

A mechanism for shape coexistence- *under construction*

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gsb=
oblate

8⁺ 2749

5⁺ 2613

2nd=
prolate

4⁺ 2114

3⁺ 1941

Q > 0

6⁺ 1782

2⁺ 1741

0.4 e b
0⁺ 1654

2⁺ 1203

0.33 e b

4⁺ 1014

25

24

50

2⁺ 456

0⁺ 508

1.16 e² b²
-0.70 e b

Q < 0

255

0⁺ 0

0.68 e² b²

⁷⁴Kr
₃₆38

8⁺ 2879

6⁺ 2763

B(E2) W u

<Q> e b

5⁺ 2452

<Q²> e² b²

2⁺ 2091

6⁺ 1859

4⁺ 1957

3⁺ 1733

2⁺ 1687

1.3 e b
0⁺ 1598

2⁺ 1221

-1.0 e b

4⁺ 1035

157

78

2⁺ 424

1.1

126

0⁺ 770

2.49 e² b²

-0.9 e b

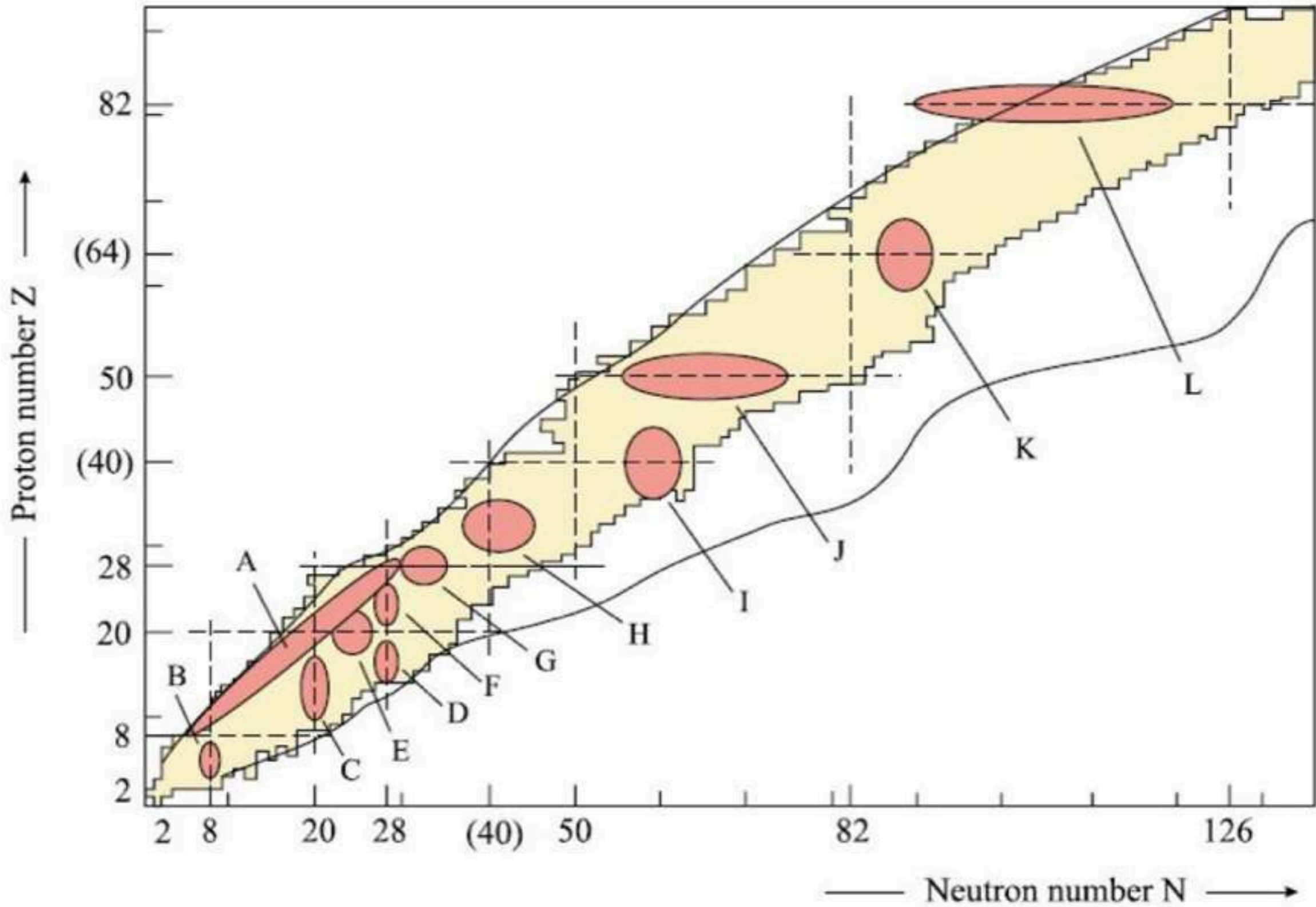
0⁺ 0

0.77 e² b²

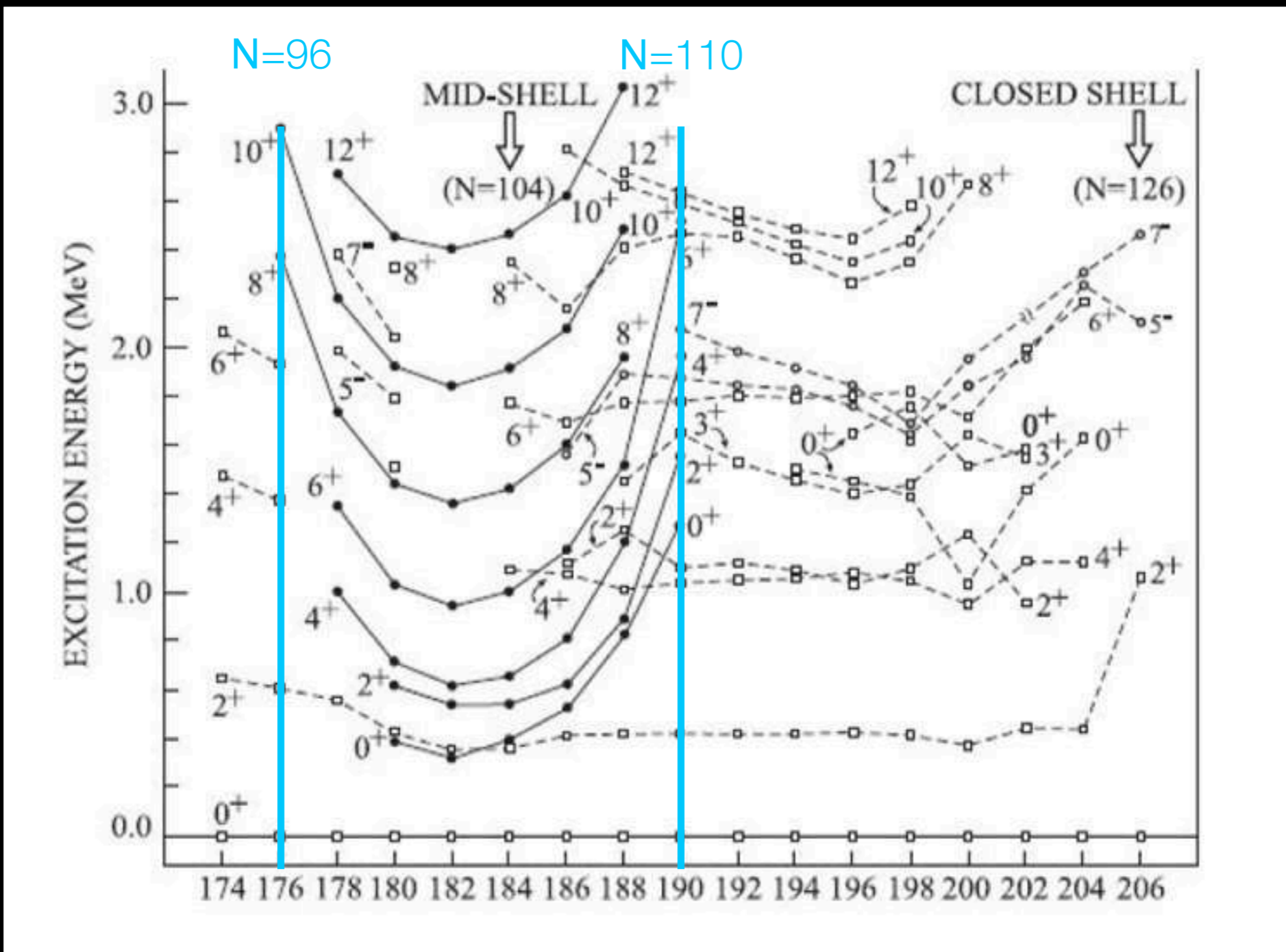
⁷⁶Kr
₃₆40

Manifestation
of shape
coexistence

Islands of coexistence

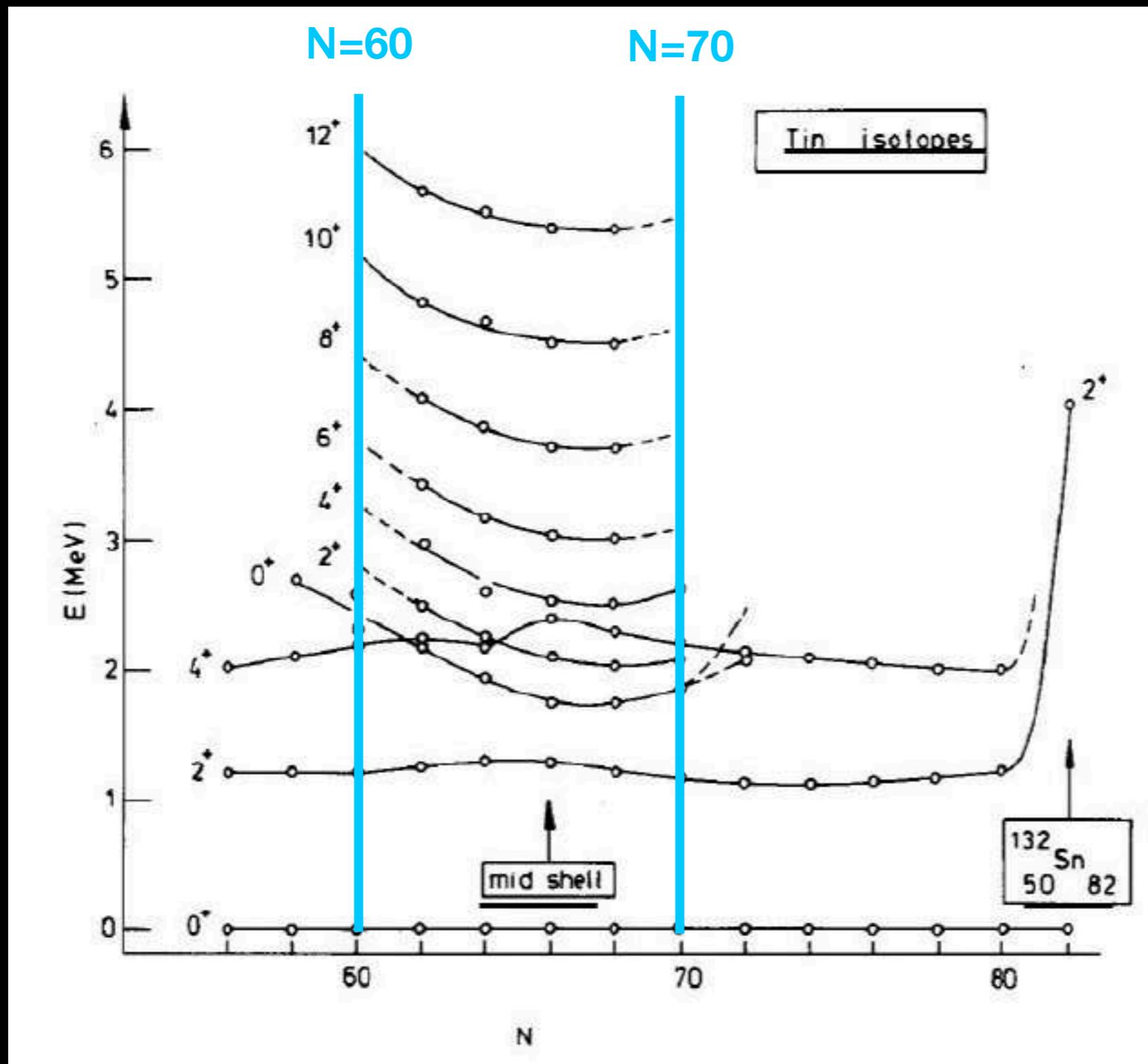


Hg isotopes



Why?

Sn isotopes



Why?

Shell model	Original Nilsson
$1g_{9/2}$	9/2[404]
	7/2[413]
	5/2[422]
	3/2[431]
	1/2[440]
$2p_{1/2}$ $1f_{5/2}$	1/2[301]
	5/2[303]
	3/2[301]
	1/2[310]
	3/2[312]
$2p_{3/2}$	1/2[321]
	7/2[303]
$1f_{7/2}$	5/2[312]
	3/2[321]
	1/2[330]
	3/2[202]
$1d_{3/2}$	1/2[200]
	1/2[211]
$2s_{1/2}$	5/2[202]
	3/2[211]
	1/2[220]
$1d_{5/2}$	1/2[101]
	3/2[101]
$1p_{3/2}$	1/2[110]
	1/2[000]
$1s_{1/2}$	

50

40

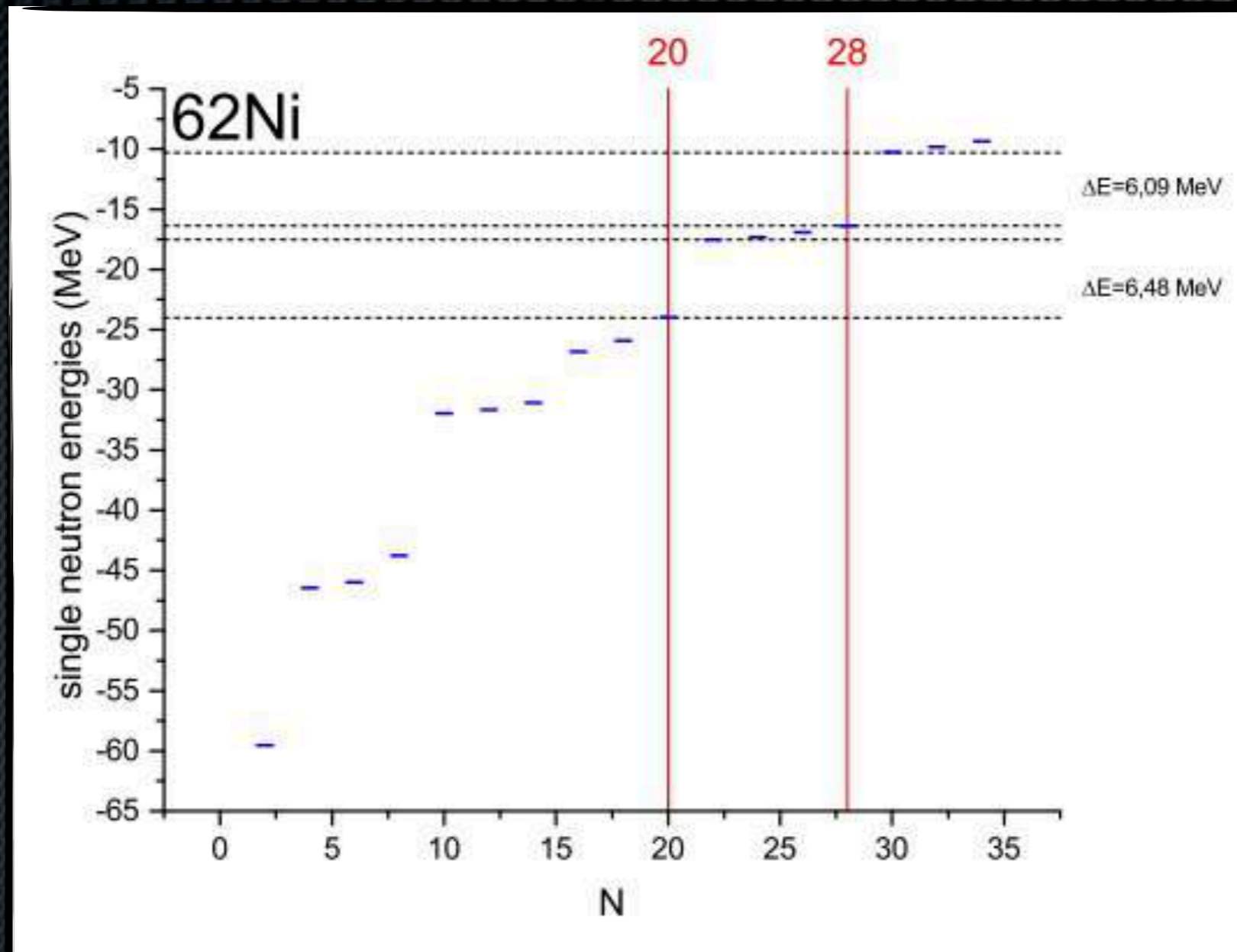
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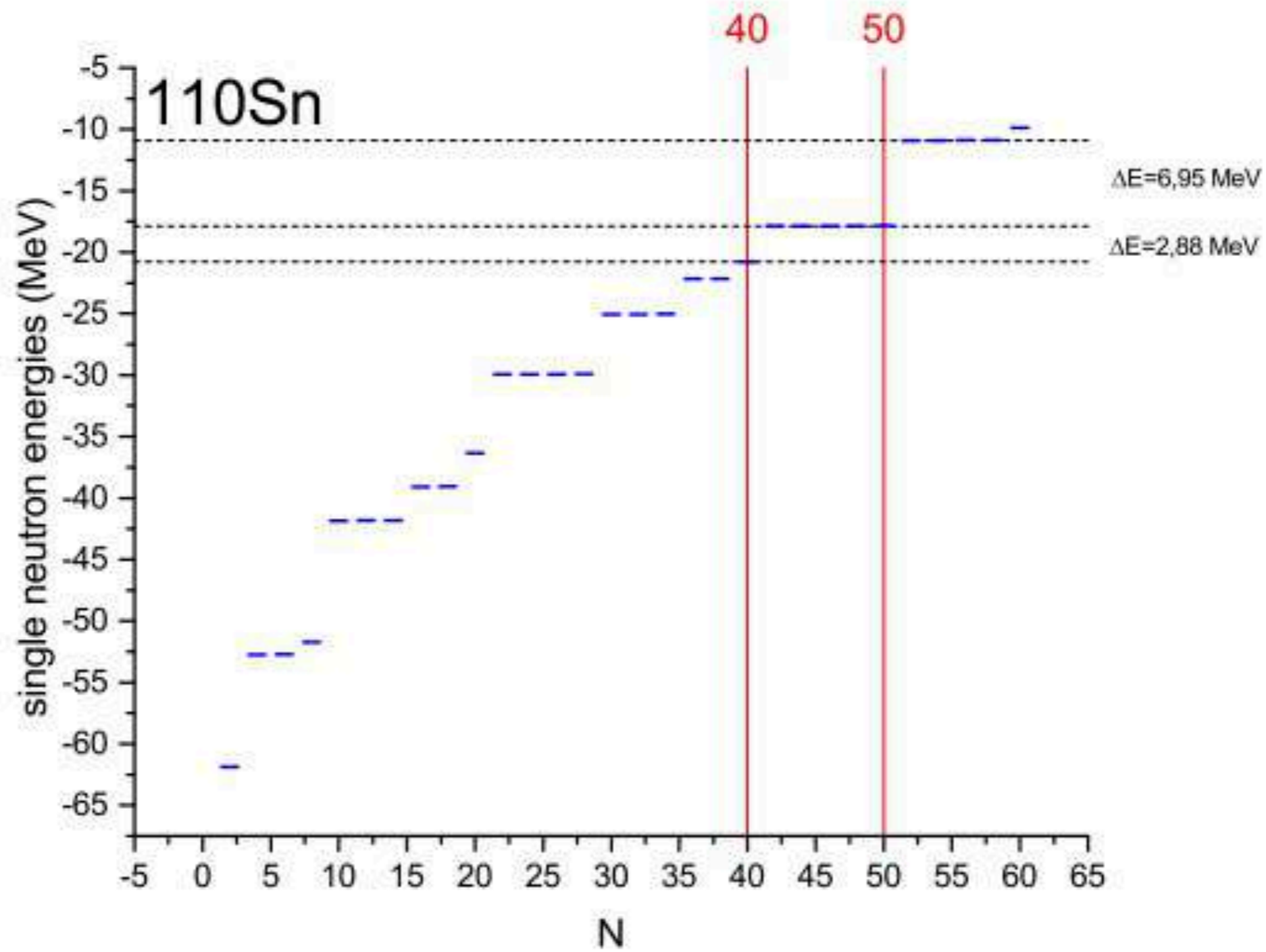
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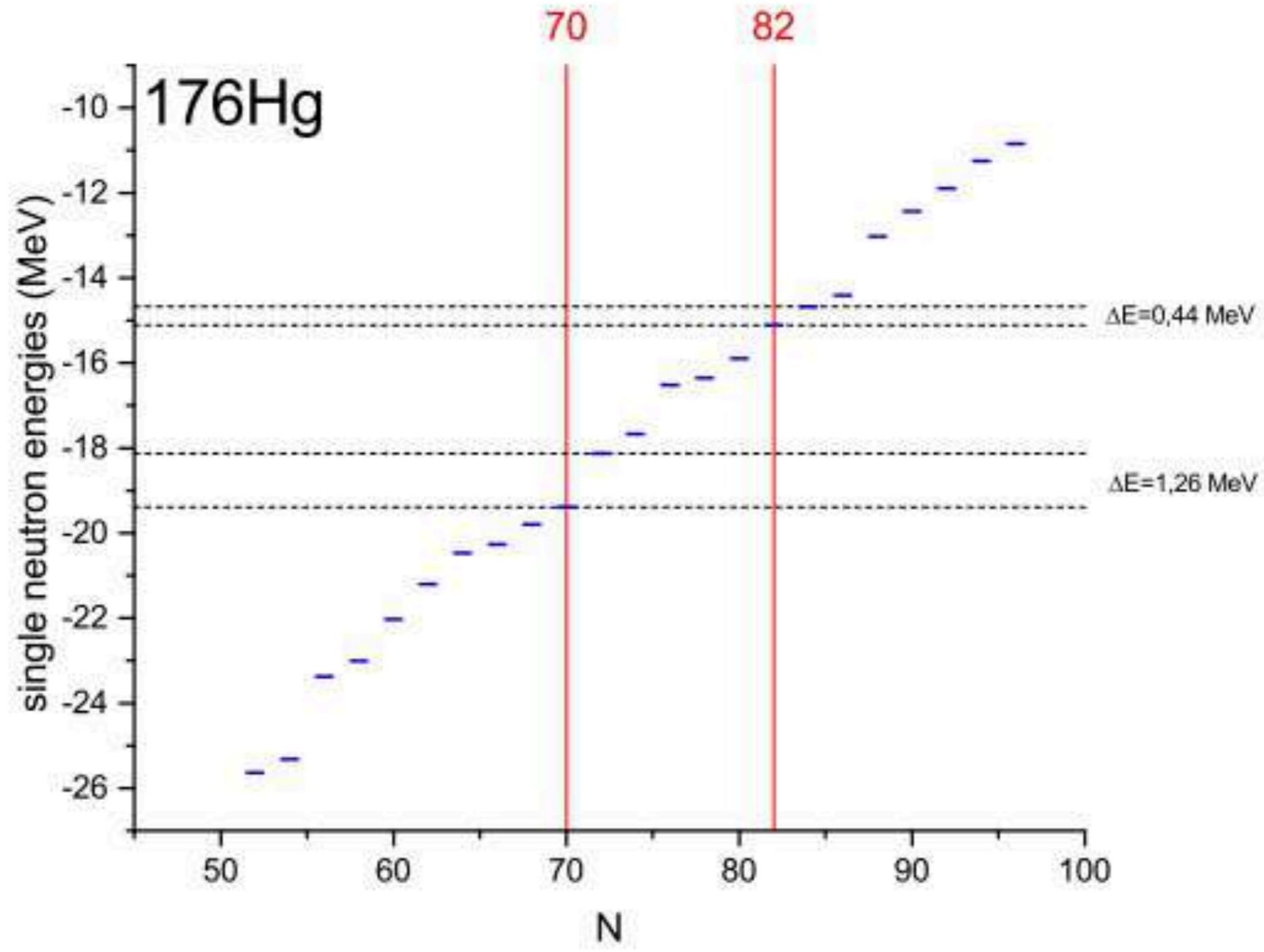
Nuclear Magic numbers

Selection rules of QQ interaction

Lowering of major energy gaps







Shell model	Original Nilsson
$1g_{9/2}$	$9/2[404]$
	$7/2[413]$
	$5/2[422]$
	$3/2[431]$
	$1/2[440]$
$2p_{1/2}$	$1/2[301]$
$1f_{5/2}$	$5/2[303]$
	$3/2[301]$
	$1/2[310]$
$2p_{3/2}$	$3/2[312]$
	$1/2[321]$
$1f_{7/2}$	$7/2[303]$
	$5/2[312]$
	$3/2[321]$
	$1/2[330]$
$1d_{3/2}$	$3/2[202]$
	$1/2[200]$
$2s_{1/2}$	$1/2[211]$
$1d_{5/2}$	$5/2[202]$
	$3/2[211]$
	$1/2[220]$
$1p_{1/2}$	$1/2[101]$
$1p_{3/2}$	$3/2[101]$
	$1/2[110]$
$1s_{1/2}$	$1/2[000]$

50

20

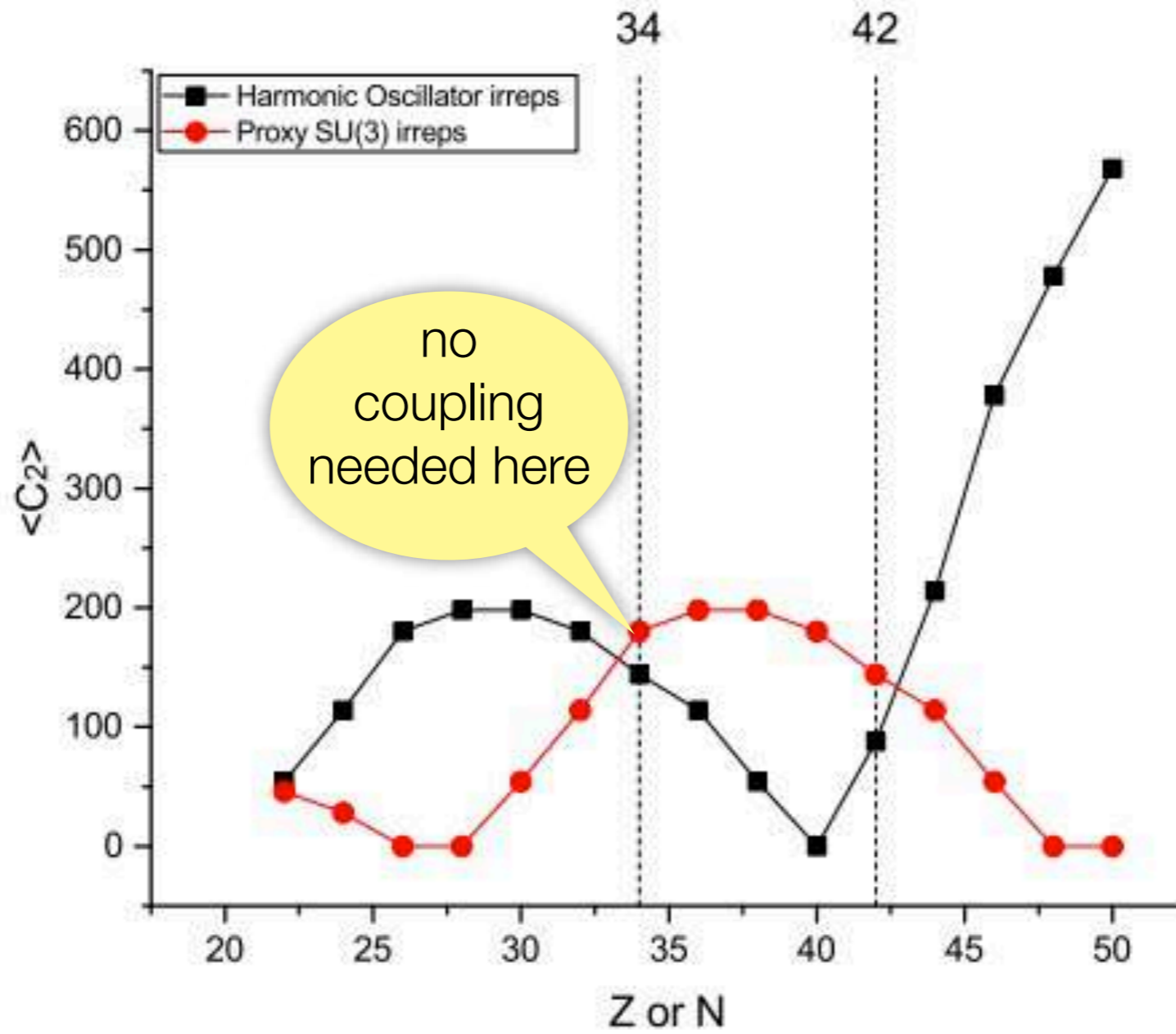
One super shell

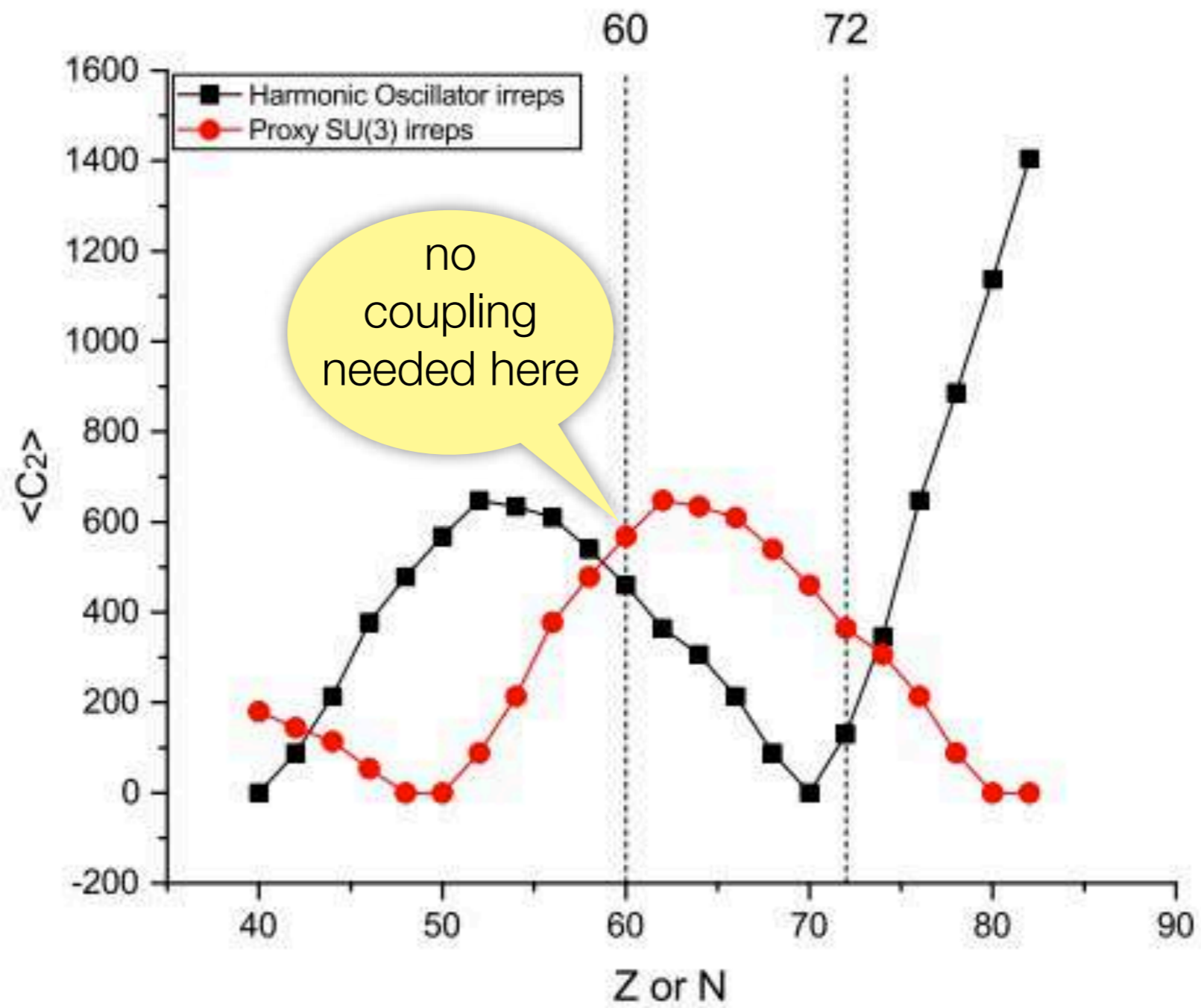
The collective quadrupole interaction

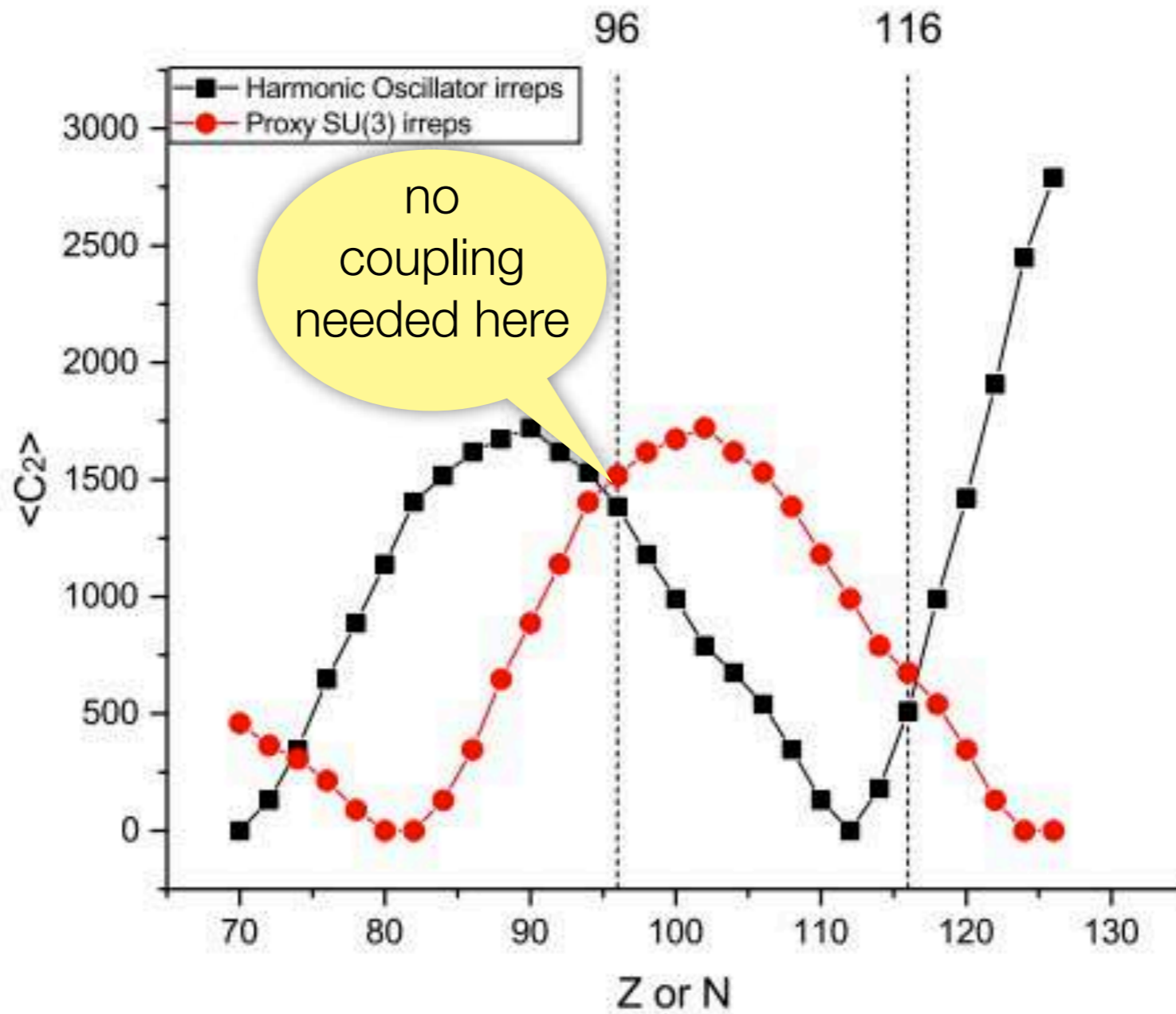
$$Q \cdot Q = 4C_2 - 3L^2,$$
$$C_2 = \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu),$$
$$\beta^2 = \frac{4\pi}{5(A\bar{r}^2)^2}(C_2 + 3)$$

$$\gamma = \arctan \left(\frac{\sqrt{3}(\mu + 1)}{2\lambda + \mu + 3} \right)$$

- (λ, μ) are the quantum numbers of SU(3). They depend on the choice of magic numbers. β is the deformation and γ is the angle which distinguishes prolate from oblate.







Z or N for shape coexistence

34-40

60-70

96-112

210-240

The 0_+ states

no particle
excitations

$$H = H_0 - \frac{\chi}{2} QQ,$$

$$0_2^+ = \frac{\chi}{2} (QQ_{nucl} - QQ_{ho}) - (N_{0,nucl} - N_{0,ho})$$

$$\frac{\chi}{2} = \frac{1\text{MeV}}{N_0}$$

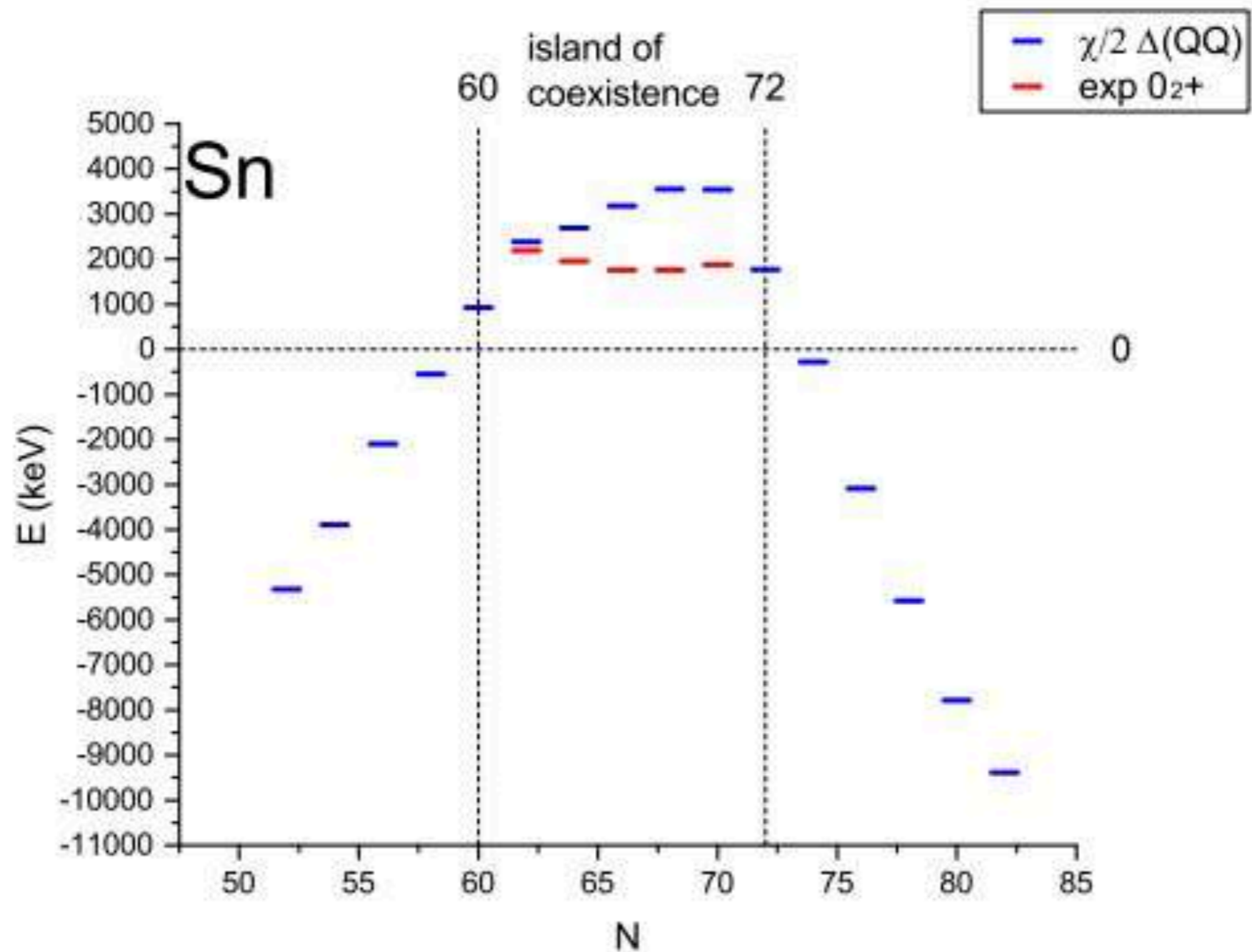
$$N_0 = \sum_{i=1}^A (n_i + \frac{3}{2}) \hbar\omega_i,$$

$$\hbar\omega_i = \frac{\hbar\omega}{N},$$

$$\hbar\omega = \frac{41\text{MeV}}{A^{1/3}}.$$

$$0_2^+ = \frac{\chi}{2} \Delta(QQ) - \Delta N_0.$$

Before the coupling of 40-70, 50-82 shells

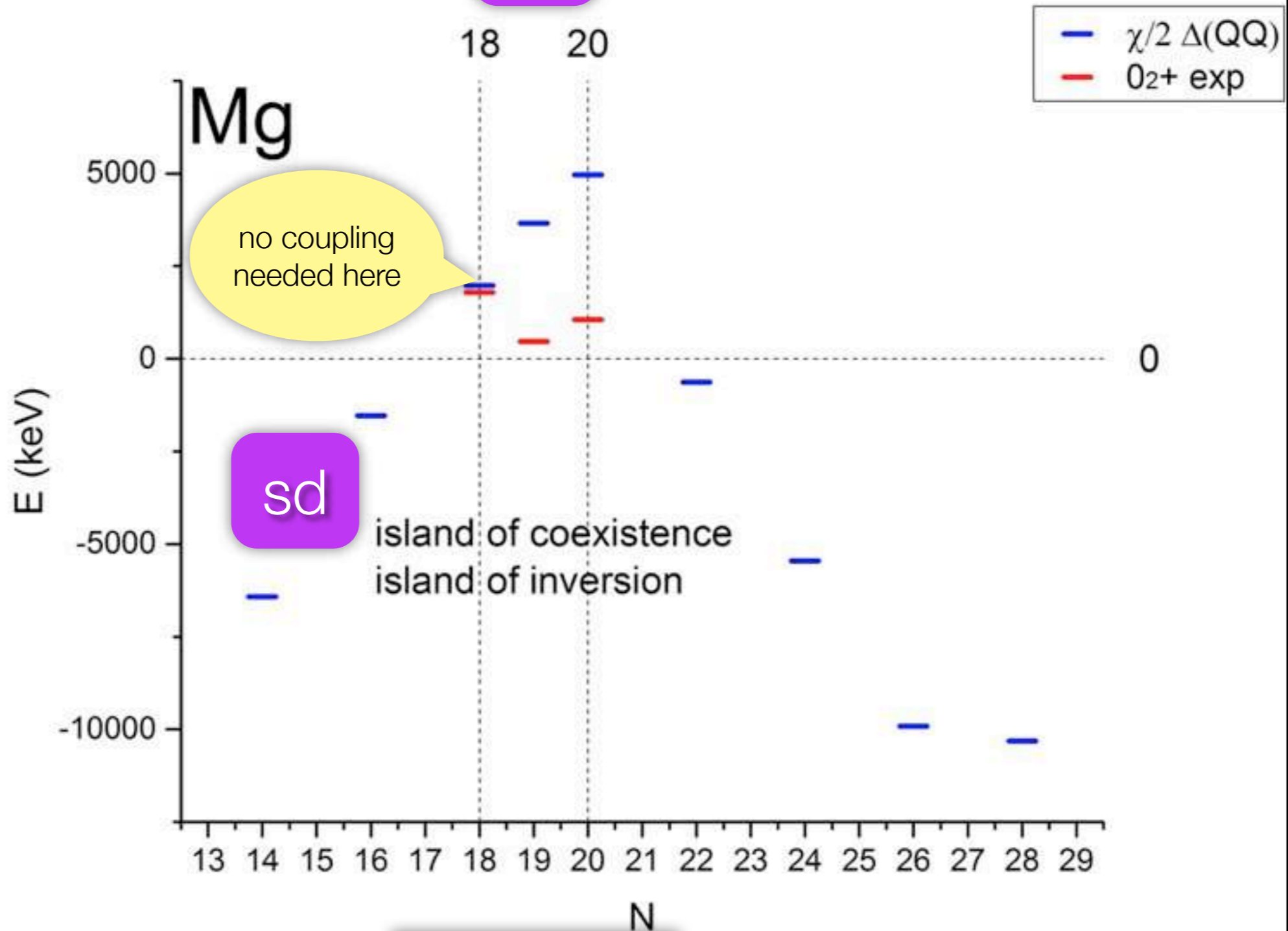


Below 28

Table 1 Magic numbers 14-28 consist of Nilsson orbitals which have a Proxy SU(3) symmetry. The Nilsson orbitals $1/2[330]$, $3/2[321]$, $5/2[312]$ different only by one quantum in the z axis with the $1/2[220]$, $3/2[211]$, $5/2[202]$ respectively. Such orbitals have differences of quantum numbers $\Delta K[\Delta N \Delta n_z \Delta \Lambda] = 0[110]$. By excluding the $7/2[303]$ orbital and replacing the rest $1f_{7/2}$ orbitals with their $0[110]$ counterparts of the $1d_{5/2}$, the 14-28 has a restored SU(3) symmetry. The 14-28 shell appears in the isotopes of the $N \sim 20$ island of inversion. This shell is also useful for the explanation of shape coexistence in the Mg region.

shell model	$K[Nn_z\Lambda]$	$0[110]$ counterparts	algebra	magic numbers
$1f_{7/2}$	$7/2[303]$	X		28
	$5/2[312]$	$5/2[202]$		26
	$3/2[321]$	$3/2[211]$		
	$1/2[330]$	$1/2[220]$		
$1d_{3/2}$	$3/2[202]$		U(6)	
	$1/2[200]$			
$2s_{1/2}$	$1/2[211]$			
$1d_{5/2}$	$5/2[202]$	X		14
	$3/2[211]$	$3/2[101]$	U(3)	12
	$1/2[220]$	$1/2[110]$		
$1p_{1/2}$	$1/2[101]$			
$1p_{3/2}$	$3/2[101]$	X		6
	$1/2[110]$	$1/2[000]$	U(1)	4
$1s_{1/2}$	$1/2[000]$		U(1)	2

sdf



Inversion
at N=18

Conclusions

- ✦ The mechanism involves **no particle excitations**.
- ✦ The 0_2+ state is always the less deformed, coupled configuration.
- ✦ The 0_1+ state comes from the uncoupled nuclear shell, except from the island of inversion, where the 0_1+ state comes from the uncoupled harmonic oscillator shell.
- ✦ In nuclei with $C_{2\text{proxy}} \approx C_{2\text{ho}}$, $\chi/2\Delta(\text{QQ}) \approx 0_2+$. In the rest nuclei coupling of the two shells has to be done.
- ✦ **Inversion of states occurs in nuclei which traditionally follow the harmonic oscillator magic numbers** and they exhibit shape coexistence. The inversion is presented in the isotopes with shape coexistence.

Done

- ✦ The mechanism predicts that shape coexistence occurs in islands.
- ✦ The candidate nuclei for shape coexistence are predicted without parameters.

Have to be done

- ✦ Pairing interaction has to be introduced in the Proxy SU(3) symmetry. Pairing acquires a **9-(λ, μ)** symbol.
- ✦ The coupling of the shells acquires a **9-(λ, μ)** symbol. A code has to be constructed for large SU(3) irreps.
- ✦ The islands of coexistence have to be predicted after the 9-(λ, μ) symbol.
- ✦ The 0_2^+ states have to be predicted after the 9-(λ, μ) symbol.

Thank you