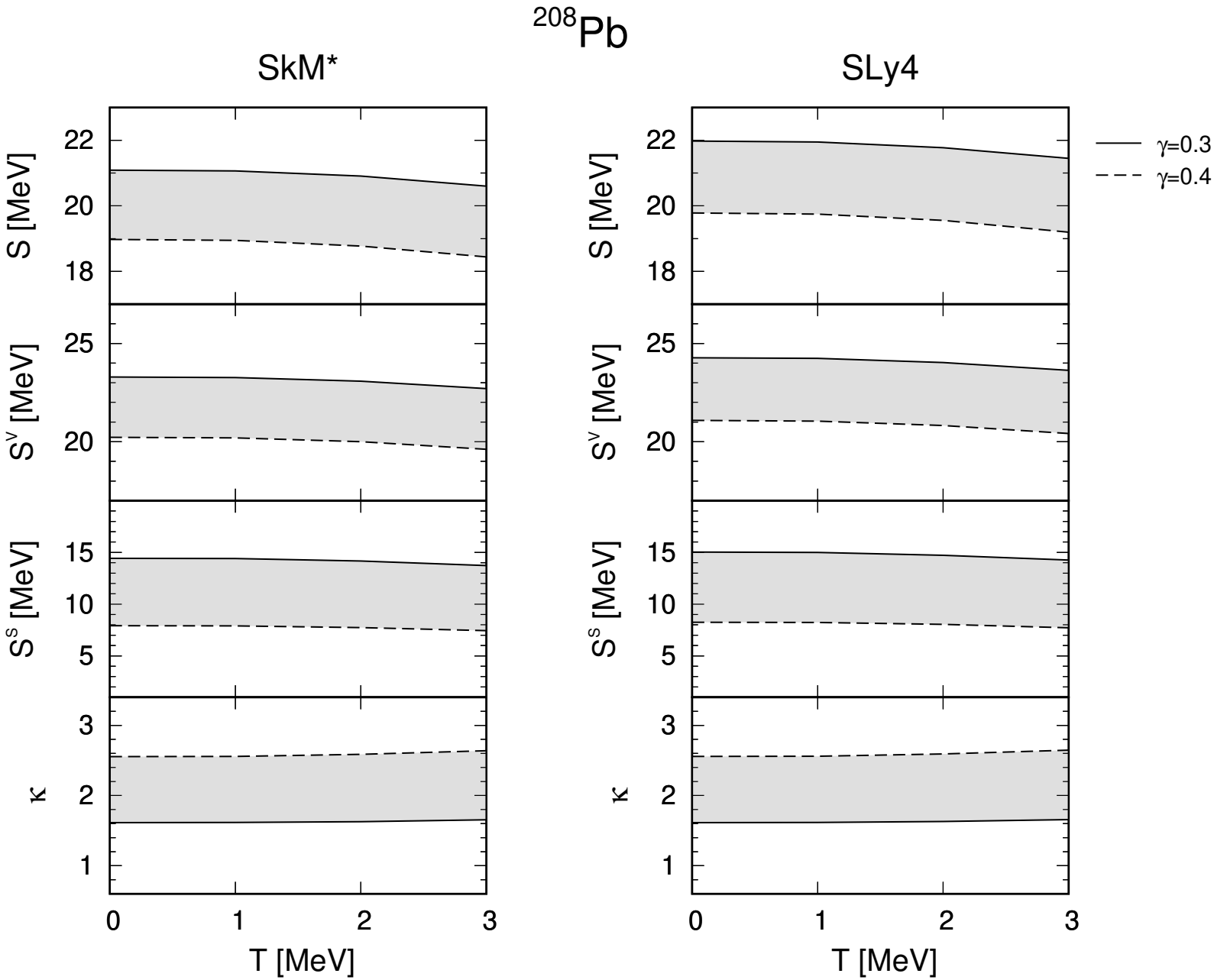
Results and Discussion



Results and Discussion



Results and Discussion



$$S_0(\rho) = S^V \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{1}$$

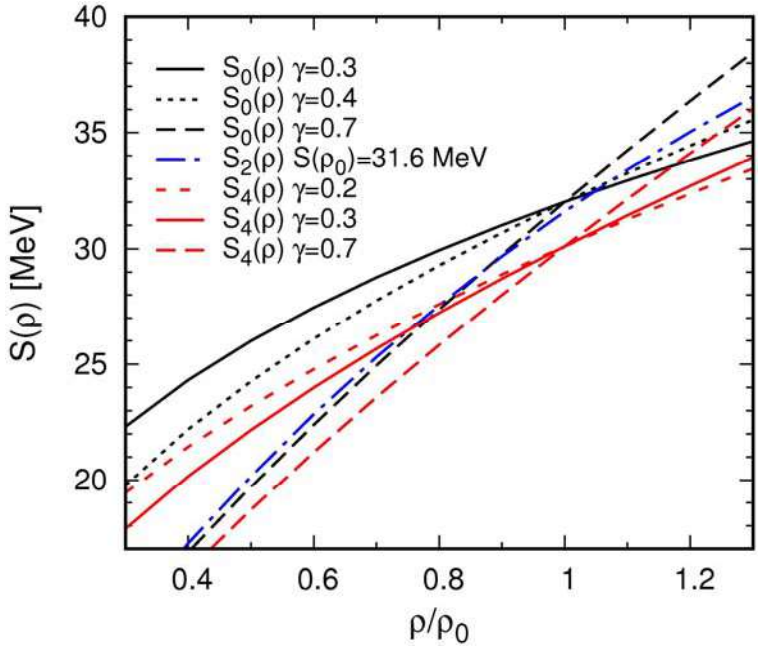
Alternative parametrizations of the density dependence of the symmetry energy

– J. Dong, W. Zuo, J. Gu, U. Lombardo, Phys. Rev. C **85**, 034308 (2012):

$$S_2(\rho) = C_k \left(\frac{\rho}{\rho_0}\right)^{2/3} + C_1 \left(\frac{\rho}{\rho_0}\right) + C_2 \left(\frac{\rho}{\rho_0}\right)^{1.52} \tag{2}$$

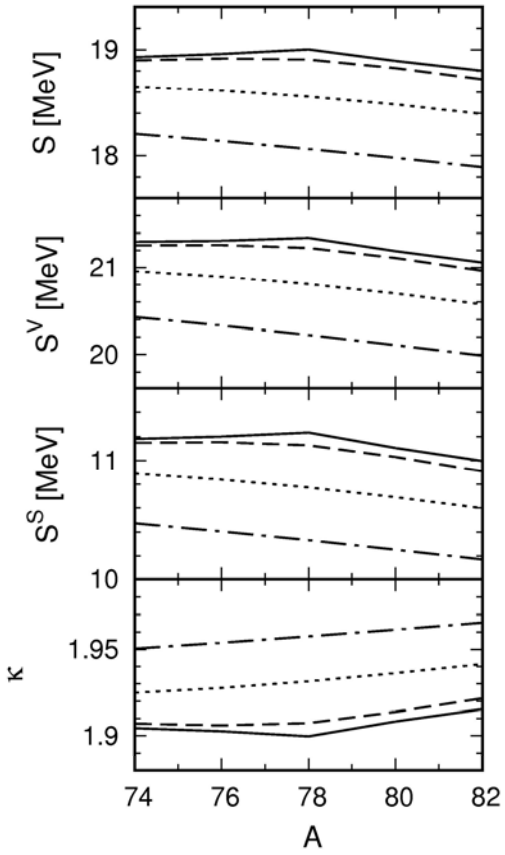
– M.B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W.G. Lynch, A.W. Steiner, Phys. Rev. Lett. **102**, 122701 (2009):

$$S_4(\rho) = 12.5 \left(\frac{\rho}{\rho_0}\right)^{2/3} + 17.6 \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{3}$$

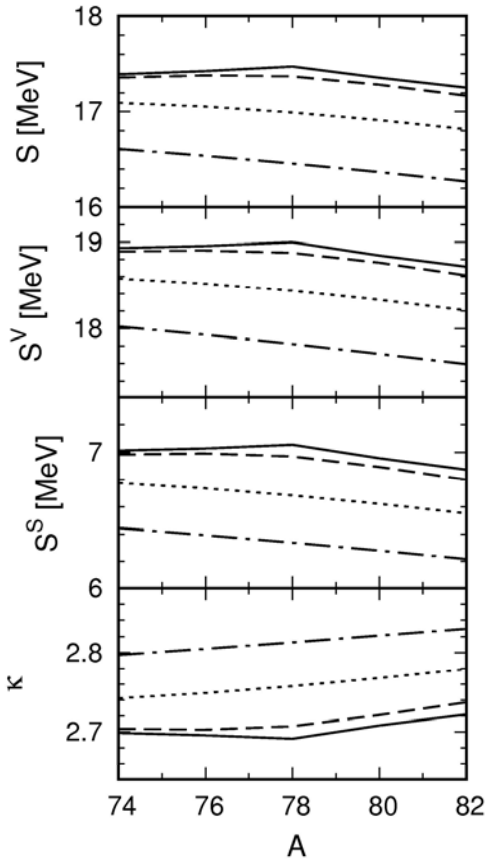


Ni isotopic chain

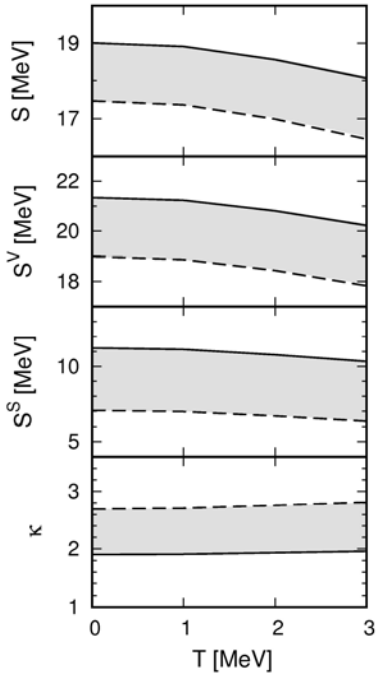
γ=0.2



γ=0.3

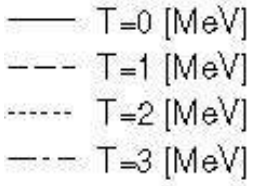


^{78}Ni



γ=0.2 (solid line)

γ=0.3 (dashed line)



Values of the parameter ***γ***, the ratio ***κ***, and the symmetry energy ***S(T)*** (in MeV) for the ^{78}Ni nucleus at T=0 MeV in the case of the density dependence of the symmetry energy $S_0(\rho)$ and by $S_4(\rho)$

γ	κ	$S(T)$	γ	κ	$S(T)$
0.3	1.66	21.35	0.2	1.90	19.00
0.4	2.64	19.08	0.3	2.69	17.48
0.5	4.09	17.20	0.4	3.74	16.23
0.7	10.67	14.26	0.7	9.82	13.58

– We study the T-dependence of the symmetry energy on the examples of Ni, Sn, and Pb isotopic chains in the interval $T = 0 - 4$ MeV

- Local Density Approximation (LDA)
- T-dependent local density distributions $\rho(r, T)$
- T-dependent kinetic energy densities $\tau(r, T)$
- Skyrme-HFB method using the cylindrical transformed harmonic-oscillator basis (HFBTHO) for $\rho(r, T)$ and $\tau(r, T)$
- For $\tau(r, T)$ also Thomas-Fermi (TF) expression up to T^2 term is used
- SLy4 and SkM* Skyrme forces used
- In addition, for ^{208}Pb with $\rho(r, T)$ from extended Thomas-Fermi (ETF) method and the rigorous density functional approach (RDFA)

– Study of T-dependent rms radii of the proton and neutron distributions, as well as the formation of neutron skins in hot nuclei

Theoretical formalism

Temperature-dependent symmetry energy coefficient with Skyrme energy density functional

- For finite systems (using LDA):

$$e_{sym}(A,T) = \frac{1}{I^2 A}\int \rho(r) e_{sym}[\rho(r),T]\delta^2(r) d^3r \tag{5}$$

$I = (N-Z)/A$, $e_{sym}[\rho(r),T]$ is the symmetry energy coefficient at temperature T of infinite nuclear matter at $\rho(r) = \rho_n(r) + \rho_p(r)$,
$\delta(r) = [\rho_n(r) - \rho_p(r)]/\rho(r)$

$$e_{sym}(\rho,T) = \frac{e(\rho,\delta,T) - e(\rho,\delta=0,T)}{\delta^2}, \tag{6}$$

$e = \mathcal{E}(r,T)/\rho$, where $\mathcal{E}(r,T)$ is the total energy density of the system

- For Skyrme EDF:

$$
\begin{aligned}
\mathcal{E}(r,T) &= \frac{\hbar^2}{2m_{n,k}}\tau_n + \frac{\hbar^2}{2m_{p,k}}\tau_p \\
&+ \frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(x_0+\frac{1}{2}\right)\left(\rho_n^2+\rho_p^2\right)\right] \\
&+ \frac{1}{12}t_3\rho^\alpha\left[\left(1+\frac{x_3}{2}\right)\rho^2 - \left(x_3+\frac{1}{2}\right)\left(\rho_n^2+\rho_p^2\right)\right] \\
&+ \frac{1}{16}\left[3t_1\left(1+\frac{1}{2}x_1\right) - t_2\left(1+\frac{1}{2}x_2\right)\right](\nabla\rho)^2 \\
&- \frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right) + t_2\left(x_2+\frac{1}{2}\right)\right] \\
&\times \left[(\nabla\rho_n)^2 + (\nabla\rho_p)^2\right] + \mathcal{E}_c(r)
\end{aligned}
\tag{7}
$$

(for infinite homogeneous NM only the first three lines contribute)
- t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, and α are the Skyrme parameters
- We use SkM* and SLy4 Skyrme interactions

Temperature-dependent kinetic energy density

a) Using HFBTHO code [M.V. Stoitsov *et al.*; 2005, 2013]
b) Using Thomas-Fermi approximation [Agrawal *et al.*; 2014] or its expression up to T^2 term [Lee, Mekjian; 2010]

$$\tau_q(r,T) = \frac{2m}{\hbar^2}\varepsilon_{K_q} = \frac{3}{5}(3\pi^2)^{2/3}\left[\rho_q^{5/3} + \frac{5\pi^2 m_q^2}{3\hbar^4}\frac{1}{(3\pi^2)^{4/3}}\rho_q^{1/3}T^2\right] \tag{14}$$

Temperature-dependent densities

$\rho_q(\vec{r},T)$ are calculated using the HFBTHO code

$$\int \rho_q(\vec{r},T)d\vec{r} = Q, \qquad Q = Z, N \tag{15}$$

- mean square radii:

$$< R_{\rm q}^2 >= \frac{\int r^2\rho_{\rm q}(\vec{r},T)d\vec{r}}{\int \rho_{\rm q}(\vec{r},T)d\vec{r}} \tag{16}$$

- rms radii: $R_{\rm q} =< R_{\rm q}^2 >^{1/2}$
- Neutron skin thickness:

$$\Delta R = R_{\rm n} - R_{\rm p} \tag{17}$$

- Density of ^{208}Pb [A.N. Antonov *et al.*; 1989]

a) Fermi-type in ETF [Brack; 1984, 1985]

$$\rho_{ETF}(r,T) = \rho_0(T)\left\{1+\exp\left[\frac{r-R(T)}{\alpha(T)}\right]\right\}^{-\gamma(T)} \tag{18}$$

It reproduces THF results up to $T = 4$ MeV.

b) Symmetrized Fermi determined with RDFA [M.V. Stoitsov *et al.*; 1987]

$$\rho_{SF}(r,T) = \rho_0(T)\frac{\sinh[R(T)/b(T)]}{\cosh[R(T)/b(T)]+\cosh[r/b(T)]} \tag{19}$$

Relationships for calculations of T-dependent symmetry energy coefficient

- The problem: how to calculate $e(\rho, \delta = 0, T)$?

We introduce other definitions of $e_{sym}(A, T)$ in LDA, analyzing two possibilities:

1)

$$I^2 e_{sym}(A,T) = \int d\vec{r} \left[\frac{\mathcal{E}(\rho_A(r), \delta, T)}{A} - \frac{\mathcal{E}(\rho_{A1}(r), \delta = 0, T)}{A1} \right] \qquad (20)$$

$\rho_A(r, Z, N)$; $\quad \rho_{A1}(r, A1 = 2Z, N1 = Z = A1/2)$

Examples:

- for Ni isotopic chain the nucleus $A1$ is ^{56}Ni $(Z = N1 = 28)$
- for Sn isotopic chain the nucleus $A1$ is ^{100}Sn $(Z = N1 = 50)$

^{56}Ni and ^{100}Sn are reference nuclei

2)

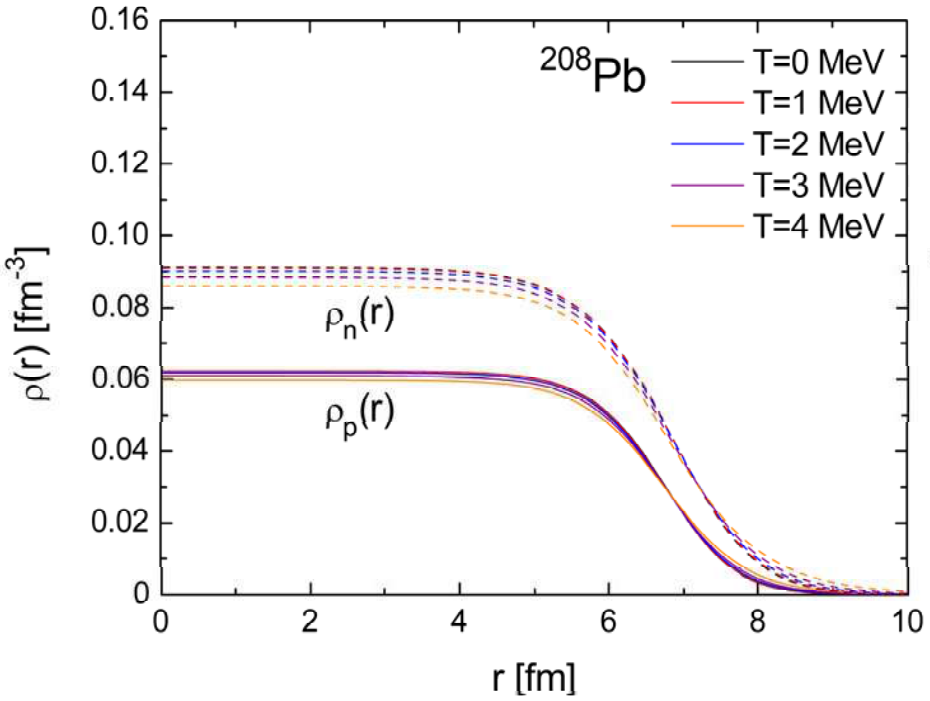
$$\begin{aligned} I^2 e_{sym}(A,T) &= \int \frac{d\vec{r}}{A} \qquad (21) \\ &\times \left[\mathcal{E}(\rho_A(r), \delta, T) - \mathcal{E}(\rho_{\bar{A}}(r), N = \bar{A}/2, Z = \bar{A}/2, \delta = 0, T) \right] \end{aligned}$$

$\bar{A} = A, A(Z,N), \bar{A}(Z = \bar{A}/2, N = \bar{A}/2)$

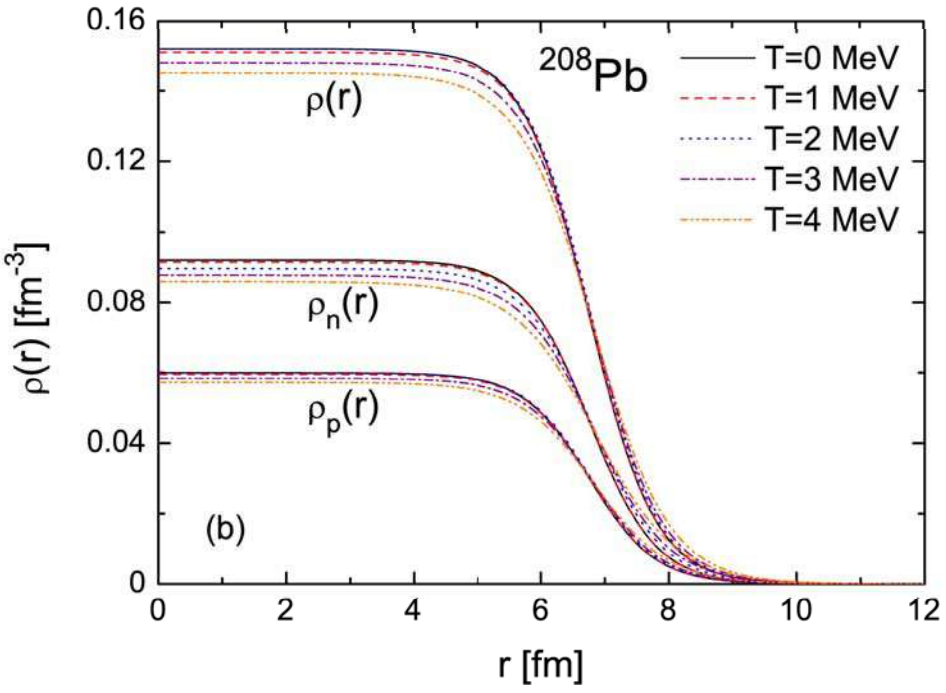
Requirement: the even-even nucleus with $N = Z = \bar{A}/2$ to be bound.

Temperature-dependent densities

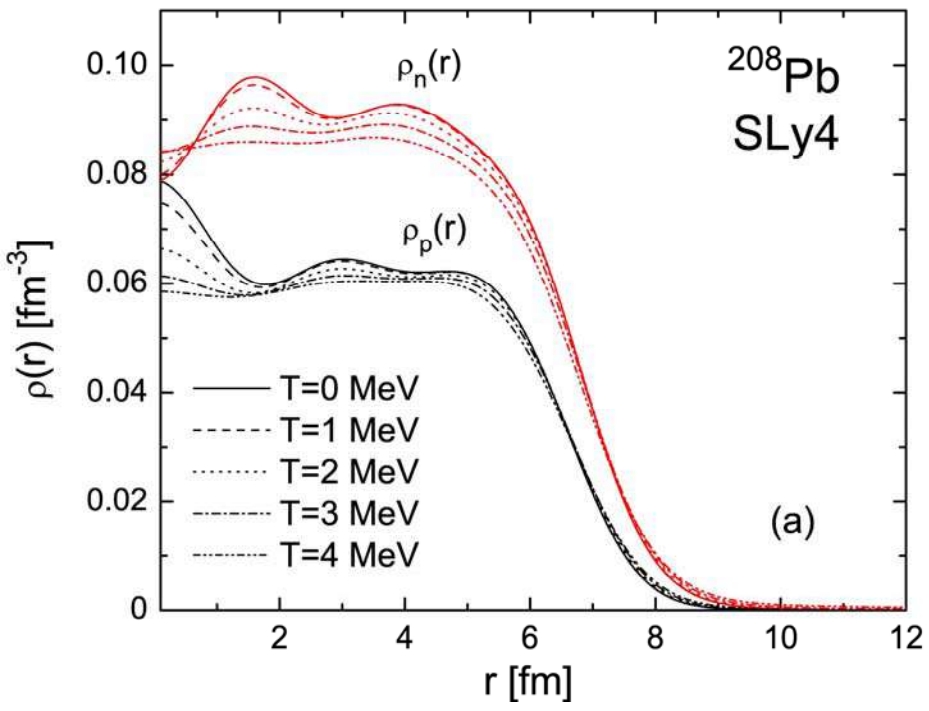
ETF

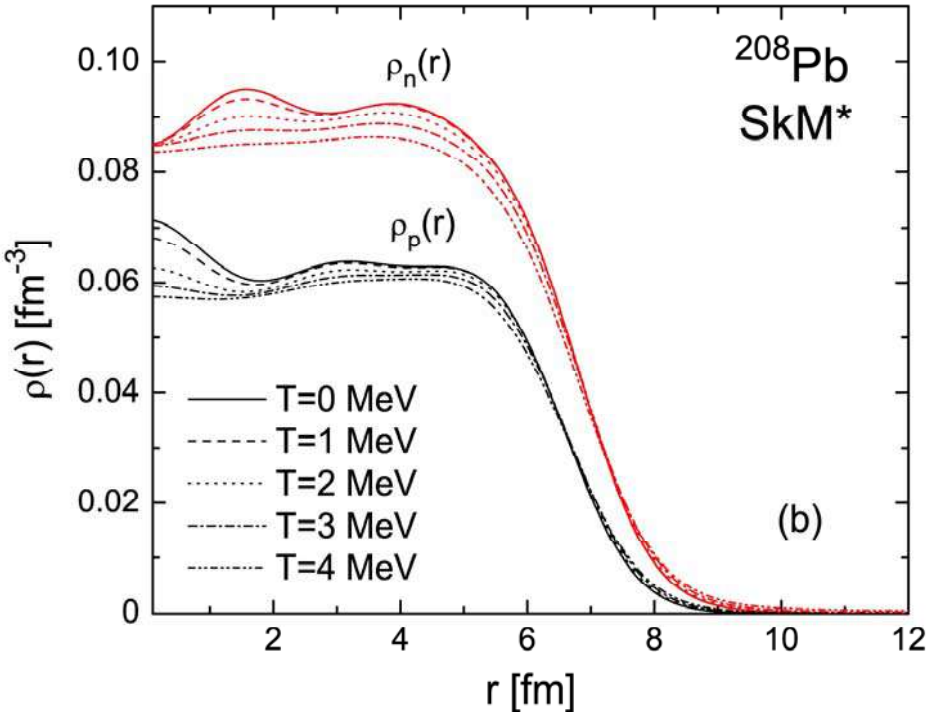


RDFA



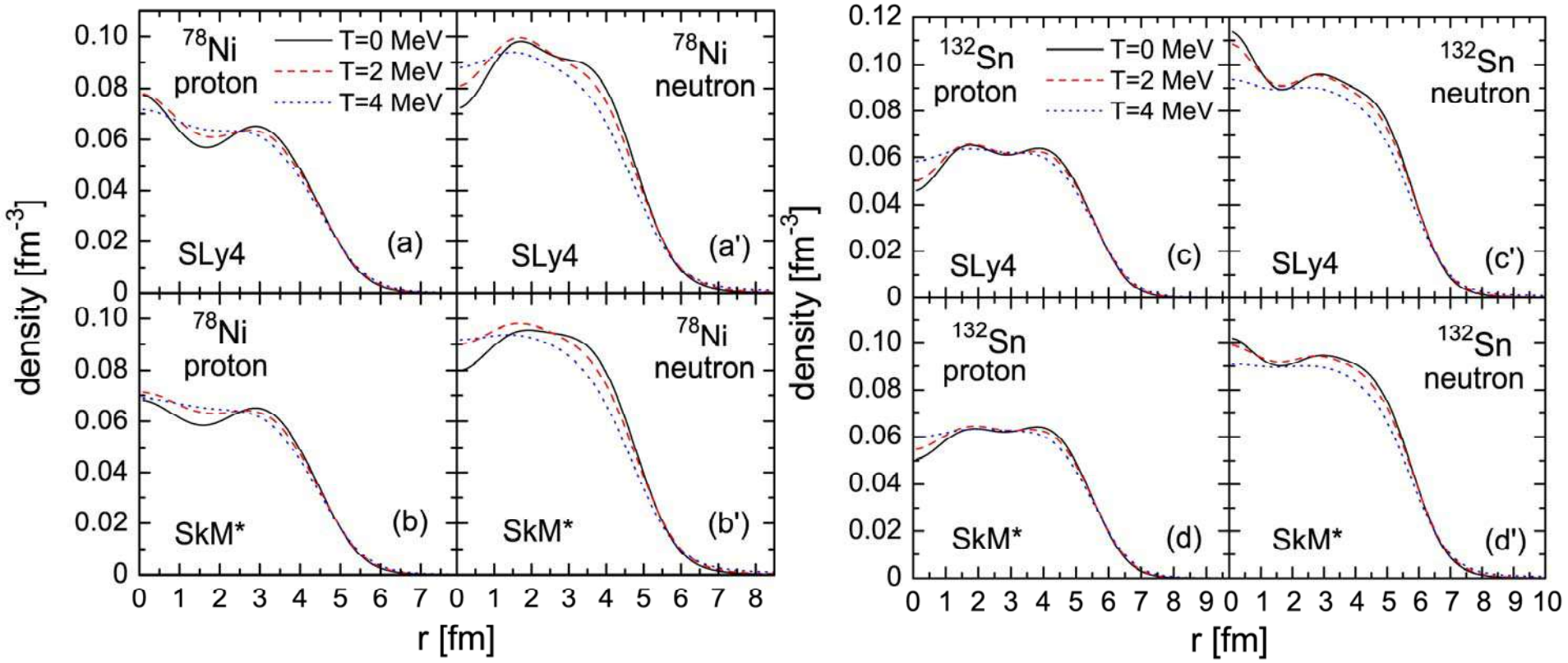
HFBTHO



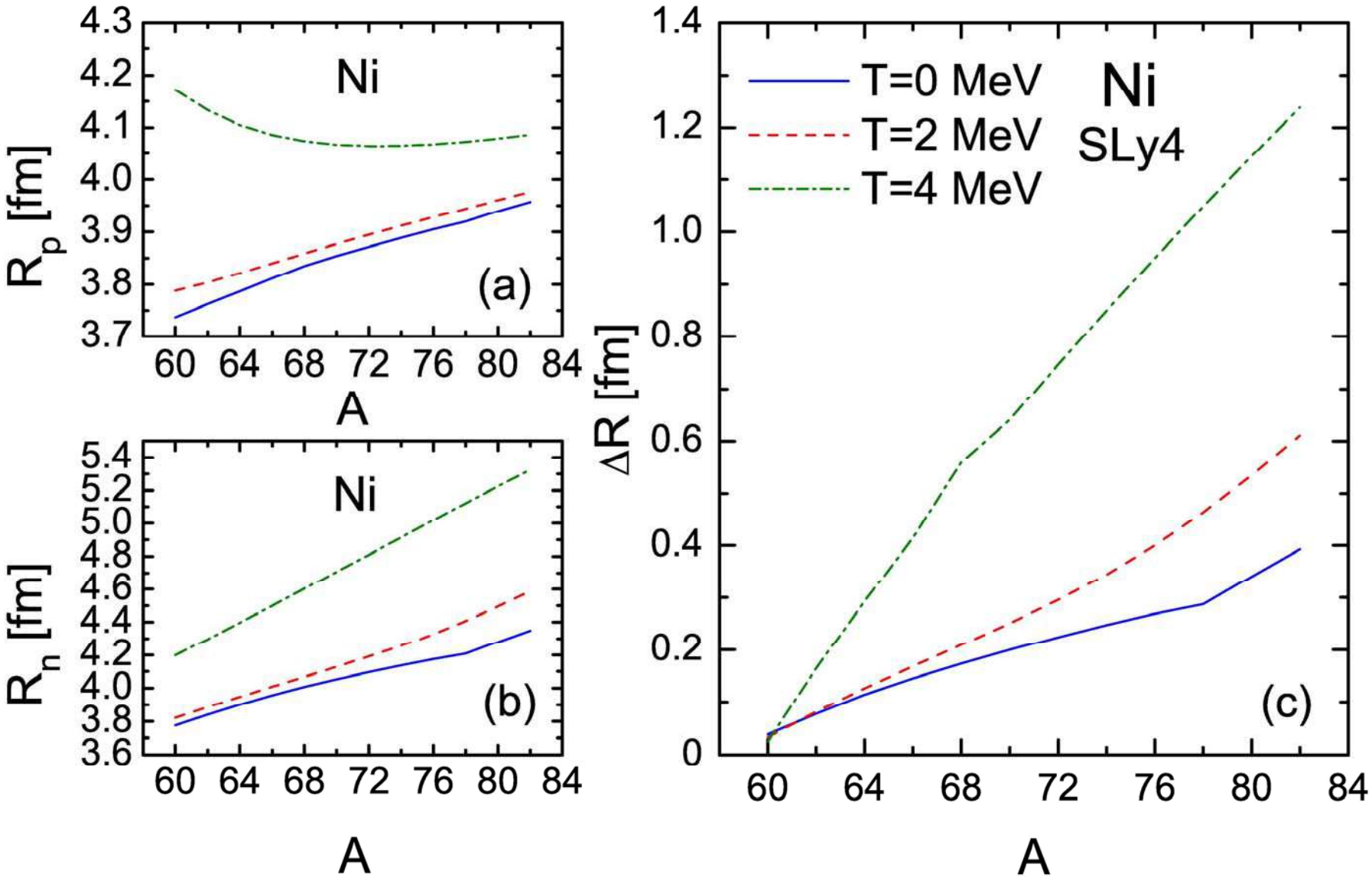


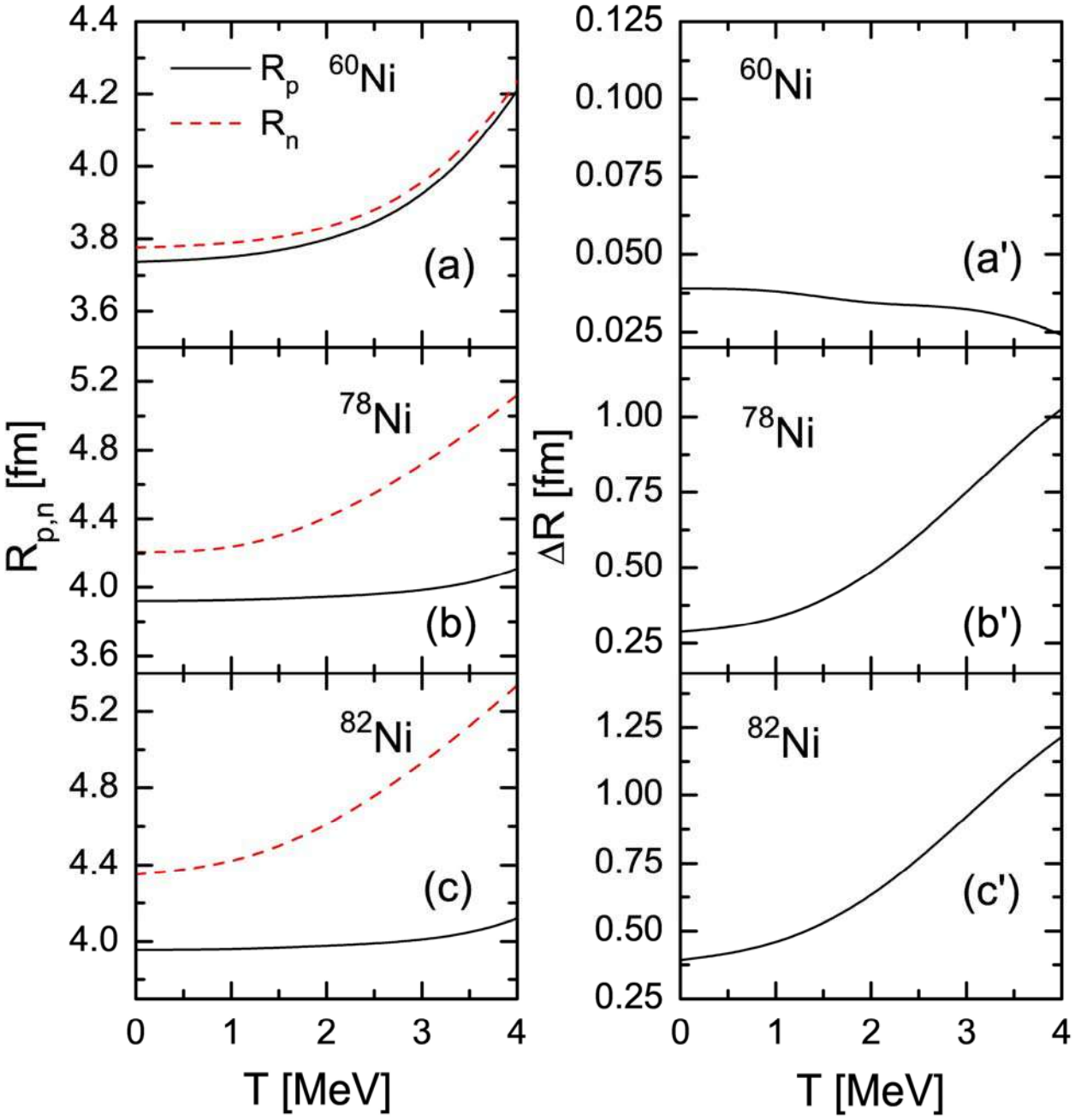
- With increasing *T* all densities decrease in the central part of the nucleus, stronger in HFBTHO case

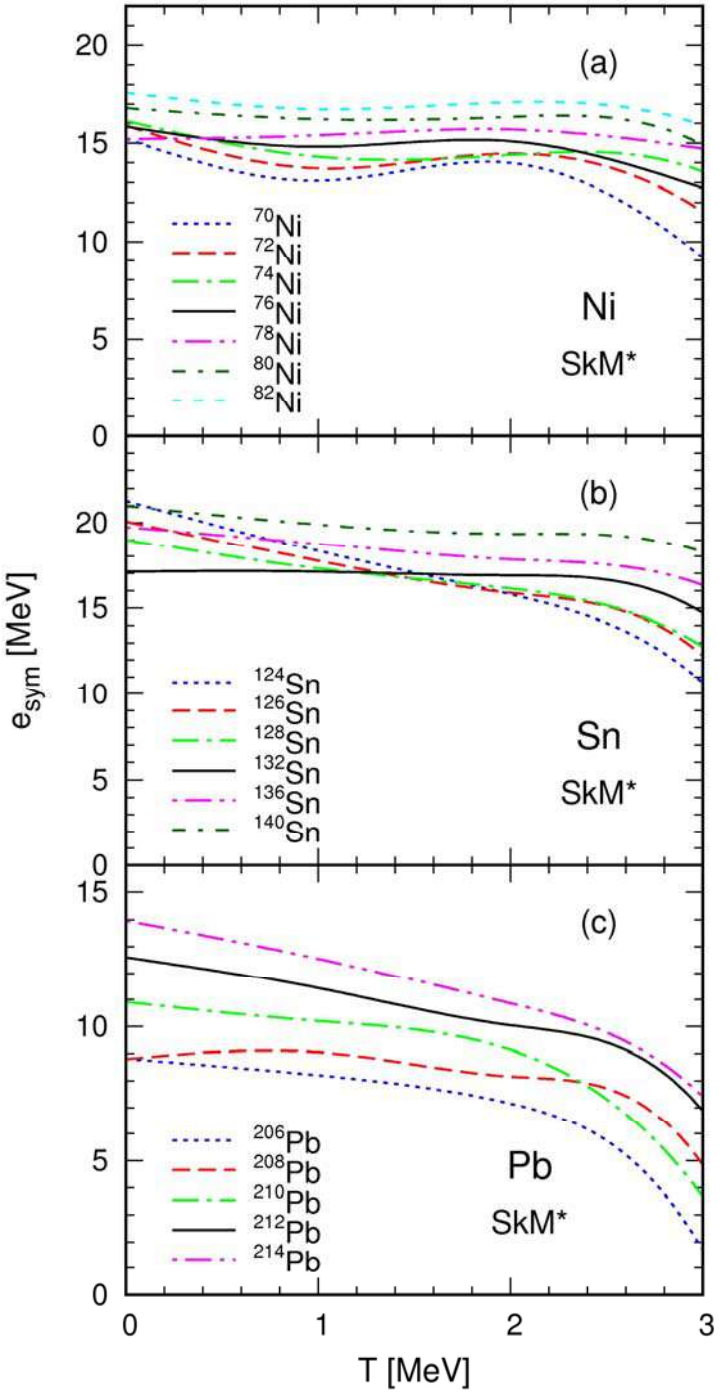
- The nuclear surface becomes more diffuse with increasing *T*



HFBTHO density distributions of protons and neutrons for **^{78}Ni** and **^{132}Sn** at T=0 MeV, T=2 MeV, and T=4 MeV obtained using the SLy4 and SkM* parametrizations


Ni
(a)
R_p [fm]
A
Ni
(b)
R_n [fm]
A
T=0 MeV
T=2 MeV
T=4 MeV
Ni
SLy4
(c)
ΔR [fm]
A


R_p
R_n
^{60}Ni
(a)
^{78}Ni
(b)
^{82}Ni
(c)
$R_{p,n}$ [fm]
T [MeV]
^{60}Ni
(a')
^{78}Ni
(b')
^{82}Ni
(c')
ΔR [fm]
T [MeV]



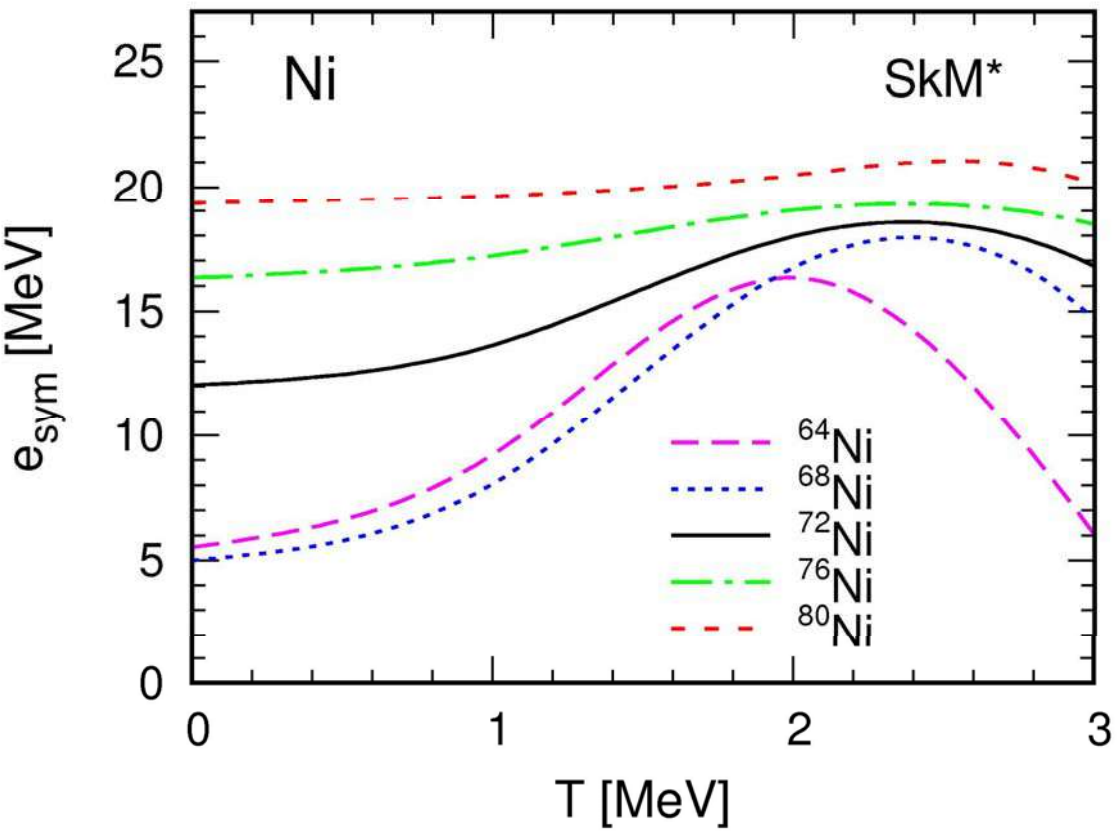
Temperature dependence of the symmetry energy coefficient

$$I^2 e_{sym}(A,T) = \int d\vec{r} \left[\frac{\mathcal{E}(\rho_A(r),\delta,T)}{A} - \frac{\mathcal{E}(\rho_{A1}(r),\delta=0,T)}{A1} \right]$$

- **Problems**: in case of δ=0 when the nucleus with Z=N1 is unbound

- for **Ni** chain: A1=56 ^{56}Ni (Z=N1=28)
- for **Sn** chain: A1=100 ^{100}Sn (Z=N1=50)
- for **Pb** chain: Z=N1=82 is unbound, again we take A1=100 (Z=N1=50)

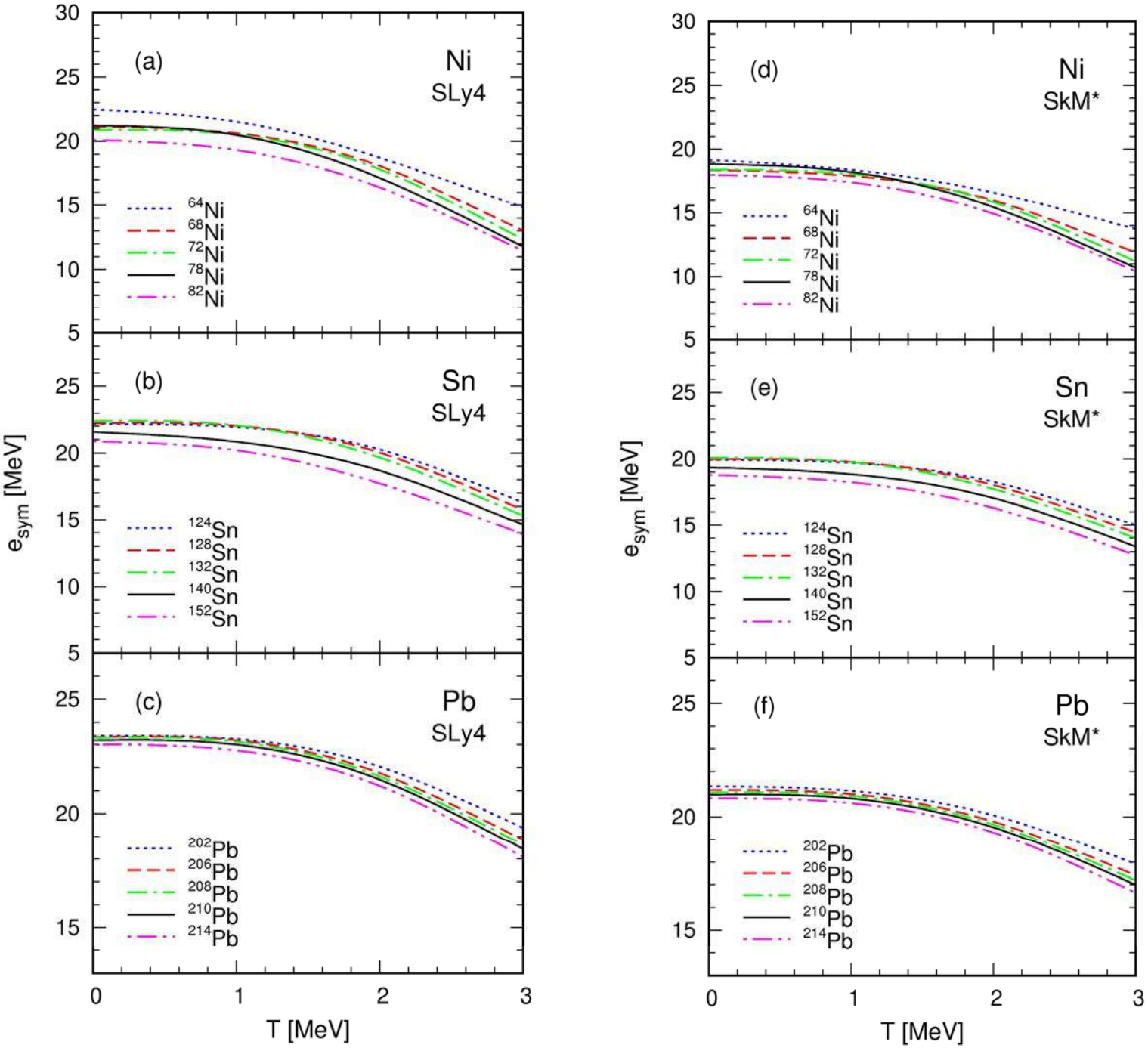
$$I^2 e_{sym}(A,T) = \int \frac{d\vec{r}}{\bar{A}} \times \left[\mathcal{E}(\rho_A(r),\delta,T) - \mathcal{E}(\rho_{\bar{A}}(r), N=\bar{A}/2, Z=\bar{A}/2, \delta=0, T)\right] \quad (23)$$

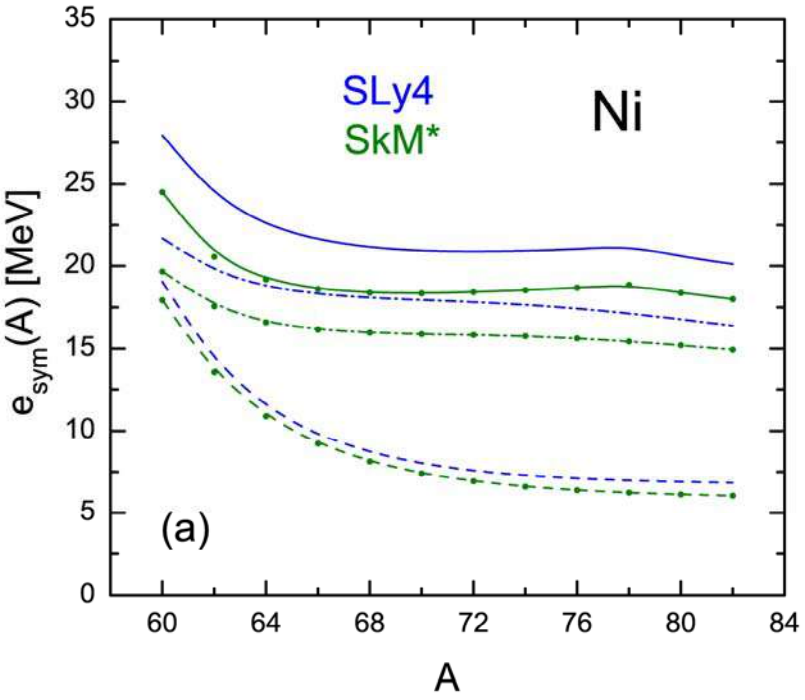


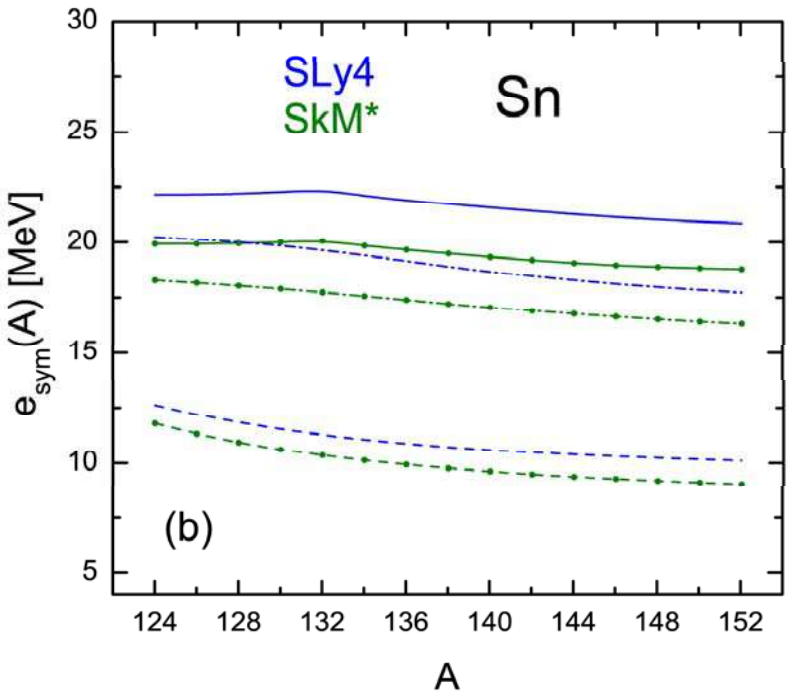
We consider:

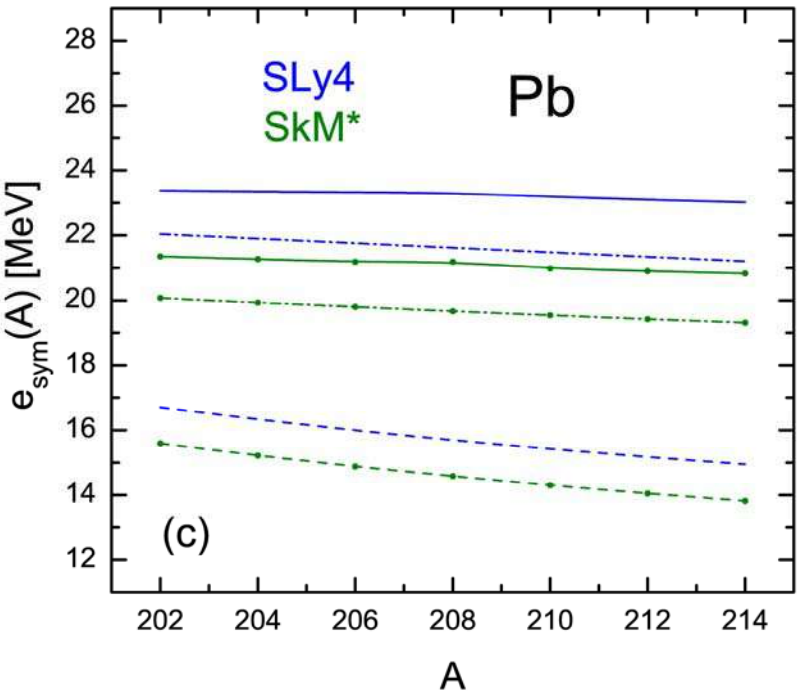
^{64}Ni [N=Z=32 (^{64}Ge)]
^{68}Ni [N=Z=34 (^{68}Se)]
^{72}Ni [N=Z=36 (^{72}Kr)]
^{76}Ni [N=Z=38 (^{76}Sr)]
^{80}Ni [N=Z=40 (^{80}Zr)]

- **Problems**: even-even nucleus with N=Z=Ā/2 (Ā=A) should be bound. This is possible only for **Ni** isotopes, but not for **Sn** and **Pb** ones. For instance: in Sn isotopes all N=Z nuclei with Ā (Ā=A) starting at 124 (N=Z=62) are unbound.


(a)
Ni
SLy4
64Ni
68Ni
72Ni
78Ni
82Ni
(b)
Sn
SLy4
124Sn
128Sn
132Sn
140Sn
152Sn
(c)
Pb
SLy4
202Pb
206Pb
208Pb
210Pb
214Pb
(d)
Ni
SkM*
64Ni
68Ni
72Ni
78Ni
82Ni
(e)
Sn
SkM*
124Sn
128Sn
132Sn
140Sn
152Sn
(f)
Pb
SkM*
202Pb
206Pb
208Pb
210Pb
214Pb
esym [MeV]
T [MeV]

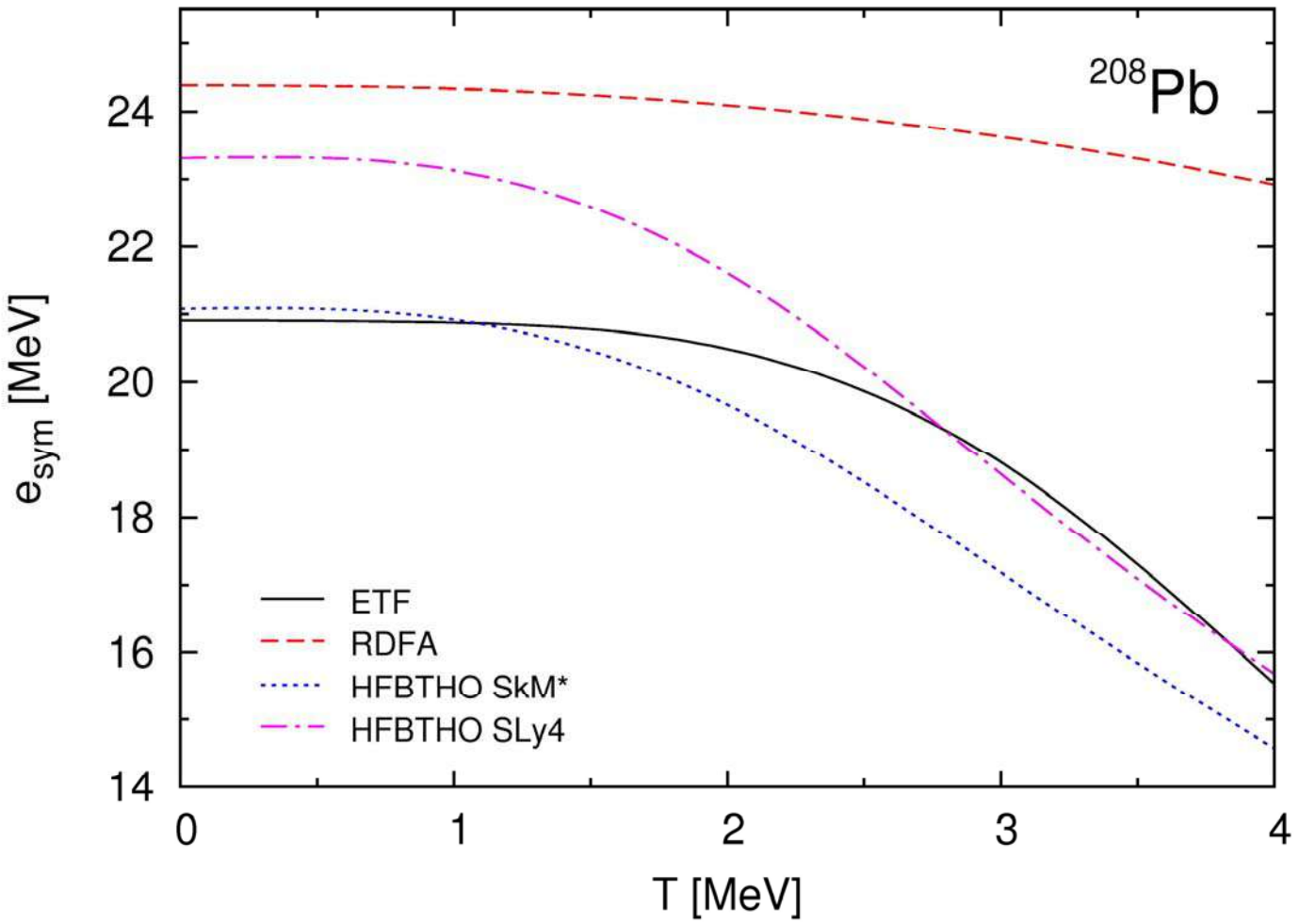






T=0 MeV: solid line
T=2 MeV: dash-dotted line
T=4 MeV: dashed line

- a kink in $e_{sym}(A)$ at *T*=0 MeV at double-magic ^{78}Ni and ^{132}Sn nuclei (no kinks for *T*=2 MeV and *T*=4 MeV) and a lack of kinks in Pb isotopic chain



- densities $\rho(T)$ from ETF, RDFA, HFBTHO (SkM* and SLy4)
- $\tau(T)$ from TF method with T^2 term

Conclusions

1. With increasing T, the quantities S, S^V, and S^S decrease, while κ slightly increases for all the isotopes in the Ni, Sn, and Pb chains and both for Skyrme forces and for all used density dependences of the symmetry energy.

2. The results for $S(T)$, $S^V(T)$, $S^S(T)$, and $\kappa(T)$ are sensitive to the choice of the density dependence of the symmetry energy $S[\rho(x,T)]$. The results obtained with values of γ between 0.30 and 0.40 agree better with the experimental information and are also close to previous results.

3. In the cases of double-magic ^{78}Ni and ^{132}Sn we observe "kinks'" for T=0 MeV in the curves of $S(T)$, $S^V(T)$, $S^S(T)$, and $\kappa(T)$, but not in the case of Pb isotopes. This effect was also observed in our previous works. It is also worth mentioning how the kinks are blurred and eventually disappear as T increases, demonstrating its close relationship with the shell structure.

4. A theoretical approach within the LDA is developed to study the *T*-dependence of the symmetry energy coefficient in finite nuclei, as well as *T*-dependent nuclear densities, rms radii, and the formation of neutron skins.

- Usage of Skyrme HFB with cylindrical transformed deformed HO basis (HFBTHO) with SkM* and SLy4 forces
- Isotopic chains of Ni, Sn, and Pb considered: densities $\rho(r,T)$ and kinetic energy densities $\tau(r,T)$ from HFBTHO and for ^{208}Pb also ETF and RDFA densities are used

5. The densities $\rho(r,T)$ decrease with T in the central region of nuclei. The neutron skin thickness grows with the increase of T for a given chain.

6. Two new definitions of $e_{sym}(A,T)$ within LDA are introduced. The results of $e_{sym}(A,T)$ **for isotopic chains** are in good agreement with theoretical predictions **for some specific nuclei** reported. The differences between our results using both definitions point out the dependence of the calculations of $e_{sym}(A,T)$ on its various definitions.

7. A comparative analysis of $e_{sym}(A,T)$ is performed using kinetic energy densities $\tau(r,T)$ from TF method up to T^2 term and density distributions $\rho(r,T)$ from HFBTHO method.The values of $e_{sym}(T)$ decrease smoothly with temperature. This is observed also in the case of ^{208}Pb with different densities $\rho(r,T)$.

THANK YOU FOR YOUR ATTENTION!