Relativistic Brueckner-Hartree-Fock Theory in Nuclear Matter and in Finite Systems

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TECHNIS

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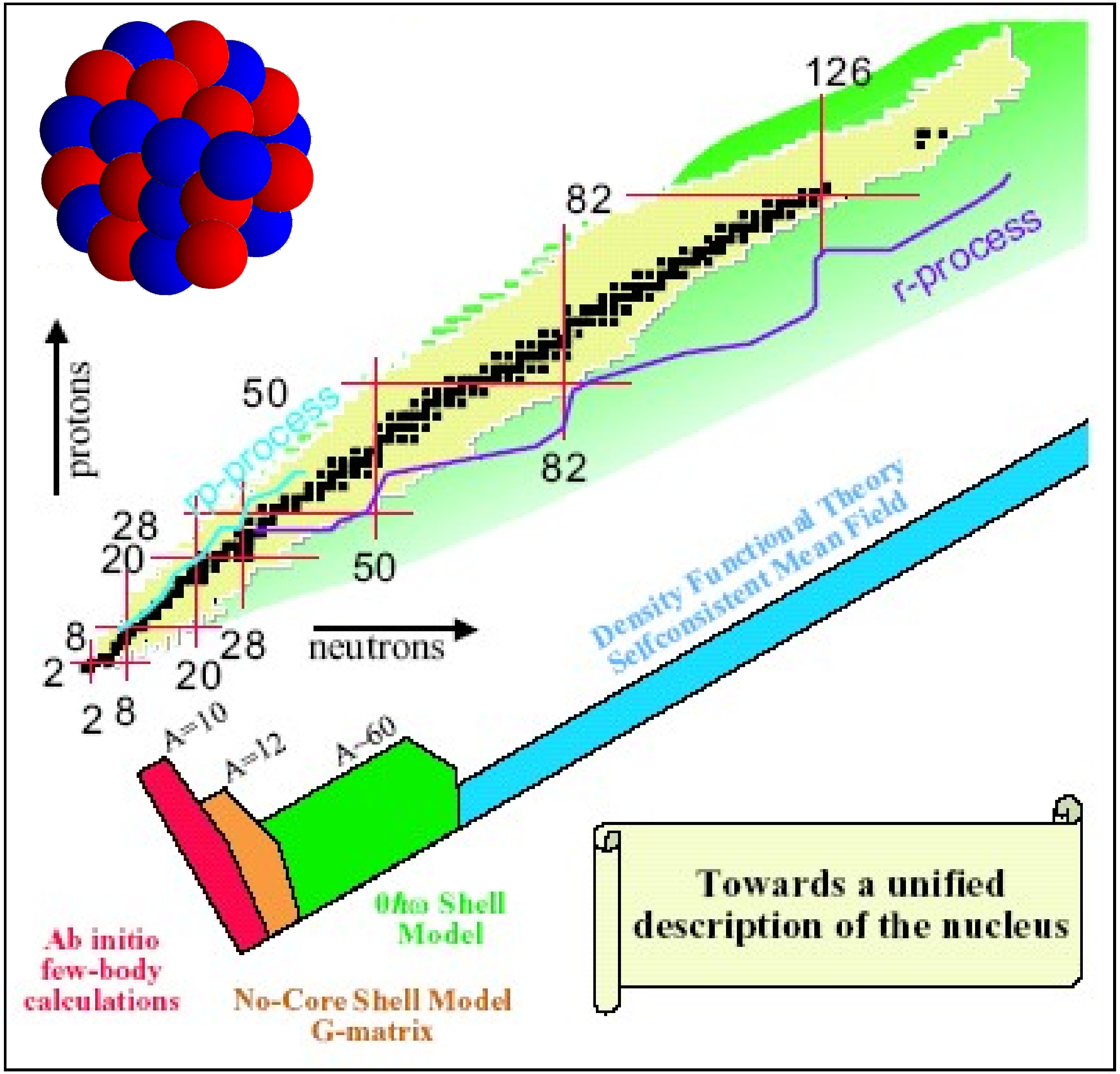
Content

- Motivation
- Relativistic Brueckner-Hartree-Fock Theory
- Applications in finite systems
- Full solution for symmetric nuclear matter

Motivation

the nuclear many-body problem:



- Ab-initio derivation of density functional theory

    first attempts of ab-initio go back to the fifties:

    Brueckner theory:

        based on mean field concept

        effective density-dep. interaction: G[ρ]

        mother of density functional theory

- Non-relativistic BHF fails: Three-body forces
- 1980: Relativistic BHF: no NNN necessary

  problems:

  a) no exact solution of RBHF in nuclear matter

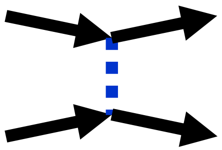
     many different approximations

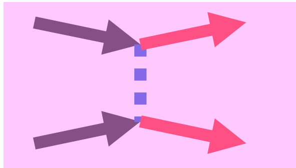
  b) no solution of RBHF in finite nuclei (tensor?)

Brueckner theory (1958):

Brueckner,Gammel, Phys. Rev. 109, 1023 (1958)

- The nucleons in the interior of the nuclear medium do not feel the same bare force V, as the nucleons feel in free space.
- They feel an effective force G.



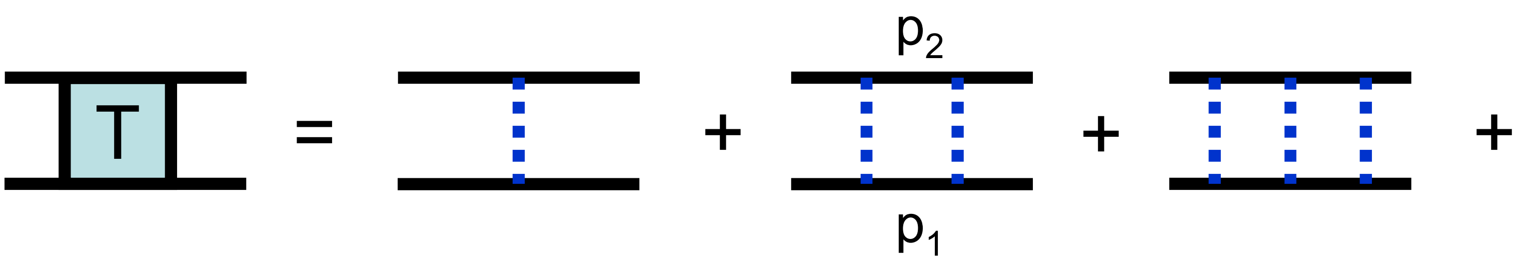


- The Pauli principle prohibits the scattering into states, which are already occupied in the medium.
- Therefore this force G(ρ) depends on the density
- This force G is much weaker than bare force V.
- Nucleons move nearly free in the nuclear medium and feel only a strong attraction at the surface (shell model)

Free nucleon-nucleon scattering:

Lippmann-Schwinger-Eq.

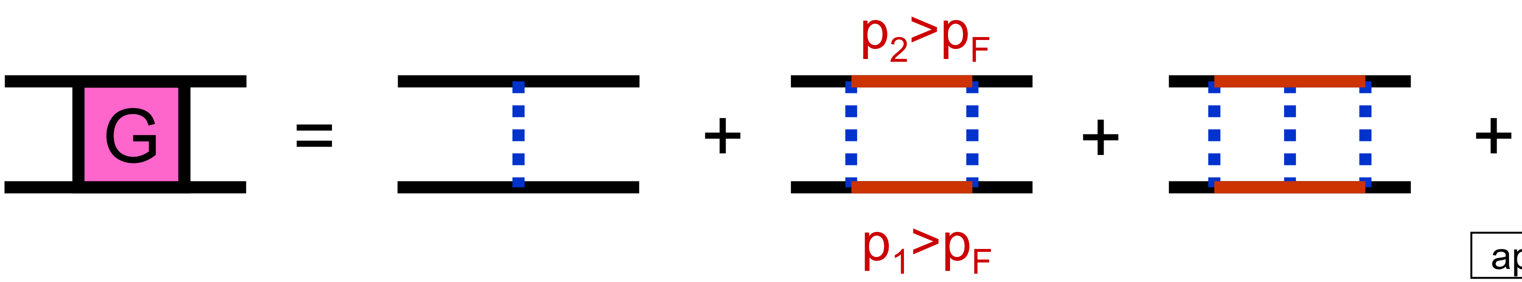
$$\langle \mathbf{k}_1\mathbf{k}_2|T|\mathbf{k}_1'\mathbf{k}_2'\rangle = \langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{k}_1'\mathbf{k}_2'\rangle + \sum_{\mathbf{p}_1\mathbf{p}_2} \langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{p}_1\mathbf{p}_2\rangle \frac{1}{E - \frac{\mathbf{p}_1^2}{2m} - \frac{\mathbf{p}_2^2}{2m} + i\eta} \langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{k}_1'\mathbf{k}_2'\rangle$$



exact

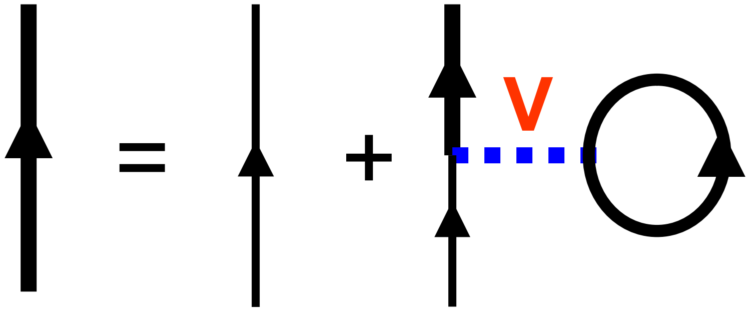
Scattering in the nuclear medium:

Bethe-Goldstone-Eq.

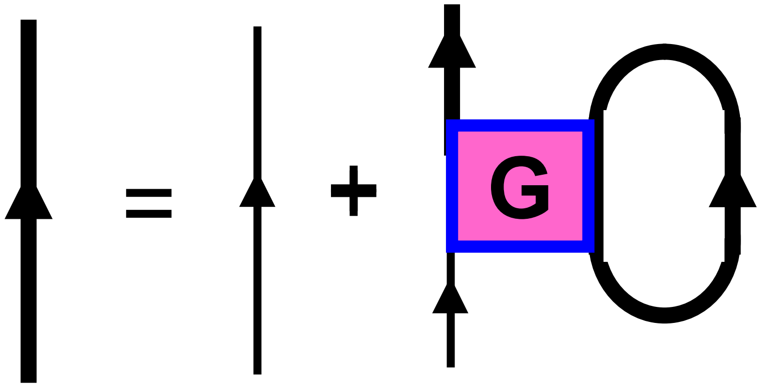


approximation

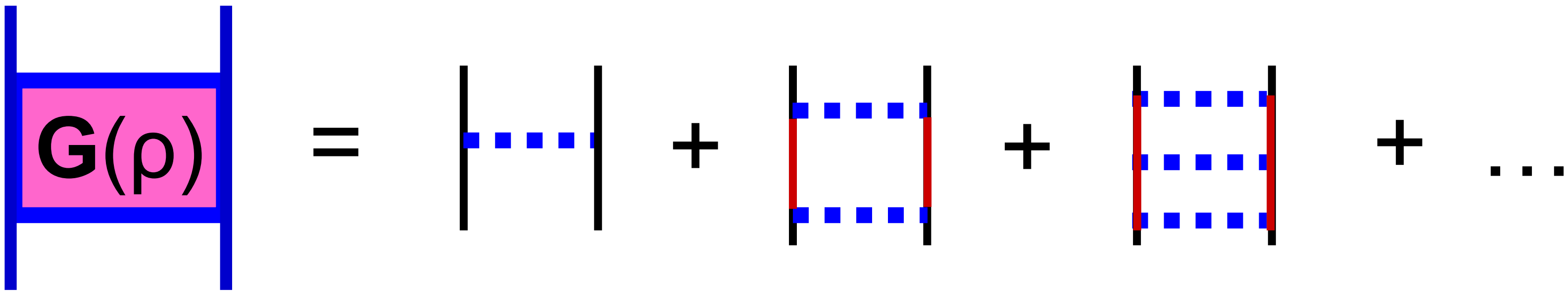
Ab-initio: Relativistic Brueckner Hartree-Fock:



Hartree-Fock

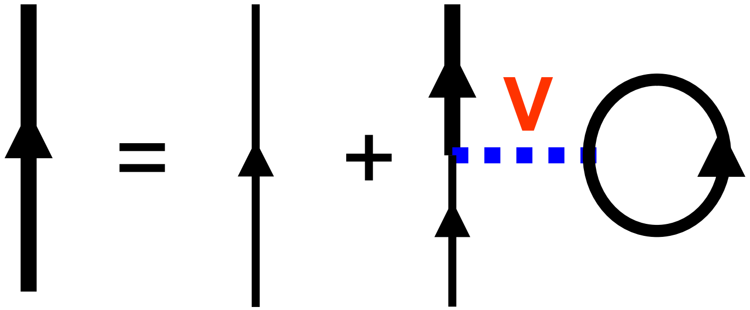


Brueckner Hartree-Fock

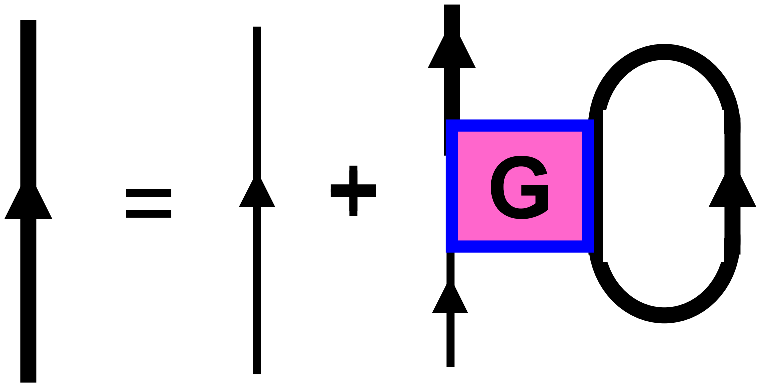


Summing up all ladder diagramms

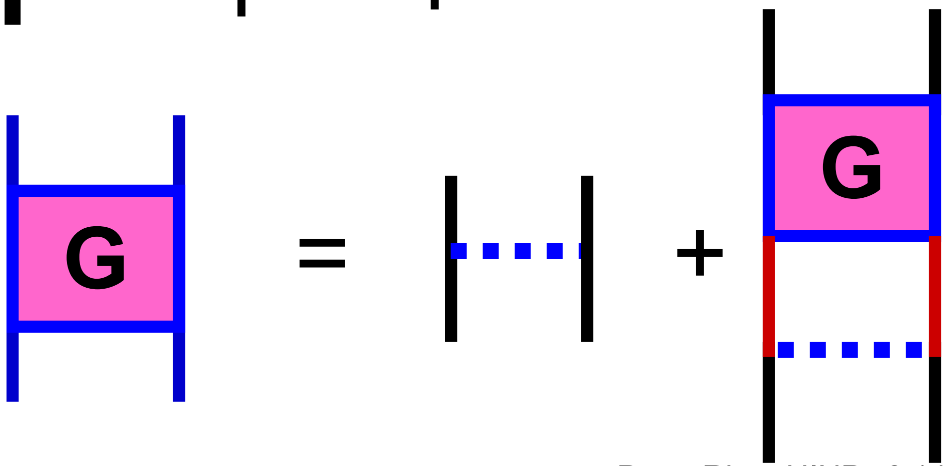
Ab-initio: Relativistic Brueckner Hartree-Fock:



Hartree-Fock



Brueckner Hartree-Fock

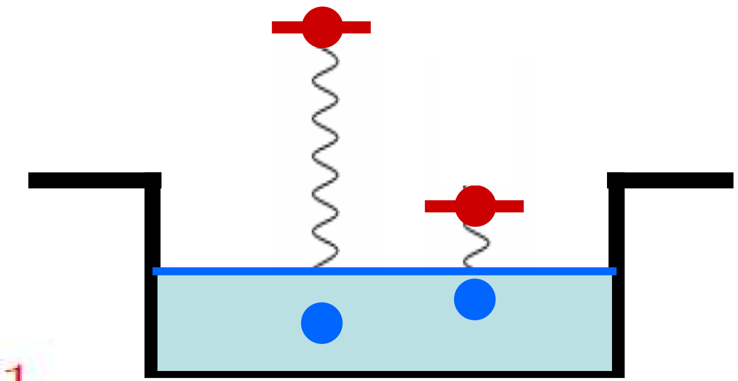


Bethe-Goldstone

Bethe-Goldstone equation:

- ω is the starting energy
- V is realistic interaction
- Q_F is the Pauli operator

$$G(\omega) = V + VQ_F \frac{1}{\omega - H_{HF}} Q_F G(\omega)$$



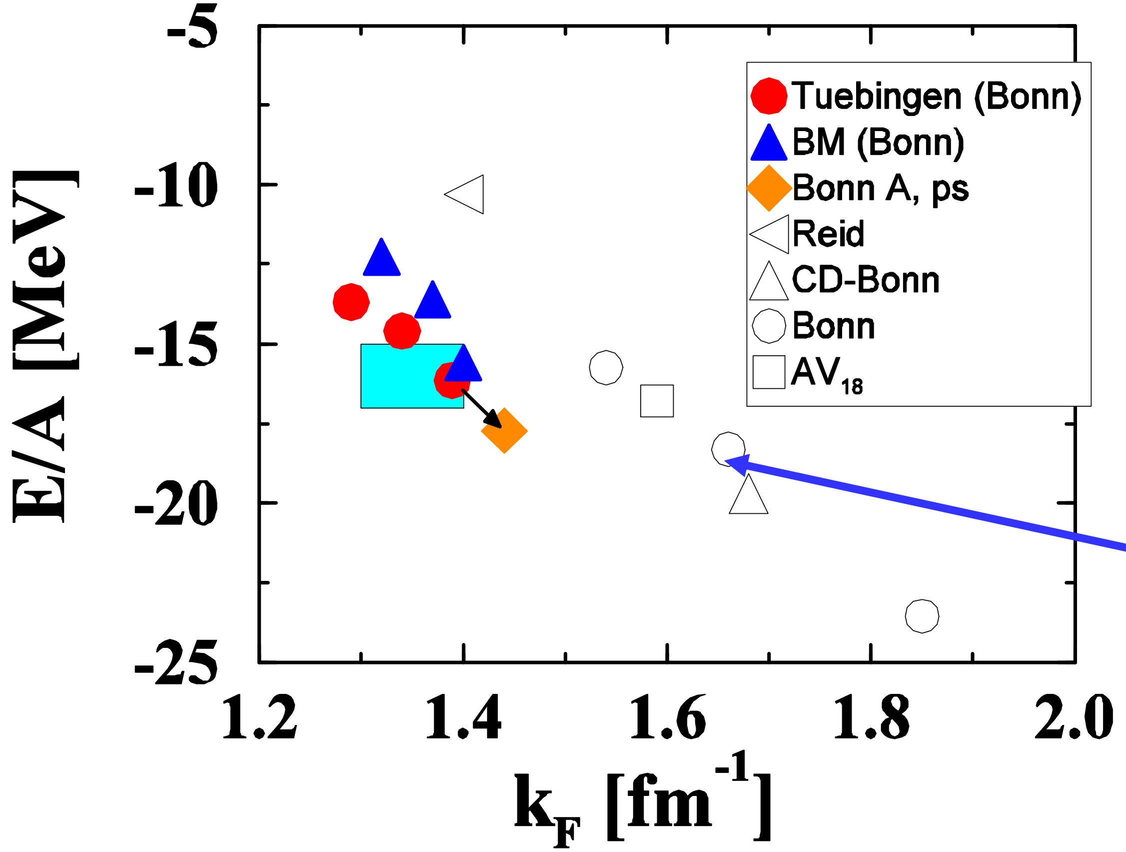
$$G(\omega) = V + VP_F(\omega)G(\omega)$$

$$P_F(\omega) = \sum_{m_1 m_2 > \varepsilon_F} |m_1 m_2\rangle \frac{1}{\omega - \varepsilon_{m_1} - \varepsilon_{m_2}} \langle m_1 m_2|$$

$$G(\omega) = \frac{1}{1 - VP_F(\omega)} V$$

Is solved in each step of the iteration

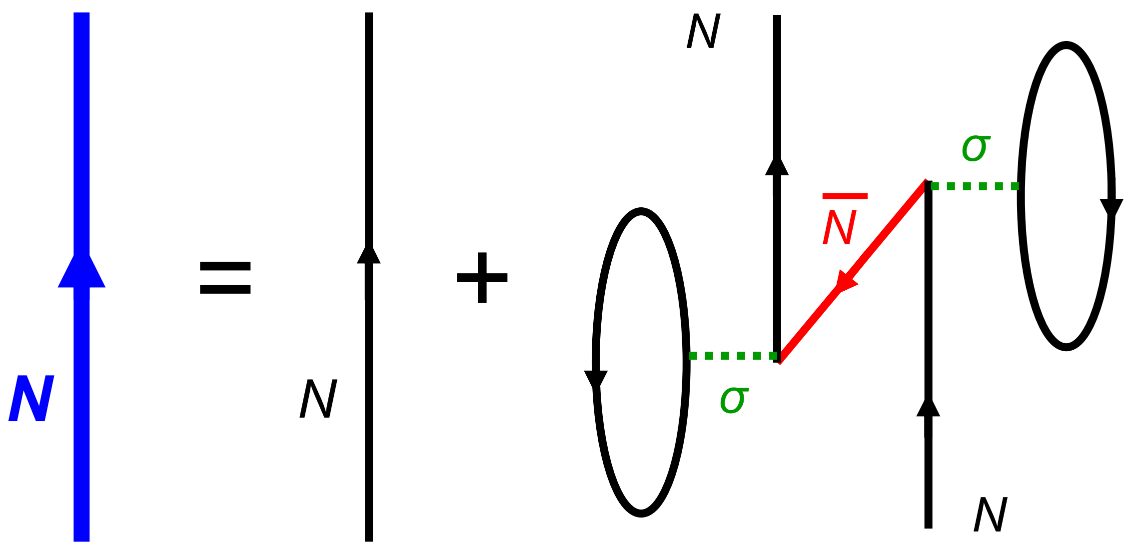
Dirac-Brueckner-Hartree-Fock in nuclear matter



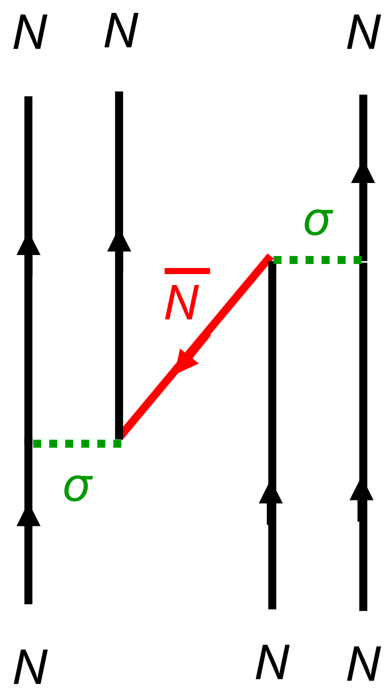
C. Fuchs, LNP (2004)

Coester-line

Non-rel. 3-body-forces and relativistic effects



eff. 3-body force



$$|\boldsymbol{u}(\boldsymbol{k},\lambda,m^*)\rangle = \alpha(m^*)|u(\boldsymbol{k},\lambda,m)\rangle + \beta(m^*)|\boldsymbol{v}(-\boldsymbol{k},-\lambda,m)\rangle$$

Dressed spinor — free spinor E>0 — free spinor E<0

$$\rightarrow \quad \frac{\Delta E}{A} \approx 4.2 \text{ MeV} \left(\frac{\rho}{\rho_0}\right)^{\frac{8}{3}}$$

Local density approximation (LDA):

1. solve the Brueckner-Hartree-Fock equations in nuclear matter at various densities ρ

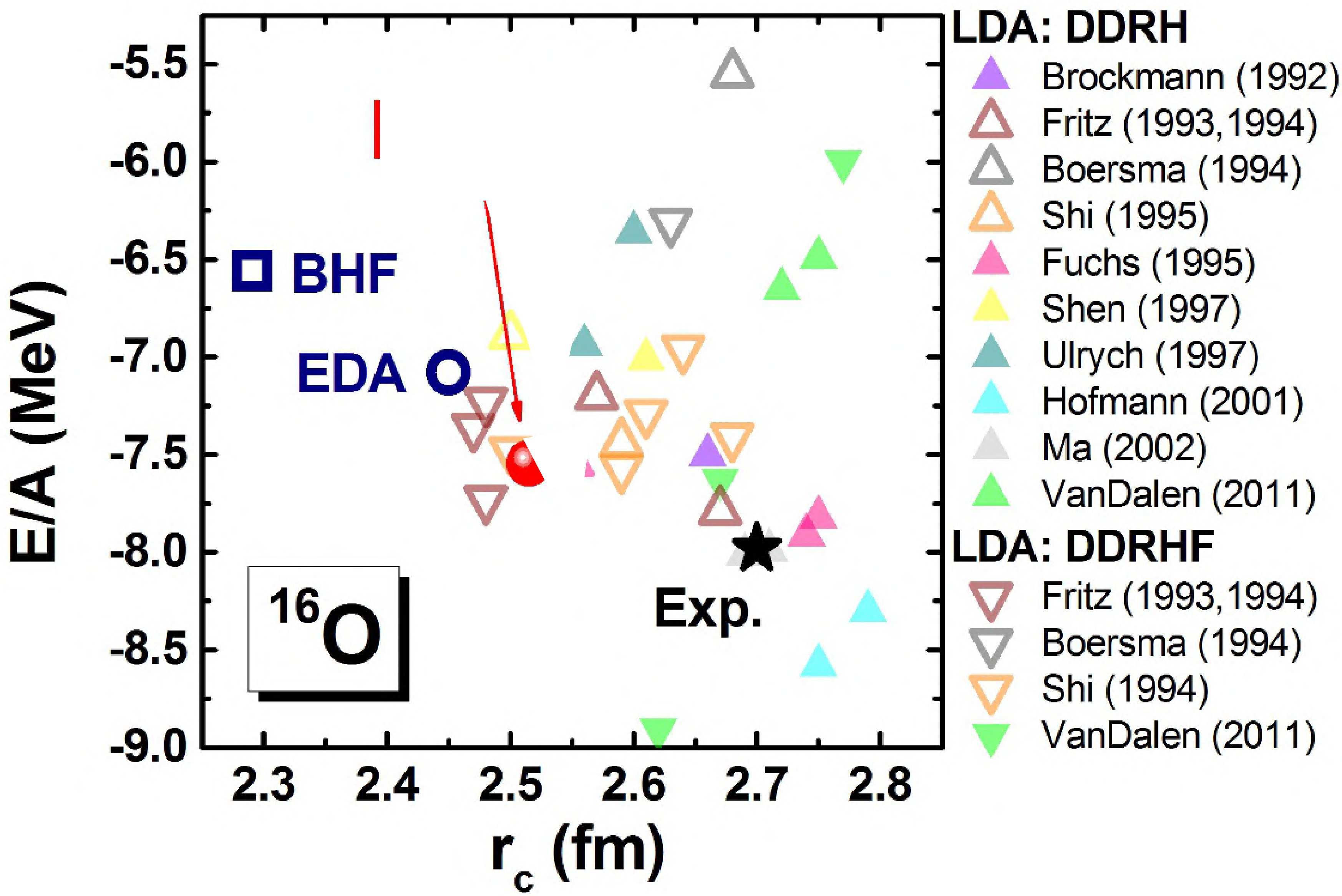
2. map the density dependent results on a Walecka model with density dependent couplings

3. this yields → $g_\sigma(\rho)$, $g_\omega(\rho)$, ….

4. but: this mapping is not unique !

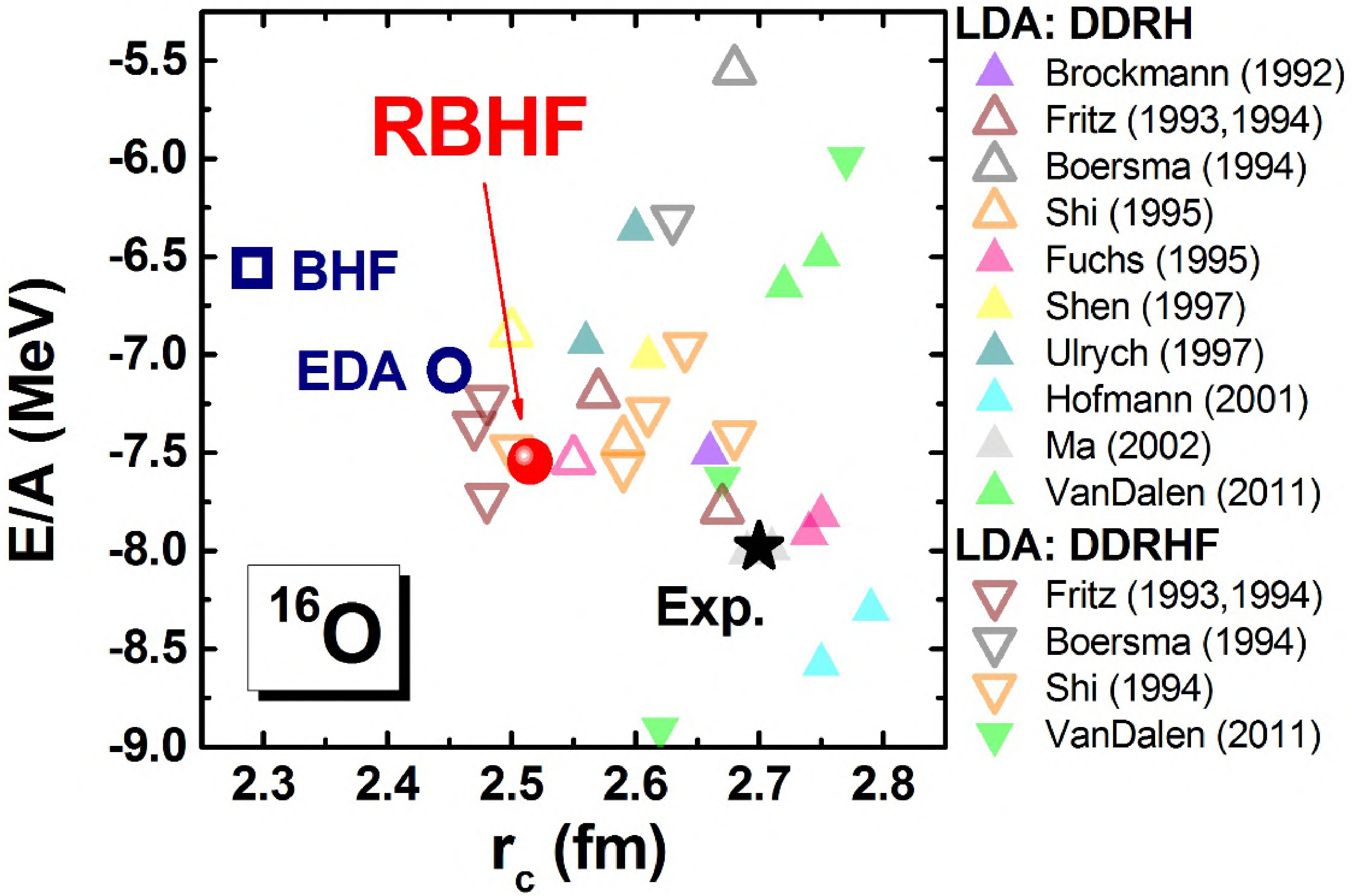
Brockmann and Toki, PRL **68**, 3408 (1992)

Relativistic BHf for finite nuclei:



Relativistic BHF for finite nuclei:

S.H. Shen et al (2017).



Problems of RBHF in finite nuclei:

1. Limitation to light spherical nuclei (^{16}O, Ca, …)
   limitation in memory
   limitation in time (no parallelization for inversion)

2. Future goal: Softening of the bare relativistic force
   relativistic V_{lowk} (derived in nuclear matter)

3. Problem (since 40 years):
   There is no full solution of RBHF in nuclear matter !

Relativistic Hartree-Fock in nucl. matter

$$H = H_0 + \Sigma = \beta M + \vec{\alpha}\vec{k} + \Sigma$$

Self-energy Σ in the Walecka model:

$$\Sigma = \beta S + V_0 + \vec{\alpha}\vec{V} = \begin{pmatrix} S + V_0 & \vec{\sigma}\vec{V} \\ \vec{\sigma}\vec{V} & -S + V_0 \end{pmatrix}$$

Self-energy Σ in BHF:

$$\Sigma_{12} = \sum_{34} G[\rho]_{1324}\rho_{43} = \begin{pmatrix} \Sigma^{++} & \Sigma^{+-} \\ \Sigma^{-+} & \Sigma^{--} \end{pmatrix}$$

Conventional solution of RBHF in nucl. Matter:

Thompson-equation: (3D reduction of the Bethe-Salpeter Equation)

$$T^{++++}(E) = V^{++++} + V^{++++} \frac{1}{E - E_{kin}} T^{++++}(E)$$

Bethe-Goldstone equation

$$G^{++++}(W) = V^{++++} + V^{++++} \frac{Q}{W - E_{56}} G^{++++}(W)$$

Self energy:

$$\Sigma_{12}^{++} = \sum_{34} G_{1324}^{++++} \rho_{43}^{++} \qquad \Sigma^{-+} = ???, \qquad \Sigma^{--} = ???$$

Approximations for Σ^{-+}, Σ^{--} …

Perturbation theory: Anastasio et al, PRC 23 (1981)

Projection onto Lorentz invariants: Horowitz et al NPA 464 (1987)

Greens-function techniques: Weigel et al, PRC 38 (1988)

Momentum dependence of $\Sigma^{++}(p)$ is used to determine S and V_0
Brockmann et al, PRC 42 (1990)

Effective DBHF-method, Schiller et al, EPJA 11 (2001)

Full solution: De Jong, Lenske PRC 58 (1998)
Katayama et al, PLB 747 (2015)

Full solution for G⁺⁺⁺⁺, G⁺⁻⁺⁺, G⁻⁻⁺⁺, …

$$G^{-+++}(W) = V^{-+++} + V^{-+++}\frac{Q}{W - E_{56}}G^{++++}(W)$$

$$^0G_J^{-+++} = {}^0V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}}\frac{Q_{\rm av}}{W - 2E_{\rm av}}\left[{}^0V_J^{-+++}\cdot{}^0G_J^{++++} + {}^2V_J^{-+++}\cdot{}^3G_J^{++++}\right],$$

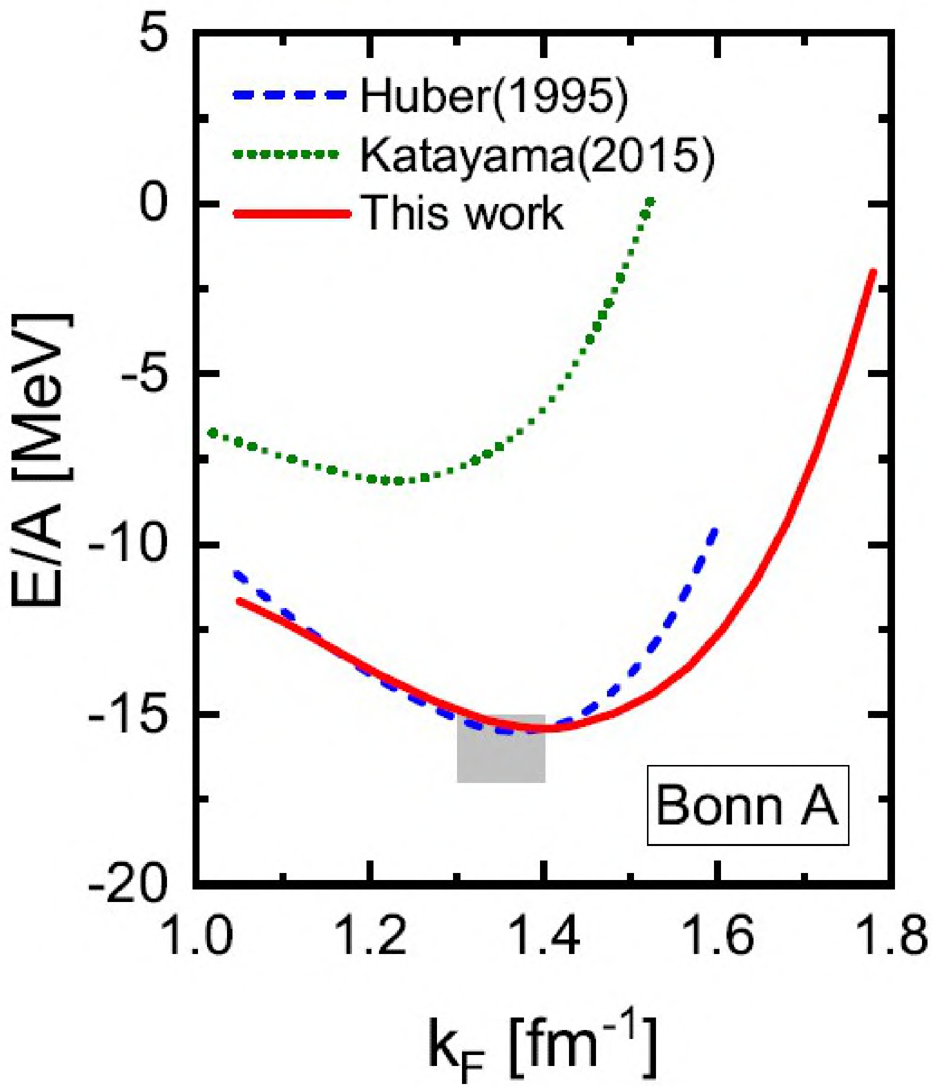
$$^1G_J^{-+++} = {}^1V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}}\frac{Q_{\rm av}}{W - 2E_{\rm av}}\left[{}^3V_J^{-+++}\cdot{}^2G_J^{++++} + {}^1V_J^{-+++}\cdot{}^1G_J^{++++}\right],$$

$$^2G_J^{-+++} = {}^2V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}}\frac{Q_{\rm av}}{W - 2E_{\rm av}}\left[{}^0V_J^{-+++}\cdot{}^2G_J^{++++} + {}^2V_J^{-+++}\cdot{}^1G_J^{++++}\right],$$

$$^3G_J^{-+++} = {}^3V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}}\frac{Q_{\rm av}}{W - 2E_{\rm av}}\left[{}^3V_J^{-+++}\cdot{}^0G_J^{++++} + {}^1V_J^{-+++}\cdot{}^3G_J^{++++}\right],$$

Sibo Wang, Ziang Zhao, P.R. and Jie Meng, PRC in print

Results for symmetric nuclear matter:



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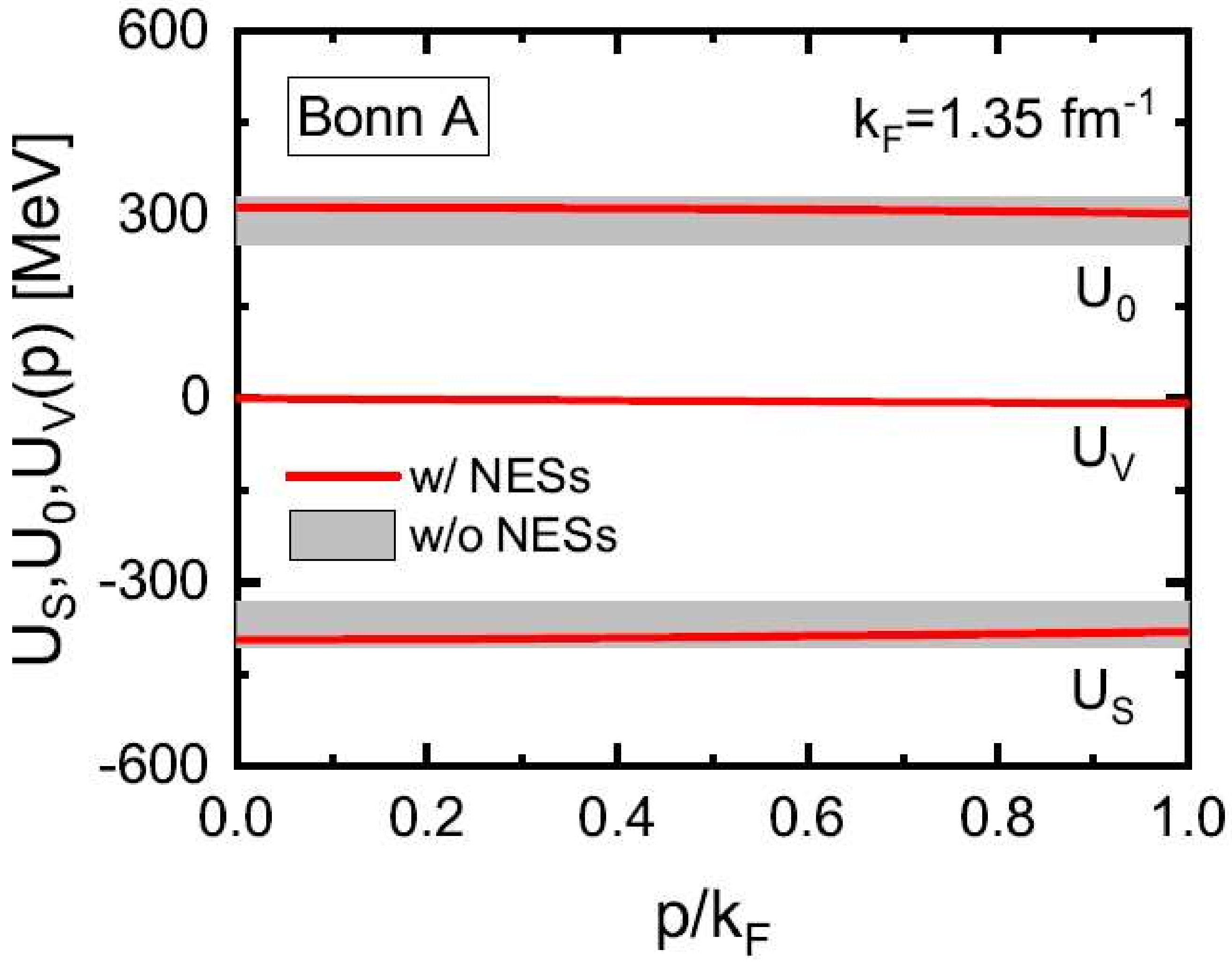
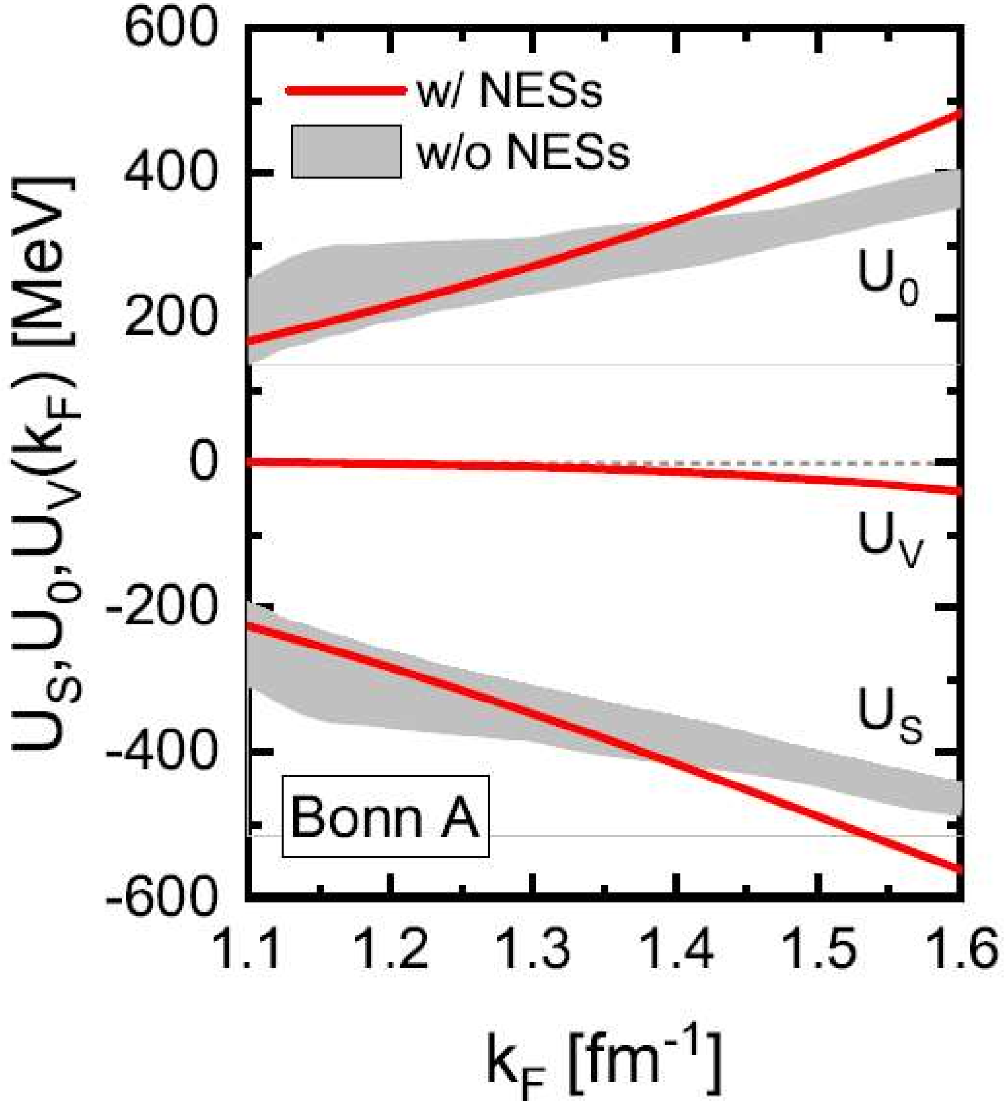
Momentum dependence:

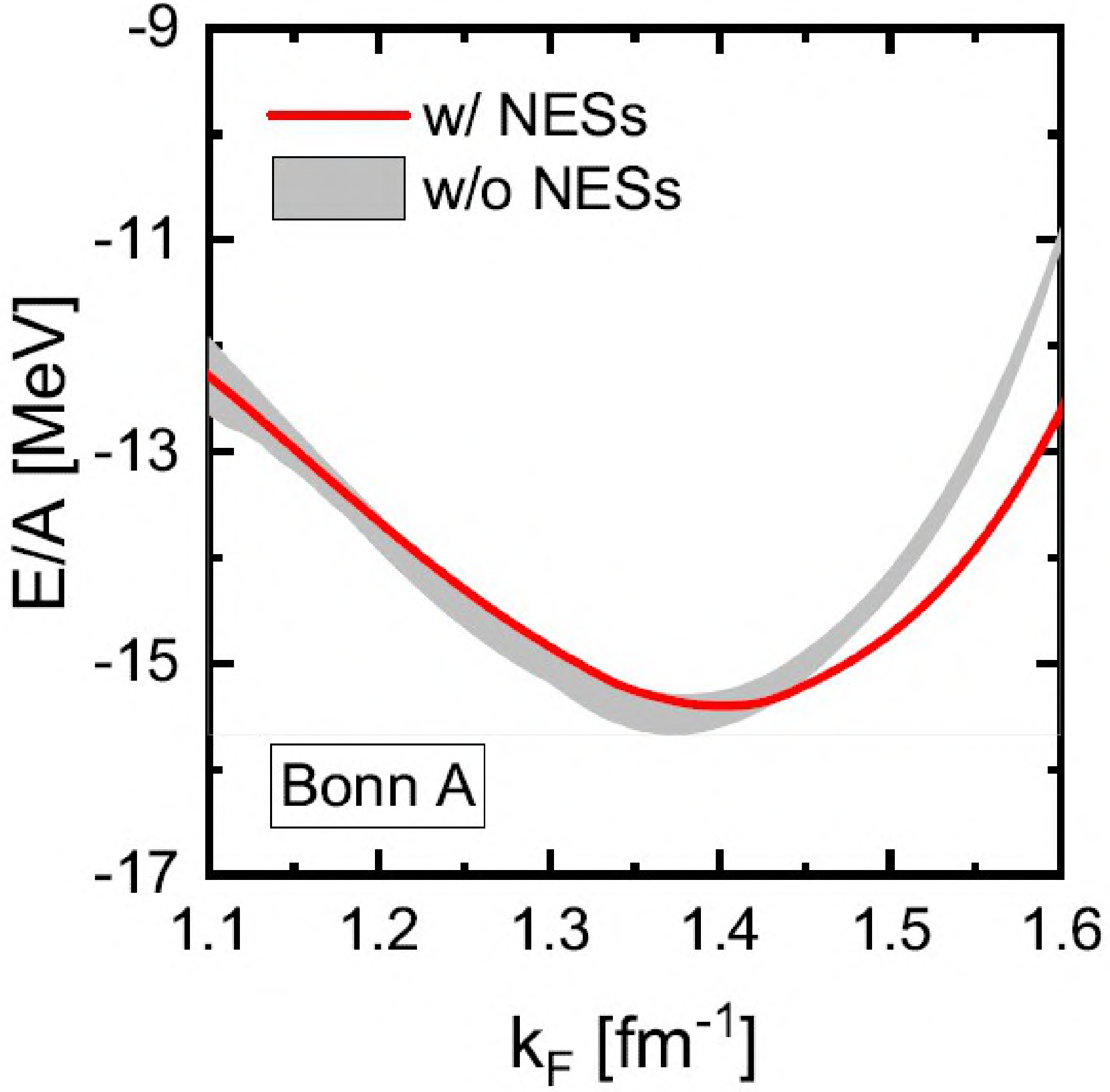
$$\Sigma(p) \;=\; \beta U_S(p) + U_0(p) + \vec{\alpha}\hat{\vec{p}}U_V(p)\ldots$$



Density dependence:



Equation of state:



Properties of symmetric nuclear matter:

Potential	ρ_0 [fm^{-3}]	E/A [MeV]	K_∞ [MeV]	M_D^*/M
RBHF Bonn A	0.188	-15.40	258	0.55
RBHF Bonn B	0.164	-13.36	206	0.61
RBHF Bonn C	0.144	-12.09	150	0.65
BHF Bonn A	0.428	-23.55	204	
BHF Bonn B	0.309	-18.30	160	
BHF Bonn C	0.247	-15.75	103	
NL3	0.148	-16.30	272	0.60
DD-ME2	0.152	-16.14	251	0.57
DD-PC1	0.152	-16.06	230	0.58
PC-PK1	0.154	-16.12	238	0.59
PKO1	0.152	-16.00	250	0.59
Empirical	0.16 ±0.01	-16±1	240±20	

Conclusions:

- RBHF is a successful microscopic tool
- Full succesful solution in nuclear matter was missing

  This gap is now solved

  Exact results are in agreement with earlier approximations

How to improve the results?

- Other relativistic NN-forces ?
- Relativistic NNN-forces ?
- Extended Brueckner theory (3 hole lines …)?
- …

Outlook for the future:

- Full solution for assymetric nuclear matter
- simplify the calculations:

  Brueckner theory with renormalized forces ($V_{low\ k}$) …

  Local density approximation under control
- heavy nuclei and the tensor force
- open shell nuclei: pairing, deformation
- optical potential
- short range correlations