

Constraints on neutron stars equation of state, from observation of the tidal deformability of the GW170817 system

A. Kanakis-Pegios ¹ Ch. C. Moustakidis ²

¹M.Sc. Computational Physics
Aristotle University of Thessaloniki

²Department of Theoretical Physics
Aristotle University of Thessaloniki

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- Neutron Stars (NS) are among the most interesting astrophysical objects in Nature.
- Conditions of extremely high matter densities prevail in their interiors, making them a natural laboratory for extreme physics. The precise form of the equation of state (EoS) of NSs remains unknown.
- Radius: $R \approx 10 - 15 \text{ km}$, Mass: $M \approx 1 - 2M_{\odot}$, Mean density: $\rho \approx 4 \times 10^{14} \text{ g/cm}^3$, Frequency: $f \approx \text{few Hz} - 700 \text{ Hz}$, Magnetic field: $B \approx 10^{12} - 10^{18} \text{ Gauss}$.

The metric for a spherically symmetric configuration and for the static case is:

$$ds^2 = e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) - e^{\nu(r)} dt^2$$

The metric functions $\lambda(r)$, $\nu(r)$ are related to the energy distribution $\mathcal{E}(r)$ and local pressure $P(r)$ as:

$$\begin{aligned} \frac{8\pi G}{c^4} \mathcal{E}(r) &= \frac{1}{r^2} (1 - e^{-\lambda(r)}) + e^{-\lambda(r)} \frac{\lambda'(r)}{r}, \\ \frac{8\pi G}{c^4} P(r) &= -\frac{1}{r^2} (1 - e^{-\lambda(r)}) + e^{-\lambda(r)} \frac{\nu'(r)}{r}, \\ P'(r) &= -\frac{P(r) + \mathcal{E}(r)}{2} \nu'(r) \end{aligned}$$

The equilibrium of the star's matter is given by the equations above, combined with the equation of state (EOS) $P = P(\mathcal{E})$.

Tolman-Oppenheimer-Volkov (TOV) equations

The system of TOV equations of the equilibrium is given by:

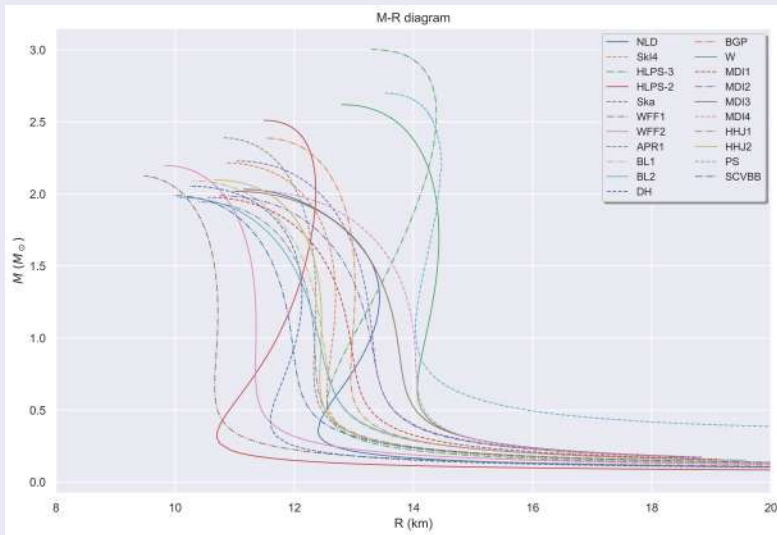
$$\frac{dP}{dr} = -\frac{G}{r^2} \left[\rho(r) + \frac{P(r)}{c^2} \right] \left[m(r) + \frac{4\pi r^3 P(r)}{c^2} \right] \left[1 - \frac{2Gm(r)}{rc^2} \right]^{-1}$$
$$\frac{dM(r)}{dr} = 4\pi^2 r^2 \rho(r), \quad \rho(r) = \mathcal{E}(r)/c^2$$

For the numerical integration of the system we set:

- $m(r) = \bar{m}(r)M_{\odot}$, $\mathcal{E}(r) = \bar{\mathcal{E}}(r)\mathcal{E}_0$, $P(r) = \bar{P}(r)\mathcal{E}_0$
- $\mathcal{E}_0 = 1 \text{ MeV fm}^{-3}$
- $\frac{GM_{\odot}}{c^2} = 1.474 \text{ Km}$
- $\frac{4\pi}{M_{\odot}c^2} = 0.7 \cdot 10^{-40} \frac{\text{s}^2}{\text{Kg}\cdot\text{Km}^2}$

M-R diagram

Results for a variety of EOS



Tidal effects during inspiral of a BNS

- GW from the late phase of a BNS are among the most significant sources for a GW detector.
- Tidal field E_{ij} induces change of quadrupole moment Q_{ij} of the neutron star.

The quadrupole moment is given by^[1]:

$$Q_{ij} = -\lambda(m)E_{ij}, \text{ with } \lambda(m) = \frac{2}{3}k_2(m)\frac{R^5}{G}$$

- The induced quadrupole moment depends on neutron star structure.
- Tidal deformability λ depends on radius R (the smaller the star, the harder to deform) and k_2 .
- The reaction of the star to the tidal field is described by the parameter k_2 , known as tidal love number.
- k_2 depends on the structure of the neutron star, i.e. its mass and equation of state.

Mathematical formalism

The tidal love number k_2 is given by^[1]:

$$\begin{aligned} k_2 = & \frac{8\beta^5}{5}(1 - 2\beta)^2[2 - y_R + (y_R - 1)2\beta] \\ & \times [2\beta(6 - 3y_R + 3\beta(5y_R - 8)) \\ & + 4\beta^3(13 - 11y_R + \beta(3y_R - 2) + 2\beta^2(1 + y_R)) \\ & + 3(1 - 2\beta)^2[2 - y_R + 2\beta(y_R - 1)] \ln(1 - 2\beta)]^{-1}, \end{aligned}$$

where $\beta = GM/Rc^2$ is the compactness parameter. The tidal love number depends also on the quantity y_R , which is determined by solving the following differential equation^[1]:

$$r \frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2Q(r) = 0, \quad y(0) = 2, \quad y_r \equiv y(R)$$

Tidal love number

$F(r)$ and $Q(r)$ are functionals of $\mathcal{E}(r)$, $P(r)$ and $M(r)$ ^[1]:

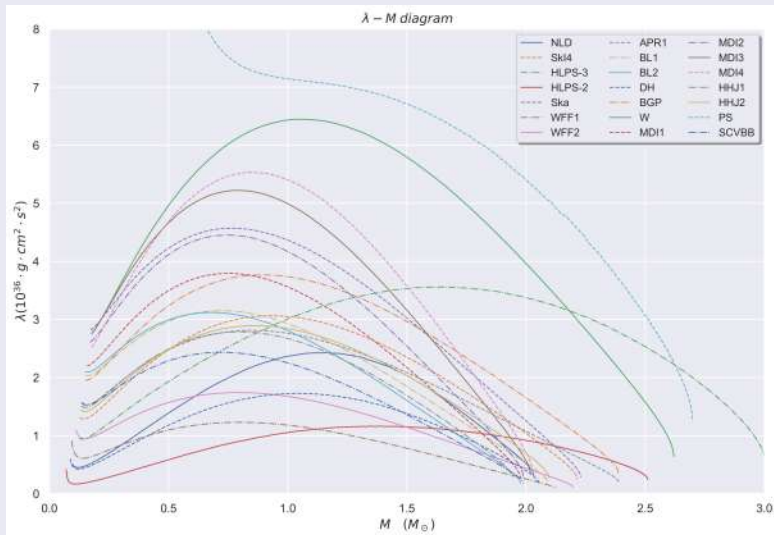
$$F(r) = \left[1 - \frac{4\pi r^2 G}{c^4} (\epsilon(r) - P(r)) \right] \left(1 - \frac{2M(r)G}{rc^2} \right)^{-1},$$

$$\begin{aligned} r^2 Q(r) &= \frac{4\pi r^2 G}{c^4} \left[5\epsilon(r) + 9P(r) + \frac{\epsilon(r) + P(r)}{\partial P(r)/\partial \epsilon(r)} \right] \left(1 - \frac{2M(r)G}{rc^2} \right)^{-1} \\ &\quad - 6 \left(1 - \frac{2M(r)G}{rc^2} \right)^{-1} \\ &\quad - \frac{4M^2(r)G^2}{r^2 c^4} \left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2} \right)^2 \left(1 - \frac{2M(r)G}{rc^2} \right)^{-2}. \end{aligned}$$

The numerical integration of these equations, combined with TOV equations, using as boundary conditions $y(0) = 2$, $P(0) = P_c$ and $M(0) = 0$ provide the mass M , the radius R of the neutron star (TOV solution), and the value $y_R = y(R)$. The compactness parameter b is determined, and therefore the tidal love number can be determined.

$\lambda - M$ diagram

Results for a variety of EOS



- During the last orbits before the neutron stars merging, the orbital phase evolution is affected by tidal deformability.
- At leading order, the tidal effects are imprinted in the gravitational-wave signal through the binary tidal deformability^[5]:

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12q + 1)\Lambda_1 + (12 + q)q^4\Lambda_2}{(1 + q)^5},$$

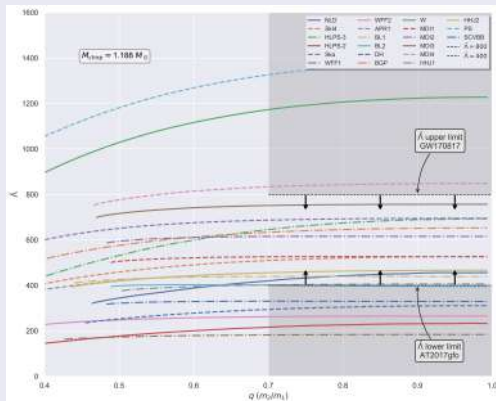
where $q = m_2/m_1 \leq 1$ is the binary's mass ratio (m_1 corresponds to the heavier component mass) and $\Lambda_1 = \Lambda_1(m_1)$ and $\Lambda_2 = \Lambda_2(m_2)$ are the tidal deformabilities of the two NS. $M_{tot} = m_1 + m_2$ is the total mass of the binary system and $M_{chirp} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$ is the chirp mass.

Constraints for the low spin case (*Abbott et al., 2017* [3])

	Low-spin priors ($ \chi \leq 0.05$)
Primary mass m_1	1.36–1.60 M_\odot
Secondary mass m_2	1.17–1.36 M_\odot
Chirp mass \mathcal{M}	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0
Total mass m_{tot}	$2.74^{+0.04}_{-0.01} M_\odot$
Radiated energy E_{rad}	$> 0.025 M_\odot c^2$
Luminosity distance D_L	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55^\circ$
Using NGC 4993 location	$\leq 28^\circ$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800
Dimensionless tidal deformability $\Lambda(1.4M_\odot)$	≤ 800

- Fixed chirp mass of $M_{\text{chirp}} = 1.188 M_\odot$.
- Binary's mass ratio: $0.7 < q < 1.0$
- Upper limit on $\tilde{\Lambda}$: $\tilde{\Lambda} \leq 800$.

$\tilde{\Lambda} - q$ for a variety of EoS



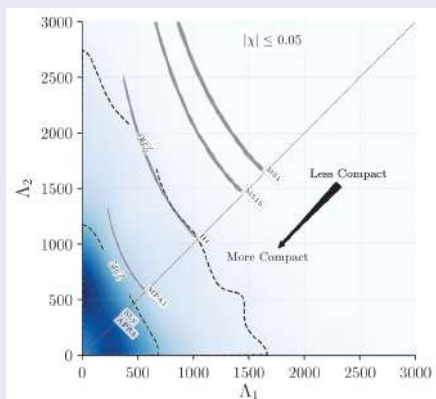
- $\tilde{\Lambda} = \tilde{\Lambda}(q, M_{chirp} = 1.188 M_{\odot}; EOS)$.
- Binary's mass ratio: $0.4 < q < 1.0$
- Upper limit on $\tilde{\Lambda}$: $\tilde{\Lambda} \leq 800$ (GW170817) [3]
- Lower limit on $\tilde{\Lambda}$: $\tilde{\Lambda} \geq 400$ from EM data (AT2017gfo) [7,8]

- $\tilde{\Lambda} = \Lambda_1 = \Lambda_2$ when $m_1 = m_2$.
- The figure shows the posterior probability densities for Λ_1, Λ_2 with 90% and 50% credible region contours
- Under the assumption that $\Lambda \simeq \alpha\beta^{-6}$ and $R_1 \simeq R_2 \simeq \hat{R}^{[5]}$:

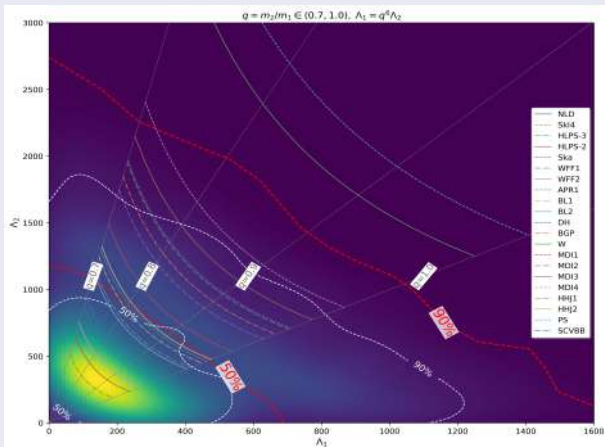
$$\Lambda_1(\tilde{\Lambda}, q) = \frac{13}{16} \tilde{\Lambda} \frac{q^2(1+q)^4}{12q^2 - 11q + 12}$$

$$\Lambda_2(\tilde{\Lambda}, q) = q^{-6} \Lambda_1$$

$\Lambda_1 - \Lambda_2$ diagram for the low spin case (Abbott et al., 2017 [3])



$\Lambda_1 - \Lambda_2$ space for a variety of EoS



- For $m_1 = m_2 = 1.36 M_\odot \Rightarrow \tilde{\Lambda} = \Lambda_1 = \Lambda_2$, unique value for each EoS.
- Posterior probability densities for Λ_1, Λ_2 using the PhenomPNRT model's data). [4,6]
- 90% and 50% credible region contours using PhenomPNRT waveform model (white contours) [4,6], and first LIGO's paper data (red contours) [3]

- GW from BNS coalescence provide strong constraints on the EoS of dense matter.
- Observations of massive NS ($1.97 \pm 0.04 M_{\odot}$ from PSR J1617-2230 and $2.01 \pm 0.04 M_{\odot}$ from PSR J0348-0432) prefer stiff EoSs [7].
- The upper limit on $\tilde{\Lambda}$ obtained from GW170817 exclude stiff EoSs which predict their values bigger than 800, such as the EoS: W, PS, MDI4.
- The lower bound on $\tilde{\Lambda}$ derived from EM observations and UV/optical analysis of GW170817 signal, exclude soft EoSs ($\tilde{\Lambda} \geq 400$).
- Further upgrade of terrestrial GW detectors will lead to stronger constraints on the bounds.

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