

Equation of state of
cold rapidly rotating
neutron stars and the
effects of the
Keplerian sequence

P.S. Koliogiannis and
Ch.C. Moustakidis

Stellar Remnants

From Static to Rapidly
Rotating Neutron Stars

Effects of the
Keplerian Sequence

Maximum frequency

The Kerr parameter

Rest Mass Sequences

Conclusions

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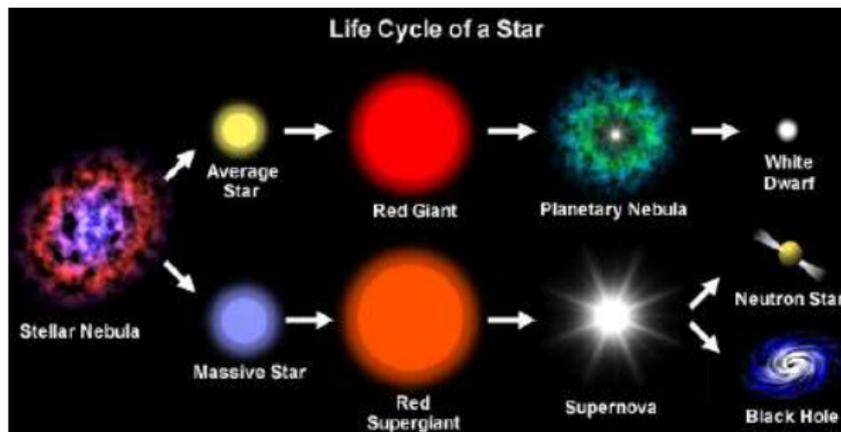
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Stellar Remnants

The remnants of supernova collapse, can take one of the three forms

- ▶ White Dwarf
- ▶ Neutron Star
- ▶ Black Hole



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Neutron Star

- ▶ Mass: $1.4-2.5 M_{\odot}$
- ▶ Eq. Radius: $10-15 \text{ km}$
- ▶ Mean density: $4 \times 10^{14} \text{ gr/cm}^3$
- ▶ Frequency: up to 2.2 kHz



The fastest observed rotating neutron star is at 716 Hz

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From Static to Rapidly Rotating Neutron Stars

Static Configuration

- ▶ Space-time metric :

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

- ▶ Perfect fluid : $T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu + p g_{\mu\nu}$
- ▶ TOV system $\rightarrow m(r), p(r), \epsilon(r)$, etc.

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Rotating Configuration

- ▶ Space-time metric :
$$ds^2 = e^\nu (1 + 2h) dt^2 - e^\lambda \left[1 + \frac{2m}{r-2M_0} \right] dr^2 - r^2 (1 + 2k) \left[d\theta^2 + \sin^2 \theta (d\phi - \omega dt)^2 \right] + O(\Omega^3)$$
- ▶ Perfect fluid : $T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu + p g_{\mu\nu}$
- ▶ TOV system $\rightarrow m(r), p(r), \epsilon(r), \omega(r) \text{ etc.}$

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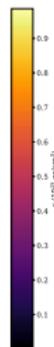
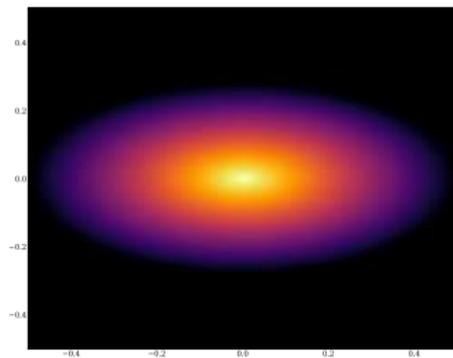
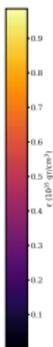
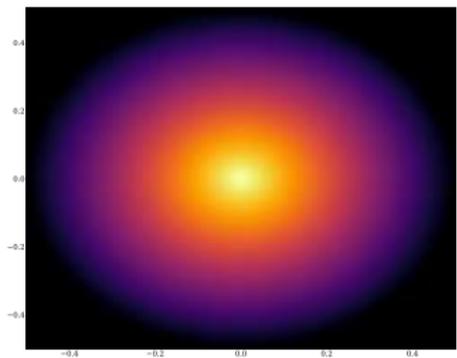
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From Static to Rapidly Rotating Neutron Stars

Differences between the static and rotating configuration

- ▶ $M_{max}^{st} < M_{max}^{rot}$
- ▶ $R_{eq,max}^{st} < R_{eq,max}^{rot}$
- ▶ $\epsilon_c^{st} > \epsilon_c^{rot}$
- ▶ The existence of the angular velocity and the Kerr parameter (only in rotating configuration)



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The knowledge of the maximum mass and the maximum frequency, can help us:

- ▶ Identify a compact object as a Black Hole
- ▶ Constrain the high density part of the Equation of State

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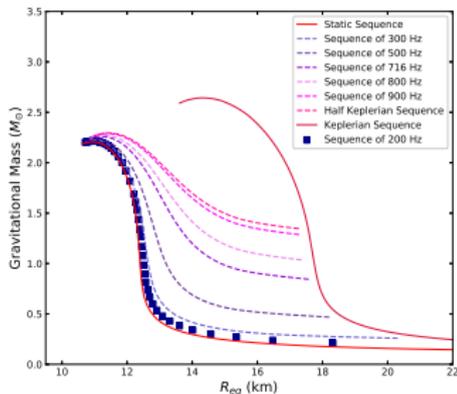
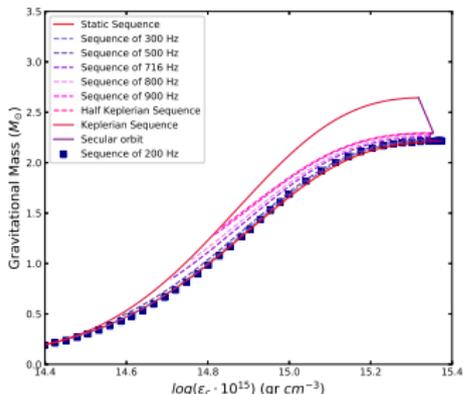
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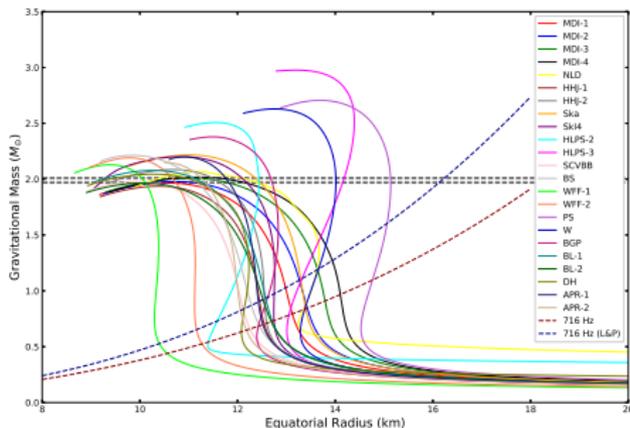
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From Static to Rapidly Rotating Neutron Stars

- ▶ We employ a total of 23 realistic EoS for neutron star matter, based on various theoretical nuclear models
- ▶ They satisfy the observed limit of $M = 1.97 \pm 0.04M_{\odot}$ and $M = 2.01 \pm 0.04M_{\odot}$
- ▶ They satisfy the observed frequency 716 Hz limit introduced by Lattimer & Prakash¹



¹ J. M. Lattimer and M. Prakash, Science 304, 536 (2004), ISSN 0036-8075, URL <https://science.sciencemag.org/content/304/5670/536>

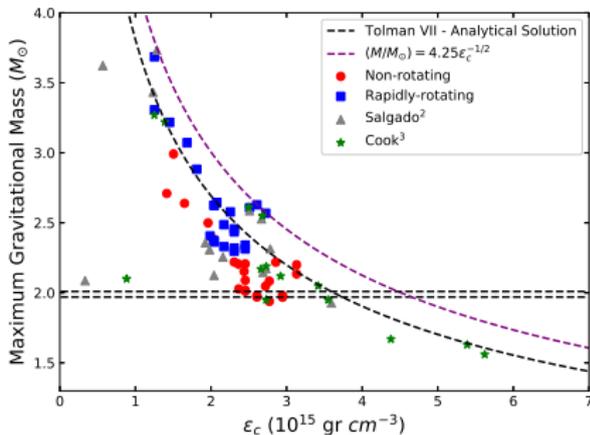


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² M. Salgado, S. Bonazzola, E. Gourgoulhon and P. Haensel, *Astron. Astrophys. Suppl. Ser.* 108, 455-459 (1994)

³ Gregory B. Cook, Stuart L. Shapiro and Saul A. Teukolsky, *The Astrophysical Journal*, Vol. 424, 823-845 (1994)



Effects of the Keplerian Sequence - Maximum frequency

- ▶ Determines the maximum rotation rate
- ▶ Depends on the gravitational mass of the star \rightarrow EoS dependent

- ▶ $f_{max}(M_{max}, R_{eq,max}) = c \left(\frac{M_{max}}{M_{\odot}} \right)^{1/2} \left(\frac{10km}{R_{eq,max}} \right)^{3/2}$

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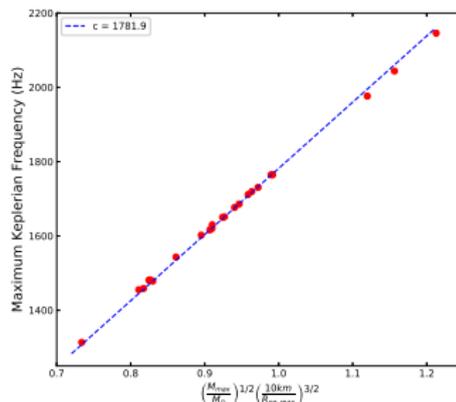
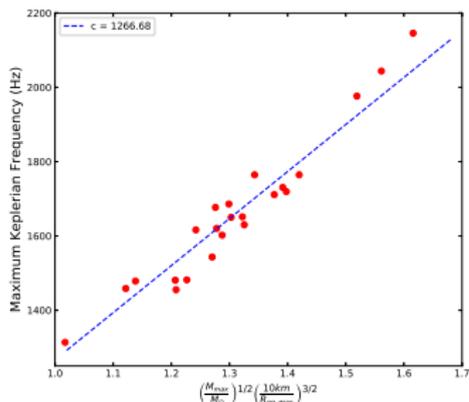
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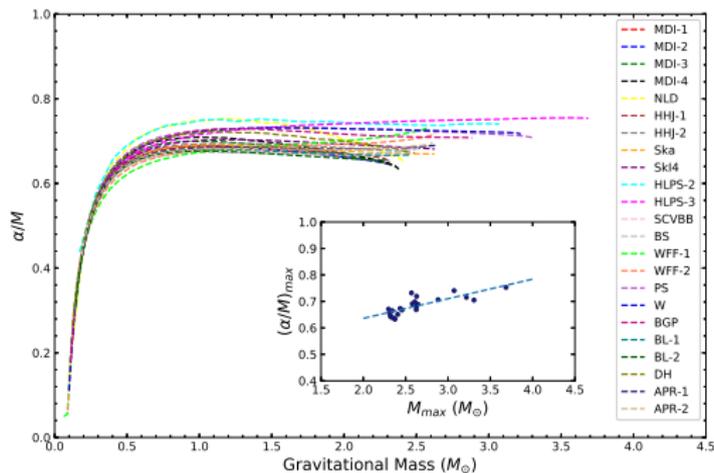
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Effects of the Keplerian Sequence - The Kerr parameter

- ▶ $j = cJ/(GM^2)$
- ▶ $(\alpha/M)_{max} \approx 0.75$
- ▶ $(\alpha/M)_{max} = 0.074M_{max} + 0.488$

$$(\alpha/M)_{max}^{B.H.} \approx 0.998$$



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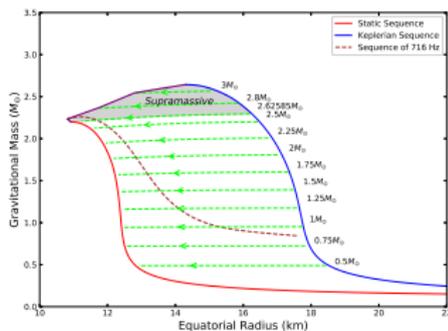
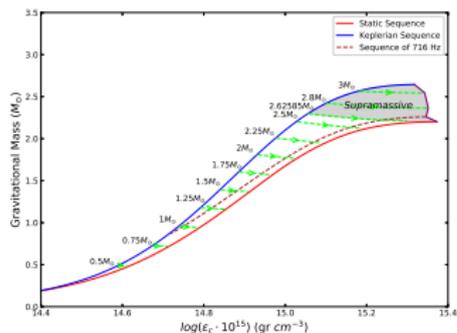
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Rest Mass Sequences

- ▶ Time evolutionary sequences
- ▶ Normal & Supramassive



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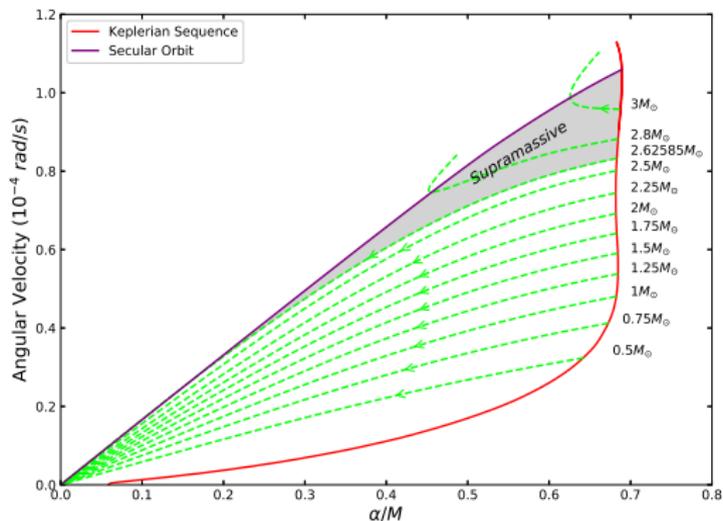
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Rest Mass Sequences

- ▶ Prior to collapse the star spins up
- ▶ Gravitation collapse



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Conclusions

- ▶ The M_{max}^{rot} and f_{max} can constrain the EoS at high densities
- ▶ The M_{max}^{rot} can provide us with the ultimate density of cold baryonic matter
- ▶ The Kerr parameter determines the final fate of the collapse of a rotating compact star
- ▶ The calculation of Kerr parameter can help us to identify a compact object as a black hole and possibly imply the existence of universal limiting value of the neutron star compactness
- ▶ Supramassive curves may provide an observable precursor to gravitation collapse to a black hole and constrain the angular momentum

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Thank you for your attention!

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