

Breaking $SU(3)$ spectral degeneracies in heavy deformed nuclei

IOANNIS ASSIMAKIS

NTUA /INPP NCSR DEMOKRITOS

Dynamical symmetries in nuclear physics

- A Hamiltonian H of a nuclear model that commutes with every element x ($[H, x] = 0$) of a Lie Algebra g is said to have symmetry g (invariant under g).
- In this case H can be written as a function of Casimir operators $C(g), C(g'), C(g'')$... of a chain of subalgebras $g \supset g' \supset g''$... of g
- Since symmetry implies degeneracy, different irreducible representations of g classify different sets of eigenstates of the system

Proxy-SU(3)

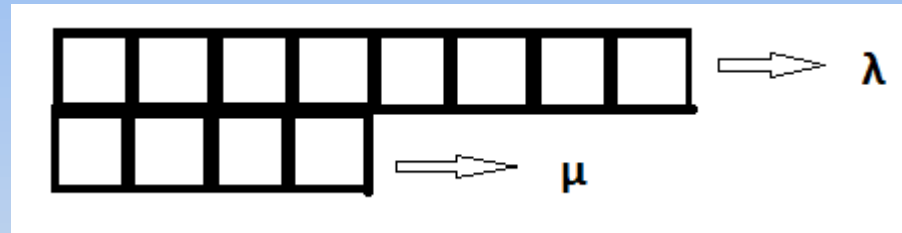
- It has been found that Nilsson orbitals that differ by $\Delta K[\Delta N \Delta n_z \Delta \Lambda] = 0[110]$ have large spatial overlaps
- Substitution of the intruder parity orbitals of each nuclear shell with their $0[110]$ partner orbitals from the next lower shell creates a shell which restores the symmetry of the harmonic oscillator i.e. SU(3)

50-82	50-82	sdg	sdg
P	P	P	P
3s1/2	1/2[400]	3s1/2	1/2[400]
2d3/2	1/2[411]	2d3/2	1/2[411]
	3/2[402]		3/2[402]
2d5/2	1/2[420]	2d5/2	1/2[420]
	3/2[411]		3/2[411]
	5/2[402]		5/2[402]
1g7/2	1/2[431]	1g7/2	1/2[431]
	3/2[422]		3/2[422]
	5/2[413]		5/2[413]
	7/2[404]		7/2[404]
1h11/2	<u>1/2[550]</u>	1g9/2	<u>1/2[440]</u>
	<u>3/2[541]</u>		<u>3/2[431]</u>
	<u>5/2[532]</u>		<u>5/2[422]</u>
	<u>7/2[523]</u>		<u>7/2[413]</u>
	<u>9/2[514]</u>		<u>9/2[404]</u>
	11/2[505]		

Nuclear shell	Harmonic oscillator shell	Symmetry
28-50	pf	U(10)
50-82	sdg	U(15)
82-126	pfh	U(21)
126-184	sdgi	U(28)

Irrep selection in the proxy-SU(3) scheme

- The eigenvalues of the Casimir operators are functions of the irreps that characterize the system
- The irreps of SU(3) are characterized by the pair (λ, μ) and can be expressed by the corresponding Young tableaux



- In proxy-SU(3) the highest weight irrep is chosen because it gives the most symmetric configuration of the total wave function

Proxy-SU(3) Hamiltonian

Including only one body and two body terms leads to a Hamiltonian with eigenvalues

$$H = \alpha L^2 + \beta I_2$$

Eigenvalues
of the
invariant
operators

$$\langle L^2 \rangle = l(l + 1)$$

$$\langle I_2 \rangle = \frac{1}{9} (\lambda^2 + \mu^2 + 3\lambda + 3\mu + \lambda\mu)$$

In this scheme bands belonging to the same irrep are degenerate

- Casimir invariants do not break the degeneracy within an irrep
- $O(3)$ scalars break the degeneracy

Bosonic models

- Ground state stands alone
- γ_1 and β_1 bands are in the next irrep

Fermionic models

- Ground state band, γ_1 band and first $K = 4$ band are in the first irrep
- β_1, γ_2 and second $K = 4$ band are in the next irrep

Including up to three body terms leads to a Hamiltonian

$$H = \alpha L^2 + \beta I_2 + \Gamma \omega$$

where $\Omega = LQL$

Omega operator matrix elements

$$\begin{aligned}\Omega_{KK} &= \sqrt{6}(2\lambda + \mu + 3)[l(l+1) - 3K^2], \\ \Omega_{K \pm 2K} &= -3[3(\mu \mp K)(\mu \pm K + 2)(l \pm K + 2) \\ &\quad \times (l \pm K + 1)(l \mp K)(l \mp K - 1)/2]^{1/2}.\end{aligned}$$

Including up to four body terms leads to a Hamiltonian

$$H = \alpha L^2 + \beta I_2 + \delta \Lambda$$

where $\Lambda = LQQL$

Lambda operator matrix elements

$$\begin{aligned} \Lambda_{KK} &= 2(2\lambda + \mu + 3)^2 [l(l+1) - 3K^2] \\ &\quad - 18K^4 + 6K^2 [5l(l+1) - 3] \\ &\quad - 12l^2(l+1)^2 - 72l(l+1) \\ &\quad - 3(\mu - K)(\mu + K + 2) [l(l+1) - 3K^2] \\ &\quad - 3(\mu + K)(\mu - K + 2) [l(l+1) - 3K^2], \\ \Lambda_{K\pm 2K} &= 6[(\mu \mp K)(\mu \pm K + 2)(l \pm K + 2)(l \pm K + 1) \\ &\quad \times (l \mp K)(l \mp K - 1)]^{1/2} (2\lambda + \mu \mp 3K) \end{aligned}$$

Parameter independent quantities in proxy-SU(3)

Splitting of the ground state and γ_1 bands

$$\frac{E(L_\gamma) - E(L_g)}{E(2_\gamma) - E(2_g)}$$

Splitting of the ground state and $K = 4$ bands

$$\frac{E(L_K) - E(L_g)}{E(4_K) - E(4_g)}$$

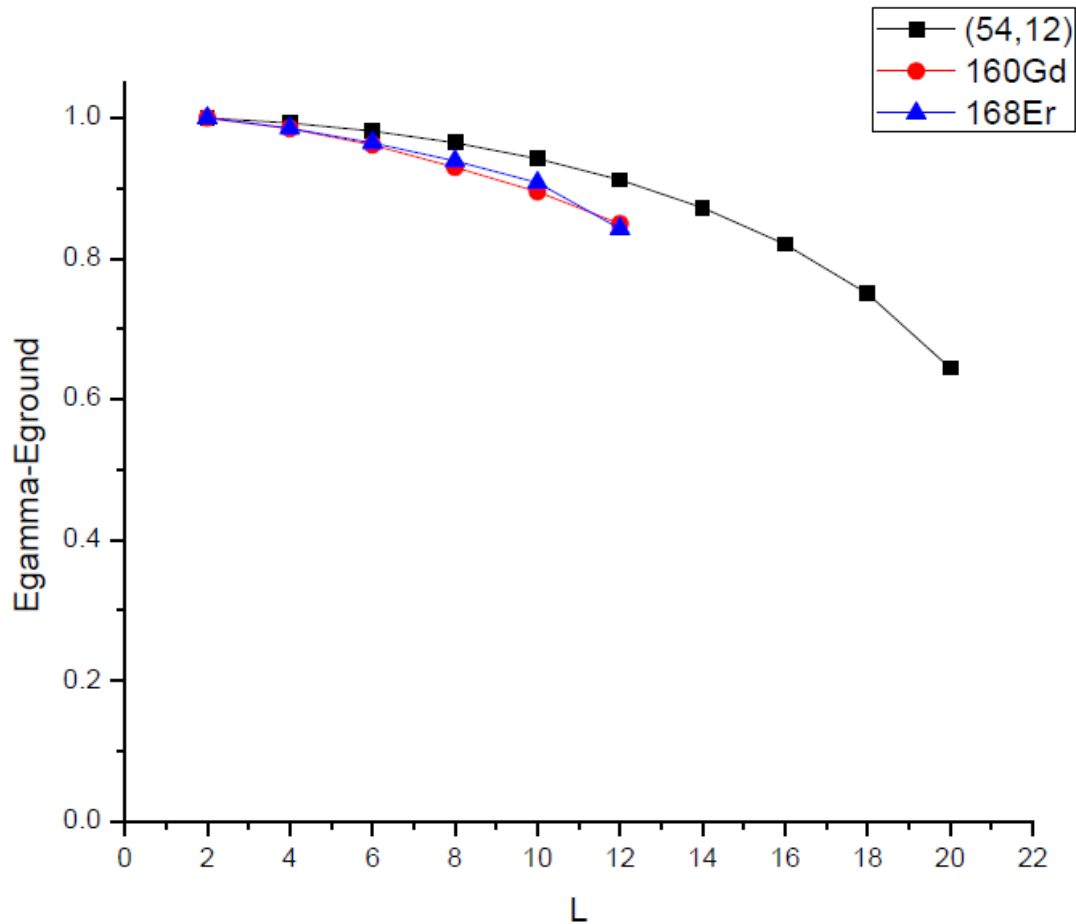
Higher 0^+ bandheads

$$\frac{E(0_n) - E(0_1)}{E(0_2) - E(0_1)}$$

Odd even staggering within γ bands

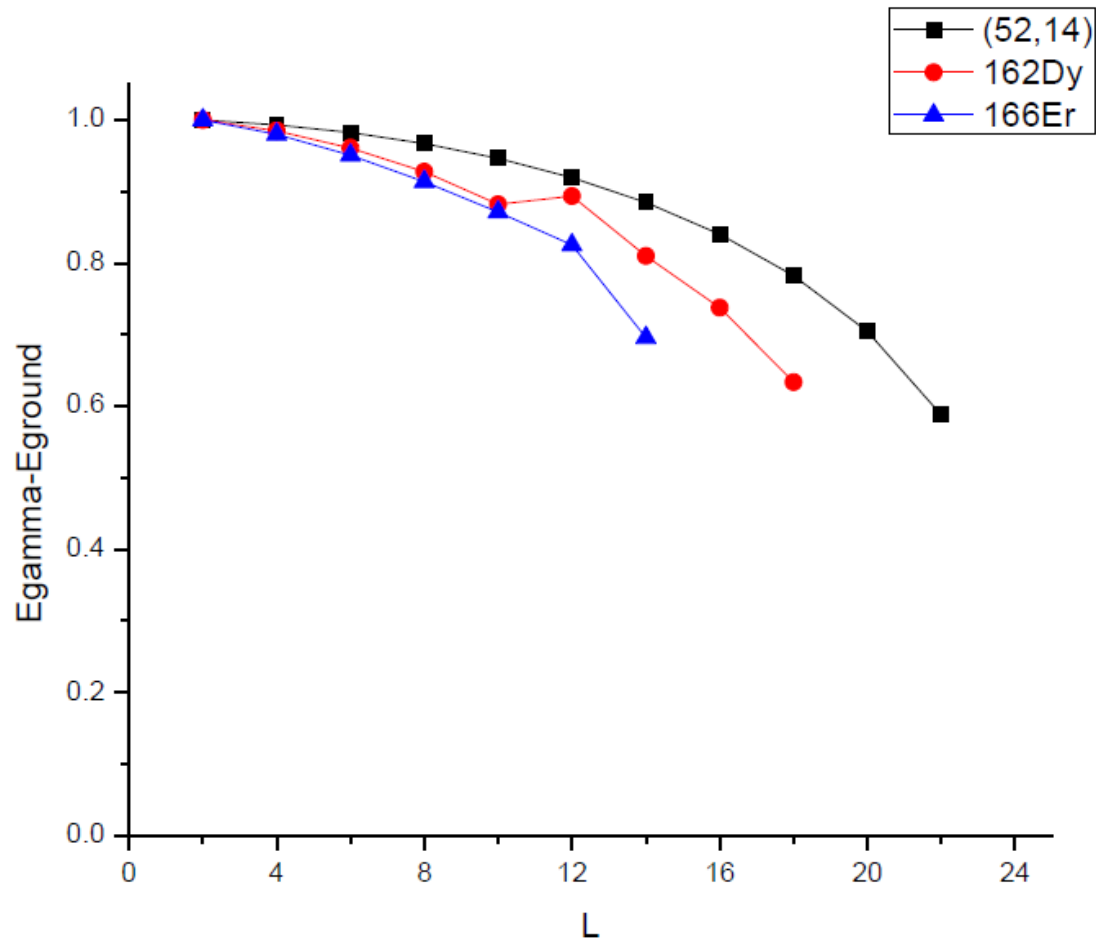
$$\frac{\Delta E(L)}{\Delta E(3)}$$

Splitting of the ground state and γ_1 bands



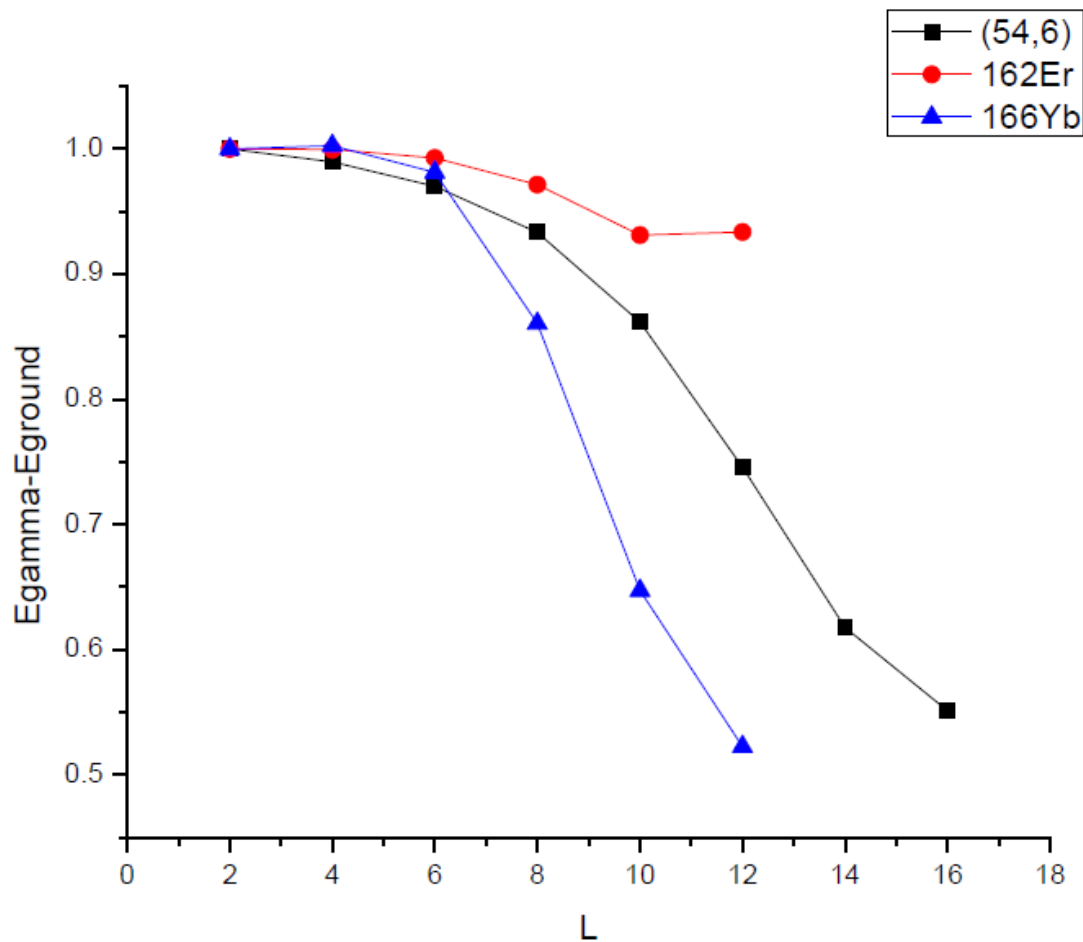
The ground state and γ_1 bands of both ^{160}Gd and ^{168}Er are characterized by the (54,12) irrep in proxy-SU(3).

Splitting of the ground state and γ_1 bands



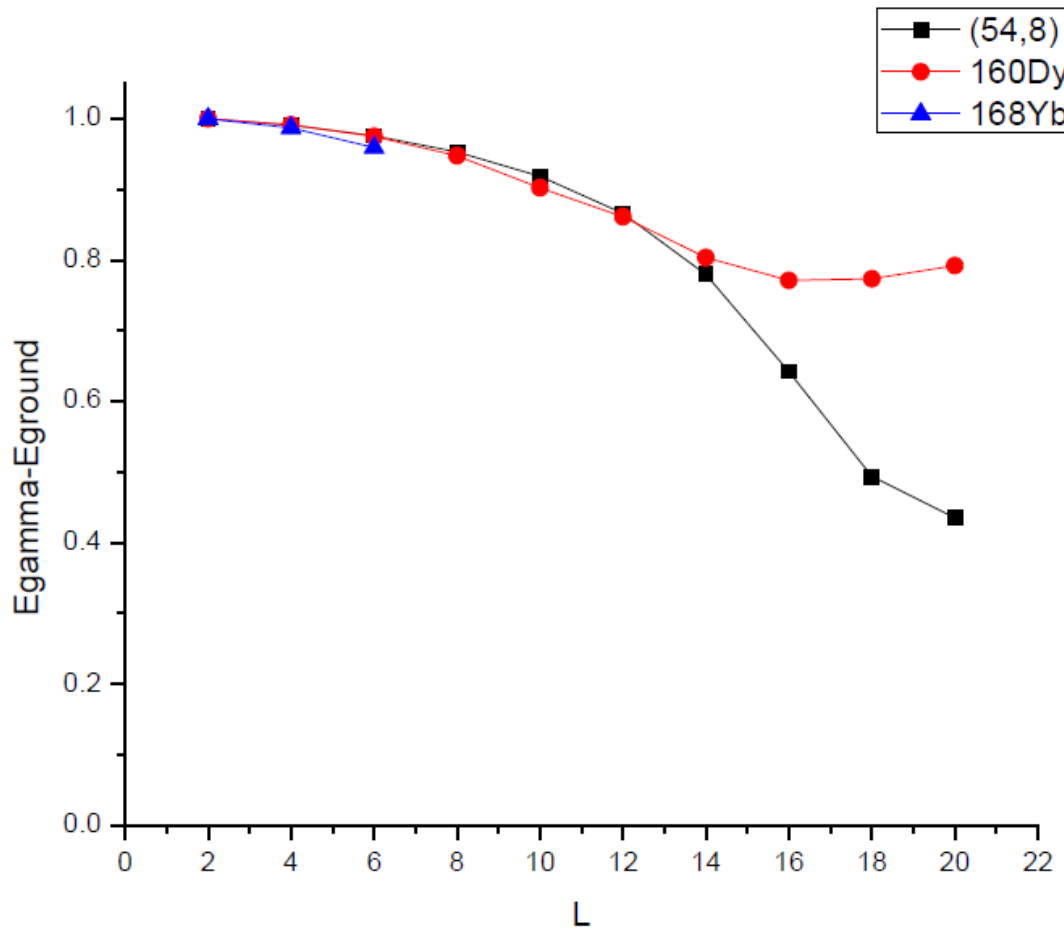
The ground state and γ_1 band of both ^{162}Dy and ^{166}Er are characterized by the (52,14) irrep in proxy-SU(3).

Splitting of the ground state and γ_1 bands



The ground state and γ_1 band of both ^{162}Er and ^{166}Yb are characterized by the (54,6) irrep in proxy-SU(3).

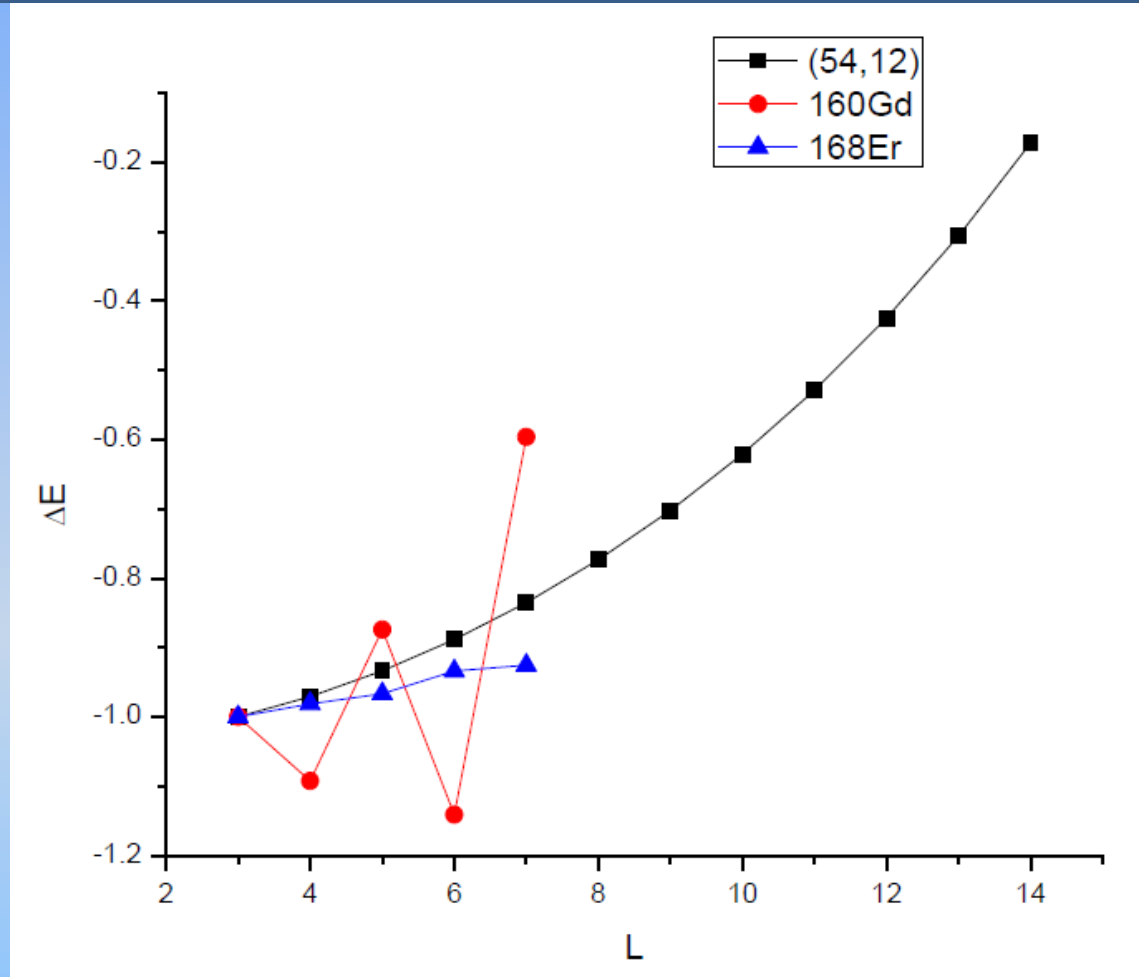
Splitting of the ground state and γ_1 bands



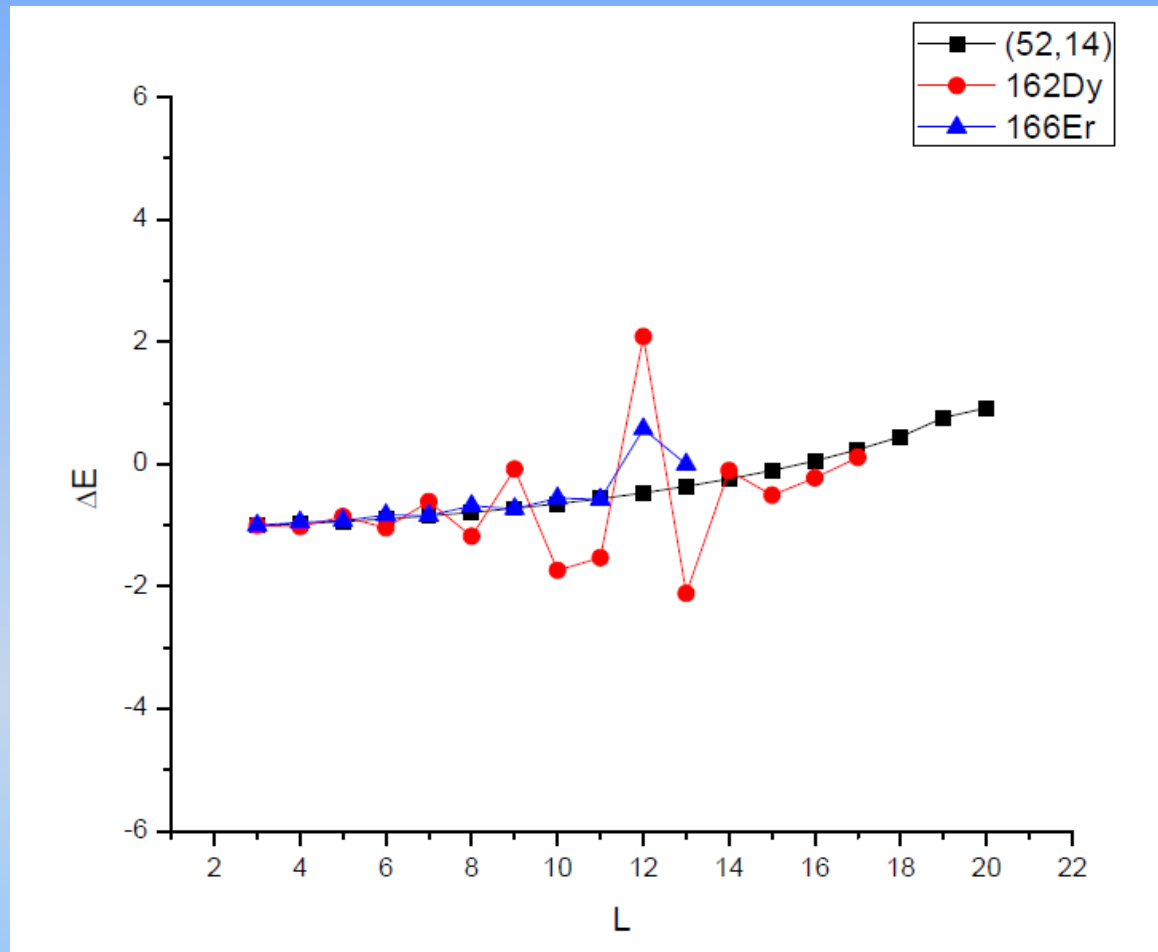
The ground state and γ_1 band of both ^{160}Dy and ^{168}Yb are characterized by the (54,8) irrep in proxy-SU(3).

Odd even staggering within γ bands

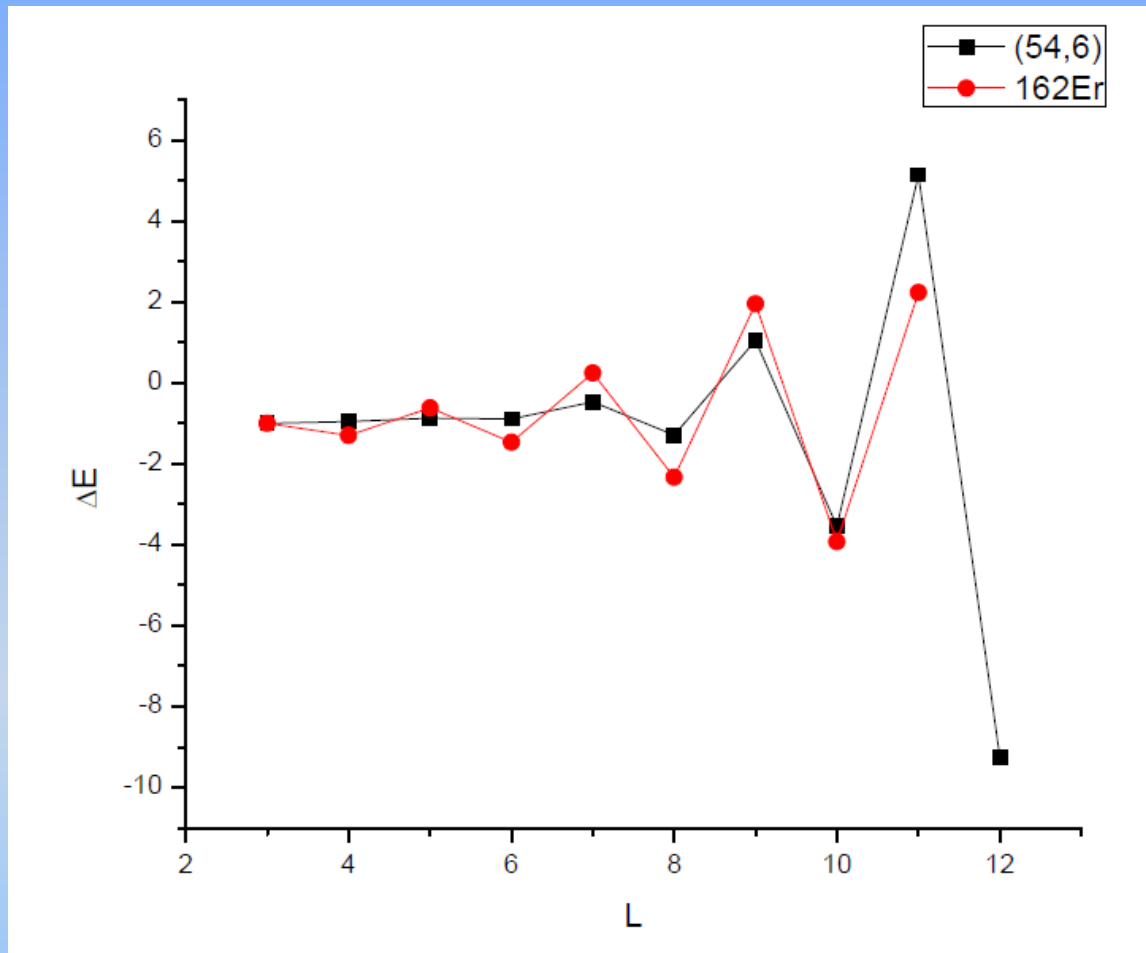
$$\frac{\Delta E(L)}{\Delta E(3)} \text{ where } \Delta(L) = E(L) - \frac{(L+1)E(L-1) + LE(L+1)}{2L+1}$$



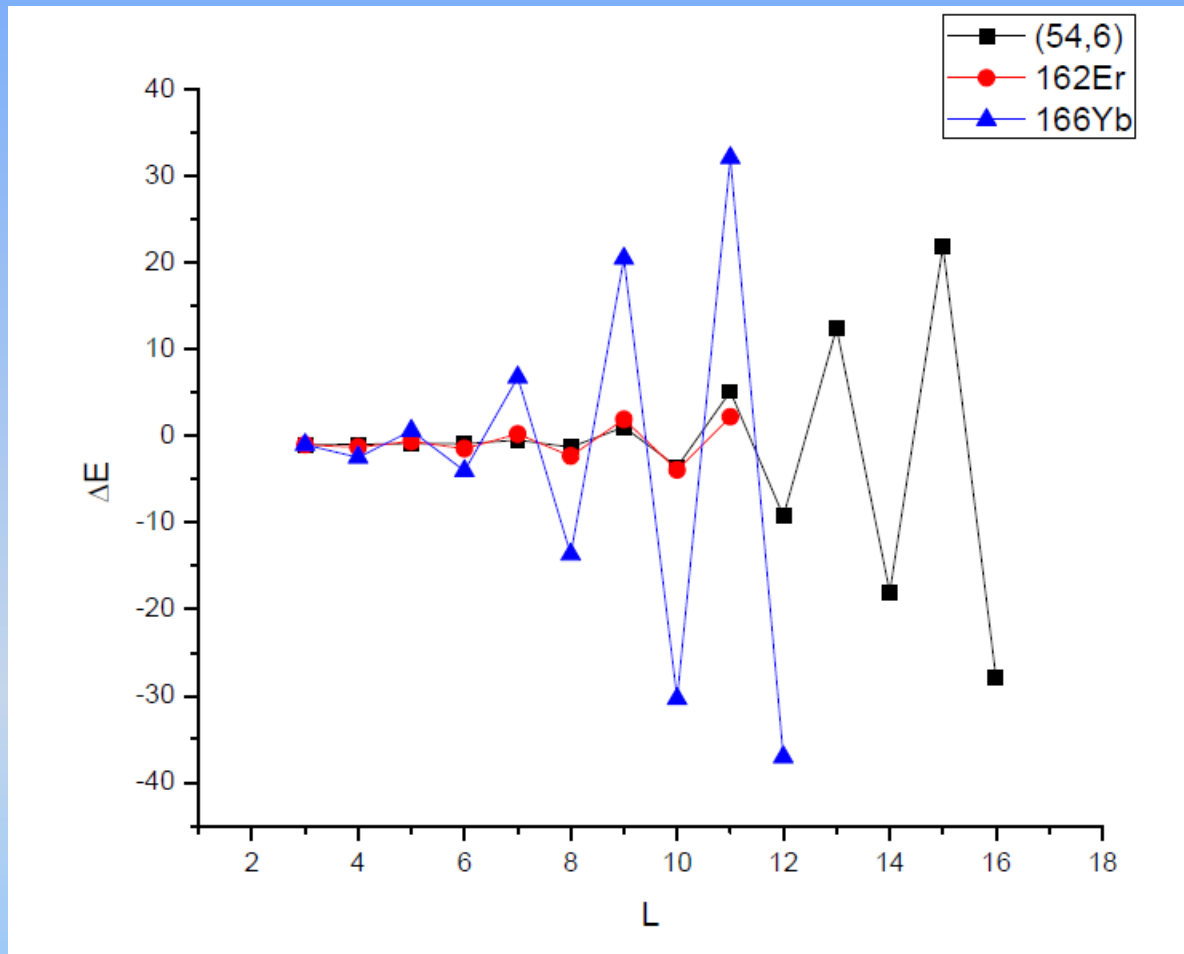
Odd even staggering within γ bands



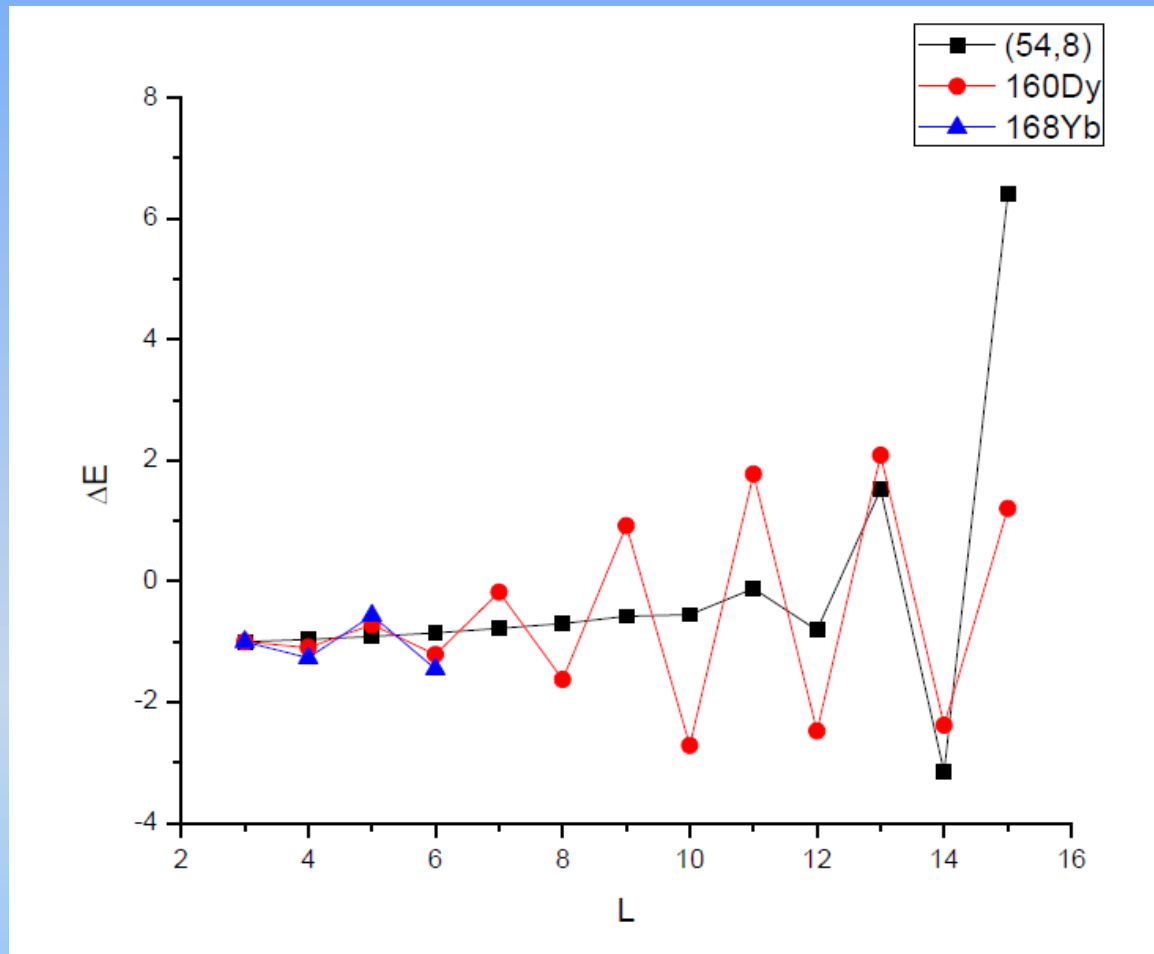
Odd even staggering within γ bands



Odd even staggering within γ bands



Odd even staggering within γ bands



Thank you for your time!!