Proxy-SU(3)

Part I:
A focus on the highest weight irreps of the Elliott SU(3) symmetry

Part II:
Binding energies with the Proxy-SU(3) symmetry

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INPP, DEMOKRITOS

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ΕΣΠΑ
πρόγραμμα για την ανάπτυξη

Part I:
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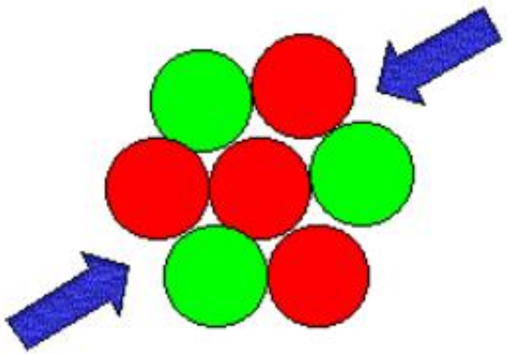
Basic properties of the strong force



Strong force binds the nucleus

- **Tensor force:** → Spin – isospin dependence; → d >2 fm
- **Spin – isospin independent attraction:** → 1 fm < d < 2 fm
- **Strong repulsive core:** → Pauli Principle; → d < 1 fm

The many – particle wave functions

Antisymmetric many-particle wave function

Spatial wave function-most symmetric

Spinor-most antisymmetric

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Test of Wigner's Spin-Isospin Symmetry from Double Binding Energy Differences

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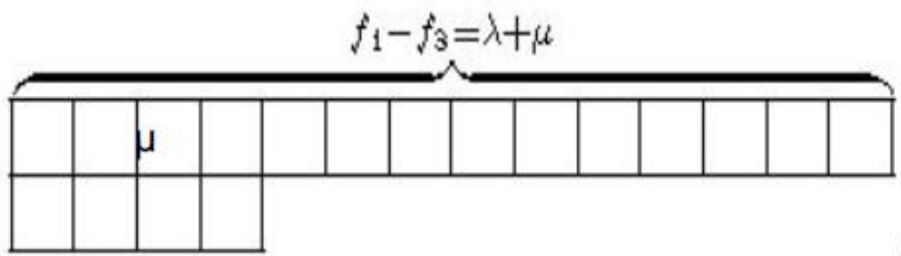
(Received 21 December 1994)

The most symmetric spatial wave function corresponds to the maximum Binding Energy.

The Elliott SU(3) irreps



The Elliott SU(3) wave functions derive from
The spatial part of the many-particle wave function

The Elliott SU(3) wave functions are many-quanta wave functions. Each box is a quantum in the z, x, or y cartesian axis.

The λ+μ represent the quanta, which are symmetric upon their interchange

The μ are nor symmetric, neither antisymmetric

An example

$$\begin{array}{|c|c|}\hline z & z \\ \hline x \\ \cline{1-1}\end{array}\ ,\ \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$$

$$\Phi_{spatial} = \sqrt{\frac{1}{6}}(2\phi_z(1)\phi_z(2)\phi_x(3) - \phi_z(1)\phi_x(2)\phi_z(3) - \phi_x(1)\phi_z(2)\phi_z(3)),$$

The quanta 1, 2 are symmetric upon their interchange, but 3 is nor symmetric, neither antisymmetric

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$$r(\lambda,\mu) = \frac{\lambda+\mu}{\lambda+2\mu}\cdot 100\%,$$

The percentage of symmetric quanta over the total number of quanta gets the maximum value for the highest weight irrep of every shell.

valence particles	λ	μ	C_2	r (%)	
1	3	0	18	100	
2	6	0	54	100	
3	7	1	81	89	
4	8	2	114	83	
5	10	1	144	92	
	7	4	126	73	
6	12	0	180	100	h.w.
	9	3	153	80	
	6	6	144	67	
7	11	2	186	87	h.w.
	8	5	168	72	
8	10	4	198	78	
9	10	4	198	78	h.w.
	7	7	189	67	
10	10	4	198	78	h.w.
	7	7	189	67	
	4	10	198	58	

Part II: Binding energies with the Proxy-SU(3) symmetry

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Method

Elliott's Hamiltonian:

$$H = H_0 - \frac{\kappa}{2} QQ$$

Where H_0 is the 3D – HO Hamiltonian for the many - particle system:

$$H_0 = \sum_{i=1}^{A} \left(\frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 r_i^2\right)$$

If $\hbar\omega = 1$ then:

$$H_0 = N_0 = \sum_{i=1}^{A} \left(\mathcal{N}_i + \frac{3}{2}\right)$$

With $\mathcal{N}_i$ being the number of the harmonic oscillator quanta of the orbit of the i^{th} *particle.*

Method

$$H = H_0 - \frac{\kappa}{2} QQ$$

Where:

$$\frac{\kappa}{2} = \frac{\hbar\omega}{4\mathrm{N}_0} \qquad \& \qquad \hbar\omega = \frac{41}{A^{1/3}}\, MeV$$

The quadrupole – quadrupole interaction in an SU(3) wave function is:

$$QQ = 4C_2 - 3\mathrm{L}(\mathrm{L}+1)$$

$$C_2 = \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)$$

Binding energy

$$\boldsymbol{BE(Z,N) = AV_0 \cdot (H_0 - \frac{\kappa}{2}QQ)}$$

$$V_0 \approx 50(1 - \frac{N-Z}{A}) \text{ MeV}$$

Where V_0 is the depth of the 3D – HO potential

The two proton or two neutron separation energies are given by:

$$\boldsymbol{S_{2p} = BE(Z,N) - BE(Z-2,N)}$$
$$\boldsymbol{S_{2n} = BE(Z,N) - BE(Z,N-2)}$$

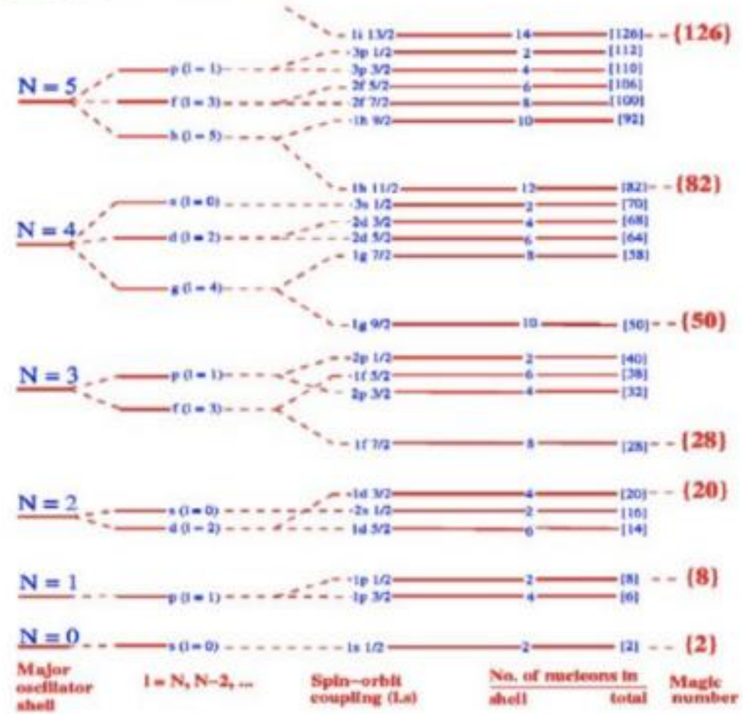
Calculation of N_0 in the HO shells

$$H_0 = N_0 = \sum_{i=1}^{A} (\mathcal{N}_i + \frac{3}{2})$$

For the proton configuration: $N_{0,Z} = 2\left(0+\frac{3}{2}\right) + 4\left(1+\frac{3}{2}\right) = 13.$

For the neutron configuration: $N_{0,N} = 2\left(0+\frac{3}{2}\right) + 6\left(1+\frac{3}{2}\right) = 18.$

$$N_0 = N_{0,Z} + N_{0,N} = 13 + 18 = 31.$$



Calculation of N_0 within the proxy-SU(3) symmetry

Within the proxy-SU(3) symmetry a unitary transformation can be applied in the intruder orbitals that reduces the number of quanta by 1.
The proxy numbers become 6-12, 14-26, 28-48, 50-80, 82-124,...

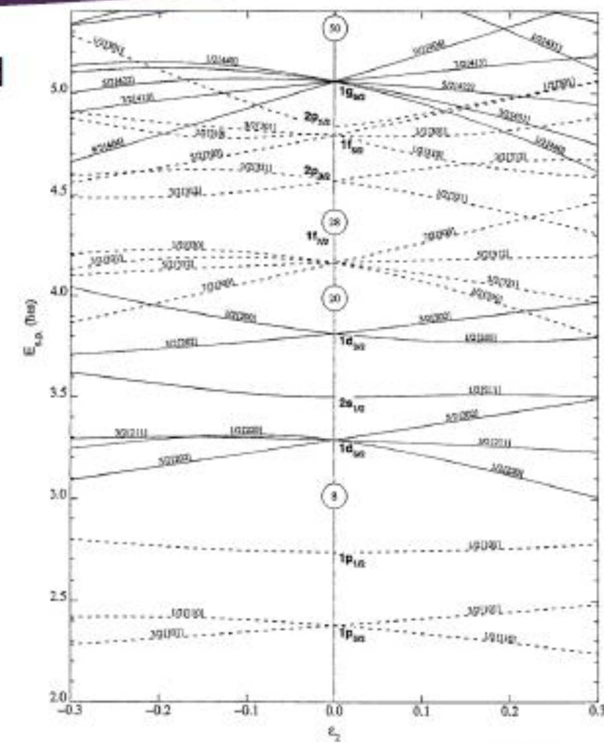
$$N_{0,so} = N_{0,core} + N_{0,val}$$

Z=6 $\quad N_{0,so} = N_{0,core} = 13$

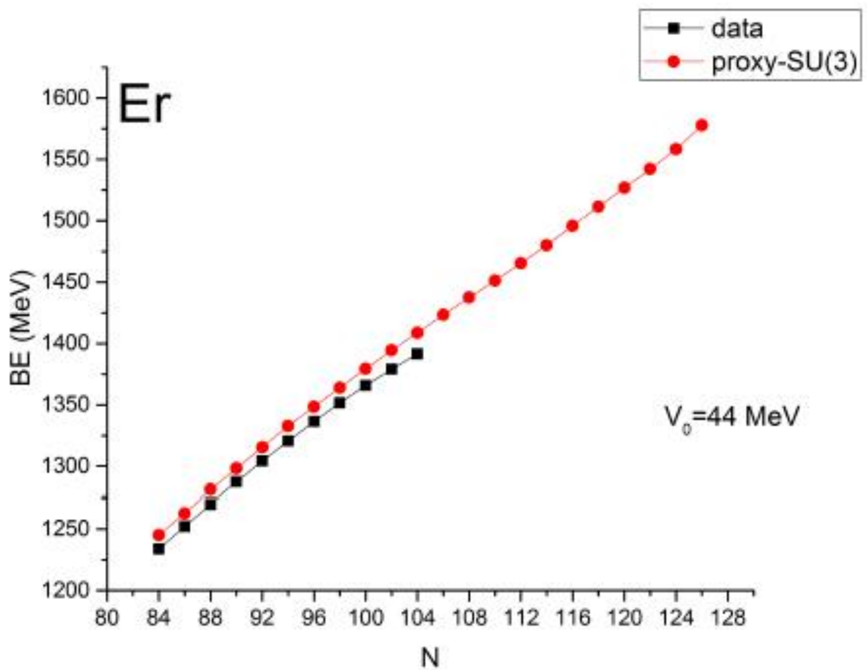
N=8 $\quad N_{0,so} = 13 + (8-6)\left(1+\frac{3}{2}\right) = 18$

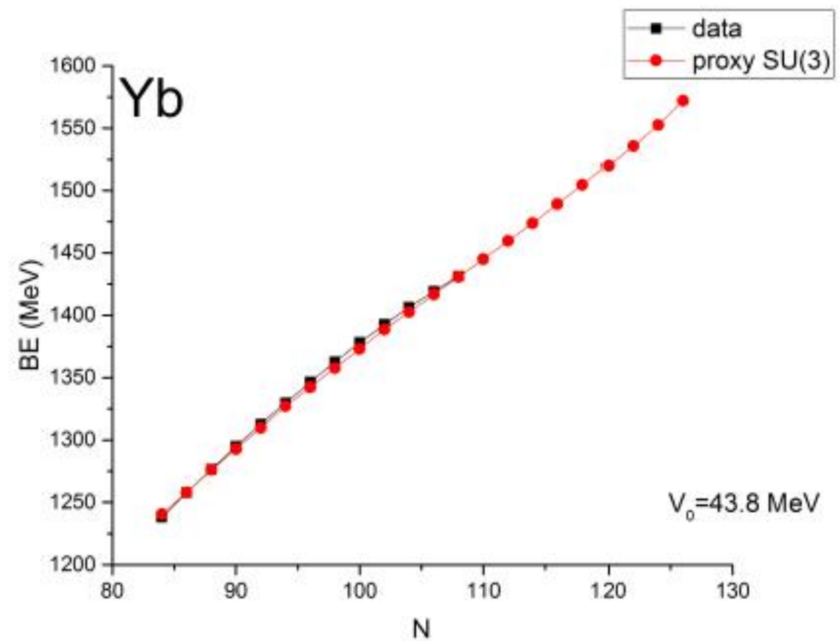
In general:

$$N_{0,so} = N_{0,core} + \sum_{i=1}^{A_{val}} \left(\mathcal{N}_{i,proxy} + \frac{3}{2}\right)$$

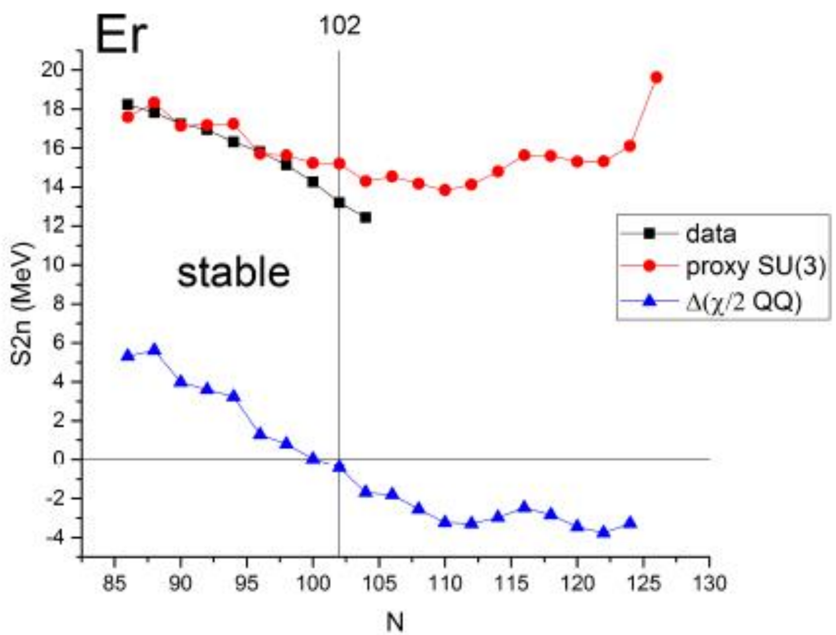


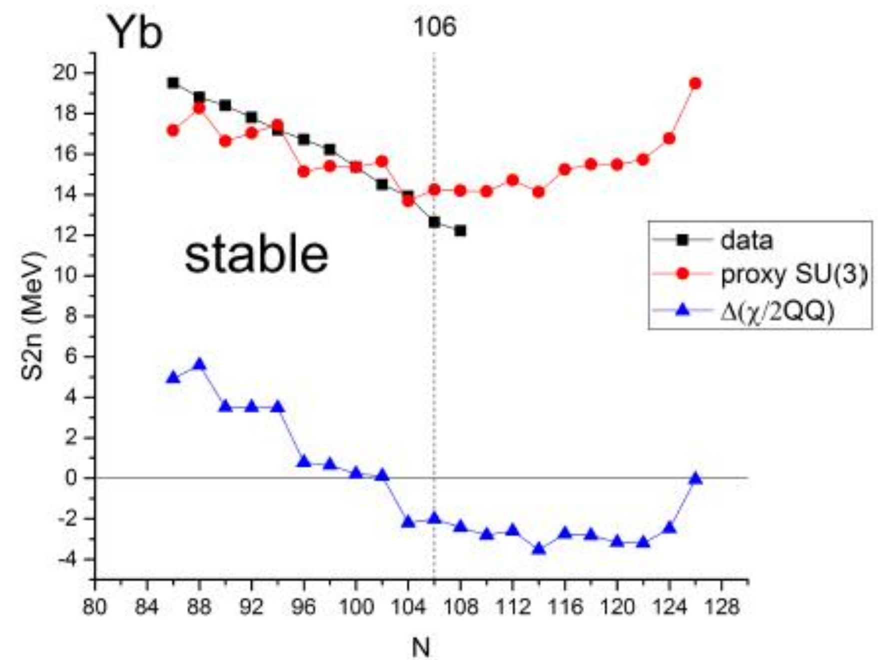
Results with proxy-SU(3) Rare Earth nuclei



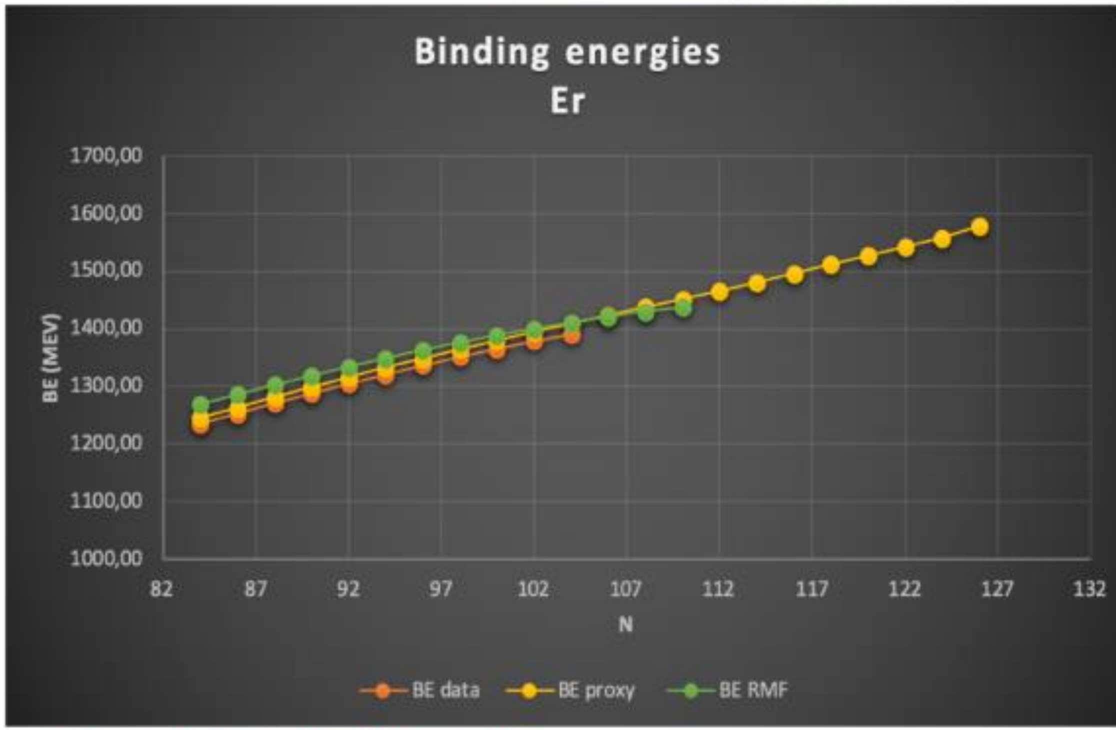


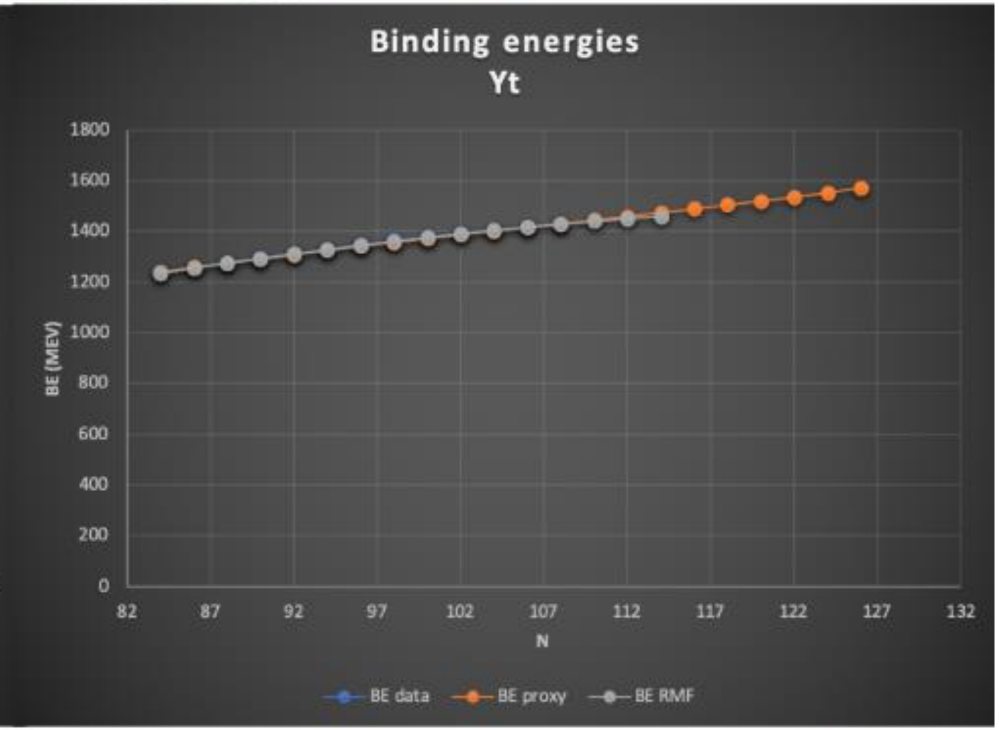
Results with proxy-SU(3) Rare Earth nuclei





Results in BE with proxy-SU(3) & RMF






A
Z
X
C
Thank
You !

Extra

The Elliott SU(3) irreps

$^{96}_{42}Mo$

- Z=42 with 14 valence protons in 7 orbitals
- [f]=[2,2,2,2,2,2,2,,0,0,0]

2 2 2 2 2 2 2 0 0 0 → S_{10} =14

2 2 2 2 2 2 0 0 0 → S_9 =12

2 2 2 2 2 0 0 0 → S_8 =10

2 2 2 2 0 0 0 → S_7=8

2 2 2 0 0 0 → S_6=6

2 2 0 0 0 → S_5=4

2 0 0 0 → S_4=2

0 0 0 → S_3=0

0 0 → S_2=0

0 → S_1=0

The weight vector w is:

w=(S_{10}-S_9, S_9-S_8, S_8-S_7, S_7-S_6, S_6-S_5, S_5-S_4, S_4-S_3, S_3-S_2, S_2-S_1, S_1)

w=(2,2,2,2,2,2,2,0,0,0)

The Elliott SU(3) irreps

$^{96}_{42}Mo$

- Z=42 with 14 valence protons in 7 orbitals
- [f]=[2,2,2,2,2,2,2,,0,0,0]

$2\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 0\ 0 \longrightarrow S_{10}=14$

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w=(2,2,2,2,2,2,2,0,0,0)

The many – body wave functions

$$H_i = \frac{p_i^2}{2M} + \frac{1}{2} M\omega^2 r_i^2 + V_{l_i s_i} \cdot l_i \cdot s_i$$

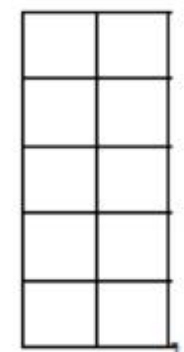
Nilsson orbitals

$K[Nn_z\Lambda]$	(n_z, n_x, n_y)
1/2[330]	(3,0,0)
3/2[321]	(2,1,0)
1/2[321]	(2,0,1)
5/2[312]	(1,2,0)
1/2[310]	(1,1,1)
3/2[312]	(1,0,2)
7/2[303]	(0,3,0)
3/2[301]	(0,2,1)
1/2[301]	(0,1,2)
5/2[303]	(0,0,3)

N=3

U(10) symmetry

Young diagram
for 10 protons
or neutrons



The Elliott SU(3) irreps

$^{96}_{42}Mo$

- Z=42 with 14 valence protons in 7 orbitals
- [f]=[2,2,2,2,2,2,2,,0,0,0]

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2 2 2 0 0 0 → S_6=6

2 2 0 0 0 → S_5=4

2 0 0 0 → S_4=2

0 0 0 → S_3=0

0 0 → S_2=0

0 → S_1=0

The weight vector w is:

w=(S_{10}-S_9, S_9-S_8, S_8-S_7, S_7-S_6, S_6-S_5, S_5-S_4, S_4-S_3, S_3-S_2, S_2-S_1, S_1)

w=(2,2,2,2,2,2,2,0,0,0)

The Elliott SU(3) irreps

$^{96}_{42}Mo$

The weight vector w is:

w=(S_{10}-S_9, S_9-S_8, S_8-S_7, S_7-S_6, S_6-S_5, S_5-S_4, S_4-S_3, S_3-S_2, S_2-S_1, S_1)

w=(2,2,2,2,2,2,2,0,0,0)

For the weight vector the summations are:

$$f_1 = \sum_{i=1}^{10} n_{z,i} = 2(3+2+2+1+1+1) = 20$$

$$f_2 = \sum_{i=1}^{10} n_{x,i} = 2(0+1+0+2+1+0+3) = 14$$

$$f_3 = \sum_{i=1}^{10} n_{y,i} = 2(0+0+1+0+1+2+0) = 8$$

Young diagram:

$[f] = (f_1, f_2, f_3) = (20,14,8).$

The Elliott SU(3) irrep (λ, μ) is given by:

$$\lambda = f_1 - f_2 \quad \& \quad \mu = f_2 - f_3$$

$$(\lambda, \mu) = (6,6)$$

A (λ, μ) irrep with μ>0 corresponds to a wave function with mixed symmetry, while a (λ,0) irrep to a totally symmetric spatial state.

We define the ratio:

$$\boldsymbol{r(\lambda,\mu) = \frac{\lambda+\mu}{\lambda+2\mu} \cdot 100\%}$$

which measures the percentage of the symmetric quanta λ+μ out of the total number of quanta λ+2μ.