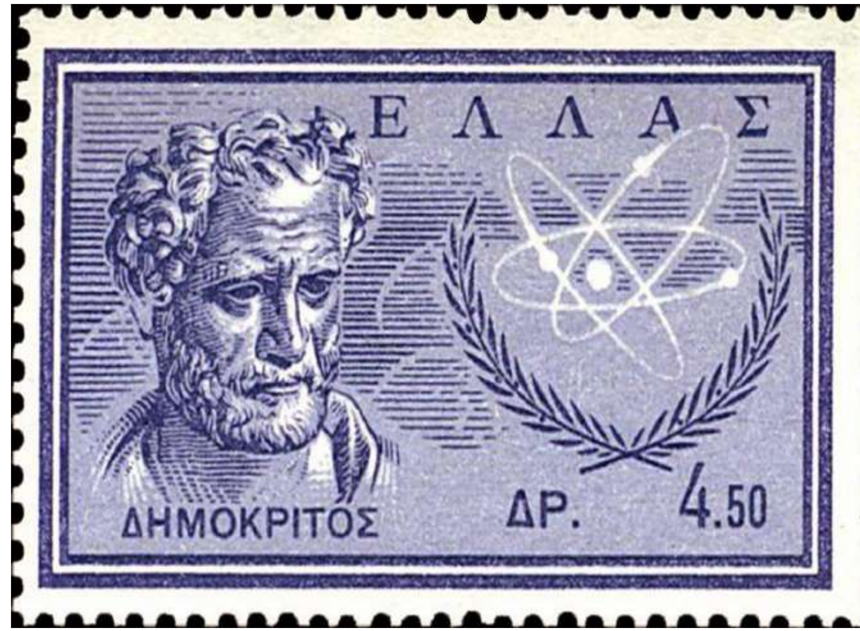
Connecting the proxy-SU(3) symmetry to the shell model

Dennis Bonatsos
INPP, NCSR Demokritos

Nuclear Research Centre Demokritos 1961



The people behind the work

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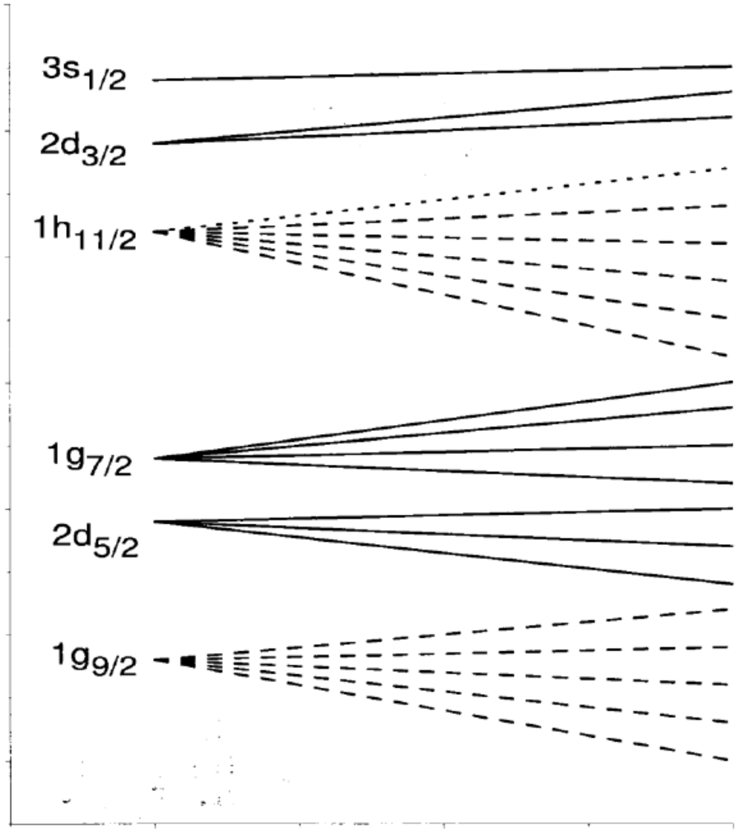
proxy-SU(3) 1h11/2->1g9/2

pseudo-SU(3) sdg -> pf



proxy-SU(3) replacements

Nilsson 0[110] pairs	ΔK[ΔN ΔNz ΔΛ]
1h11/2	1g9/2
1/2[550]	1/2[440]
3/2[541]	3/2[431]
5/2[532]	5/2[422]
7/2[523]	7/2[413]
9/2[514]	9/2[404]
11/2[505]	

50-82 shell

orbitals left out of the symmetry

pseudo-SU(3): 1/2[550], 3/2[541], 5/2[532],
7/2[523], 9/2[514], 11/2[505]

proxy-SU(3): 11/2[505] (at the top)

Nilsson model

H-7

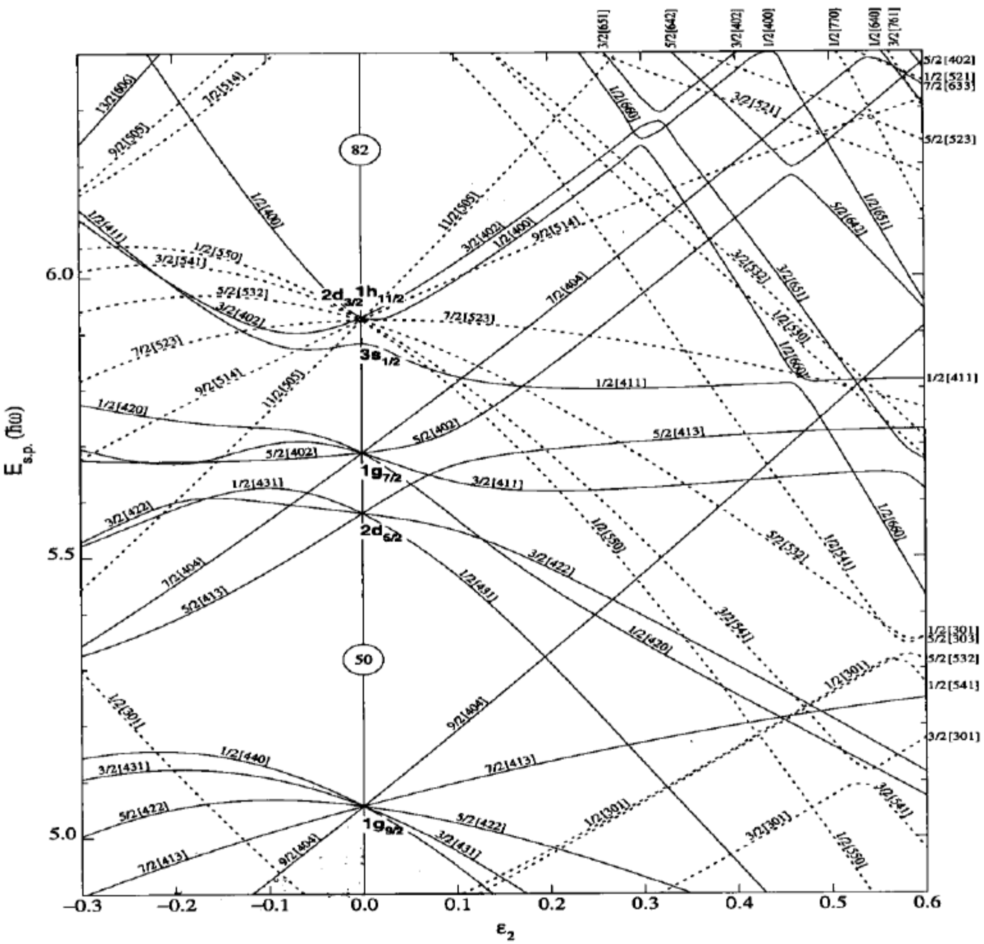


Figure 5. Nilsson diagram for neutrons, $50 \leq N \leq 82$ ($\varepsilon_4 = \varepsilon_2^2/6$).

14164

Proxy-SU(3)

Uses Nilsson 0[110] pairs   ΔK[ΔN ΔNz ΔΛ]

First used for proton-neutron interaction

R.B. Cakirli, K. Blaum, and R.F. Casten,

Phys. Rev. C 82 (2010) 061304(R)

Same angular momentum content

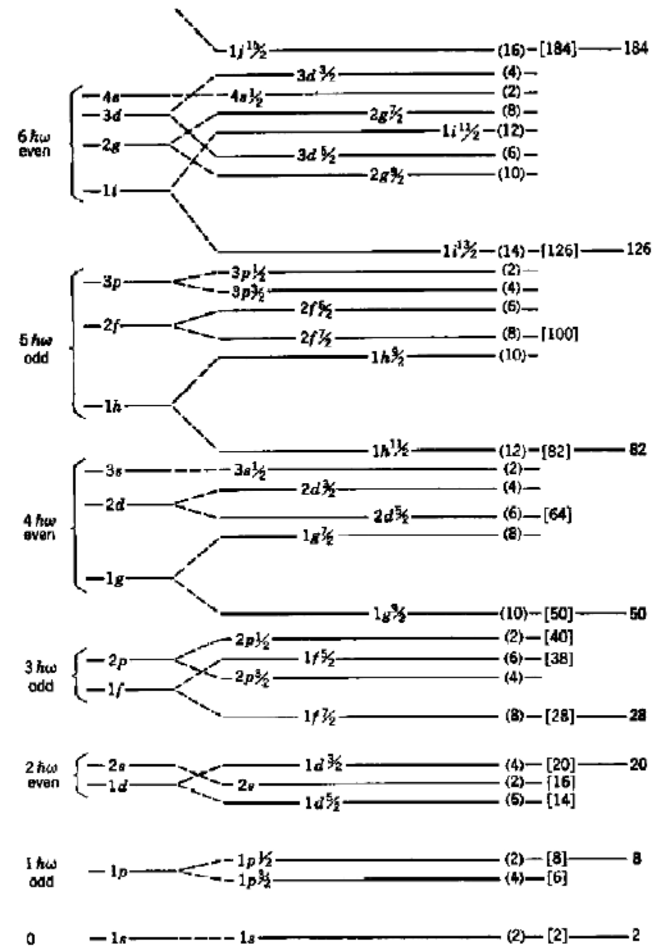
Large overlaps

D. B., S. Karampagia, R.B. Cakirli, R.F. Casten, K. Blaum, L. Amon Susam, Phys. Rev. C 88 (2013) 054309

Matrix of the Nilsson H for the sdg proton shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[440]	$\frac{3}{2}$[431]	$\frac{5}{2}$[422]	$\frac{7}{2}$[413]	$\frac{9}{2}$[404]
1/2[400]	6.28	−0.13	0	0.22	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		5.74	0	0.18	0	0	0.27	0	0	0	0	0	0	0	0
3/2[402]			6.26	0	0.16	0	0	0.19	0	0	0	0	0	0	0
1/2[420]				5.30	0	0	−0.16	0	0	0	**0.27**	0	0	0	0
3/2[411]					5.61	0	0	−0.13	0	0	0	**0.27**	0	0	0
5/2[402]						6.00	0	0	−0.09	0	0	0	**0.19**	0	0
1/2[431]							5.06	0	0	0	**0.18**	0	0	0	0
3/2[422]								5.27	0	0	0	**0.22**	0	0	0
5/2[413]									5.56	0	0	0	**0.22**	0	0
7/2[404]										5.93	0	0	0	**0.18**	0
1/2[440]											**5.73**	0	0	0	0
3/2[431]												**5.74**	0	0	0
5/2[422]													**5.82**	0	0
7/2[413]														**5.98**	0
9/2[404]															**6.22**

Shell model basis |n L J Mj>



Nilsson model — basis $K[N\ N_z\ \Lambda]$

pairs $0[110]$ — $\Delta K[\Delta N\ \Delta N_z\ \Delta\Lambda]$

shell model — basis $|n\ L\ J\ M_j\rangle$

pairs ??? — $|\Delta n\ \Delta L\ \Delta J\ \Delta M_j\rangle$

Elliott SU(3)

sd shell

J.P. Elliott, Proc. Roy. Soc. Ser. A 245 (1958) 128, 562

J.P. Elliott and M. Harvey, 272 (1963) 557

classification in terms of SU(3)

cartesian basis [Nz Nx Ny Ms]

Elliott to shell model basis

[Nz Nx Ny Ms] = R [n L M Ms]

R: unitary transformation

Davies and Krieger, Can. J. Phys. 69 (1991) 62

[n L M Ms] = C | n L J Mj>

C: Clebsch Gordan coefficients

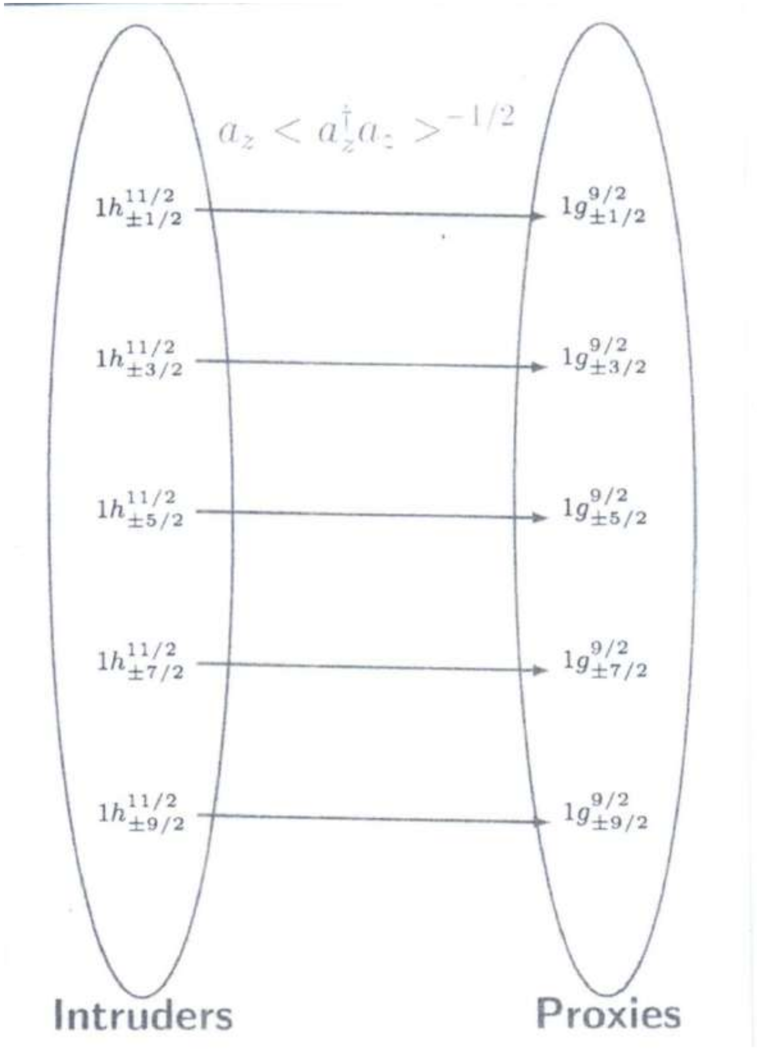
[Nz Nx Ny Ms] = R C | n L J Mj>

Elliott shell model

Table 3 The same as Table 1, but for $\mathcal{N}=2$, related the harmonic oscillator shell 8–20 (sd shell), or to the proxy-SU(3) shell 14–26

$\|n_z, n_x, n_y, m_s\rangle \|n, l, j, m_j\rangle$	$\|2s^{1/2}_{-1/2}\rangle$	$\|2s^{1/2}_{1/2}\rangle$	$\|1d^{3/2}_{-3/2}\rangle$	$\|1d^{3/2}_{-1/2}\rangle$	$\|1d^{3/2}_{1/2}\rangle$	$\|1d^{3/2}_{3/2}\rangle$	$\|1d^{5/2}_{-5/2}\rangle$	$\|1d^{5/2}_{-3/2}\rangle$	$\|1d^{5/2}_{-1/2}\rangle$	$\|1d^{5/2}_{1/2}\rangle$	$\|1d^{5/2}_{3/2}\rangle$	$\|1d^{5/2}_{5/2}\rangle$
$\|0,0,2,-\frac{1}{2}\rangle$	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{15}}$	0	$-\frac{1}{\sqrt{5}}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{10}}$	0	$-\frac{1}{2\sqrt{5}}$	0
$\|0,0,2,\frac{1}{2}\rangle$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{5}}$	0	$\frac{1}{\sqrt{15}}$	0	0	$-\frac{1}{2\sqrt{5}}$	0	$-\frac{1}{\sqrt{10}}$	0	$-\frac{1}{2}$
$\|0,1,1,-\frac{1}{2}\rangle$	0	0	0	0	0	$-i\sqrt{\frac{2}{5}}$	$\frac{i}{\sqrt{2}}$	0	0	0	$-\frac{i}{\sqrt{10}}$	0
$\|0,1,1,\frac{1}{2}\rangle$	0	0	$-i\sqrt{\frac{2}{5}}$	0	0	0	0	$\frac{i}{\sqrt{10}}$	0	0	0	$-\frac{i}{\sqrt{2}}$
$\|0,2,0,-\frac{1}{2}\rangle$	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{15}}$	0	$\frac{1}{\sqrt{5}}$	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{10}}$	0	$\frac{1}{2\sqrt{5}}$	0
$\|0,2,0,\frac{1}{2}\rangle$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{5}}$	0	$\frac{1}{\sqrt{15}}$	0	0	$\frac{1}{2\sqrt{5}}$	0	$-\frac{1}{\sqrt{10}}$	0	$\frac{1}{2}$
$\|1,0,1,-\frac{1}{2}\rangle$	0	0	$\frac{i}{\sqrt{10}}$	0	$i\sqrt{\frac{3}{10}}$	0	0	$i\sqrt{\frac{2}{5}}$	0	$\frac{i}{\sqrt{5}}$	0	0
$\|1,0,1,\frac{1}{2}\rangle$	0	0	0	$-i\sqrt{\frac{3}{10}}$	0	$-\frac{i}{\sqrt{10}}$	0	0	$\frac{i}{\sqrt{5}}$	0	$i\sqrt{\frac{2}{5}}$	0
$\|1,1,0,-\frac{1}{2}\rangle$	0	0	$\frac{1}{\sqrt{10}}$	0	$-\sqrt{\frac{3}{10}}$	0	0	$\sqrt{\frac{2}{5}}$	0	$-\frac{1}{\sqrt{5}}$	0	0
$\|1,1,0,\frac{1}{2}\rangle$	0	0	0	$-\sqrt{\frac{3}{10}}$	0	$\frac{1}{\sqrt{10}}$	0	0	$\frac{1}{\sqrt{5}}$	0	$-\sqrt{\frac{2}{5}}$	0
$\|2,0,0,-\frac{1}{2}\rangle$	$-\frac{1}{\sqrt{3}}$	0	0	$\frac{2}{\sqrt{15}}$	0	0	0	0	$\sqrt{\frac{2}{3}}$	0	0	0
$\|2,0,0,\frac{1}{2}\rangle$	0	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{2}{\sqrt{15}}$	0	0	0	0	$\sqrt{\frac{2}{3}}$	0	0

unitary transformation
|0 1 1 0> pairs |Δn ΔL ΔJ ΔMj>



Proxy-SU(3) pairs

Nilsson model basis K[N Nz Λ]

pairs 0[110] ΔK[ΔN ΔNz ΔΛ]

shell model basis |n L J Mj>

pairs |0 1 1 0> |Δn ΔL ΔJ ΔMj>

de Shalit – Goldhaber pairs

A. de Shalit and M. Goldhaber, PR 92 (1953) 1211

β transition probabilities

maximum interaction

neutrons	1i13/2	1h11/2	1g9/2	1f7/2	1d5/2
protons	1h11/2	1g9/2	1f7/2	1d5/2	1p3/2

|0 1 1 0> pairs |Δn ΔL ΔJ MΔj>

Table 1 Expansions of Nilsson orbitals $\Omega[Nn_z\Lambda]$ in the shell model basis $|Nlj\Omega\rangle$ for three different values of the deformation ϵ

$\frac{3}{2}[541]$ $\lvert Nlj\Omega\rangle$ ϵ	$\lvert 51\frac{3}{2}\frac{3}{2}\rangle$	$\lvert 53\frac{5}{2}\frac{3}{2}\rangle$	$\lvert 53\frac{7}{2}\frac{3}{2}\rangle$	$\lvert 55\frac{9}{2}\frac{3}{2}\rangle$	$\lvert 55\frac{11}{2}\frac{3}{2}\rangle$
0.05	0.0025	−0.0015	0.0641	−0.0122	0.9979
0.22	0.0371	−0.0286	0.2565	−0.0640	0.9633
0.30	0.0601	−0.0506	0.3287	−0.0922	0.9366

$\frac{3}{2}[651]$ $\lvert Nlj\Omega\rangle$ ϵ	$\lvert 62\frac{3}{2}\frac{3}{2}\rangle$	$\lvert 62\frac{5}{2}\frac{3}{2}\rangle$	$\lvert 64\frac{7}{2}\frac{3}{2}\rangle$	$\lvert 64\frac{9}{2}\frac{3}{2}\rangle$	$\lvert 66\frac{11}{2}\frac{3}{2}\rangle$	$\lvert 66\frac{13}{2}\frac{3}{2}\rangle$
0.05	−0.0002	0.0046	−0.0013	0.0821	−0.0086	0.9966
0.22	−0.0100	0.0711	−0.0278	0.3240	−0.0469	0.9418
0.30	−0.0207	0.1149	−0.0509	0.4091	−0.0687	0.9010

The Nilsson orbitals shown possess the highest total angular momentum j in their shell. The existence of a leading shell model eigenvector is evident at all deformations. See Sect. 5 for further discussion

future

shell model calculations

taking advantage of

the proxy-SU(3) symmetry

LONG VERSION

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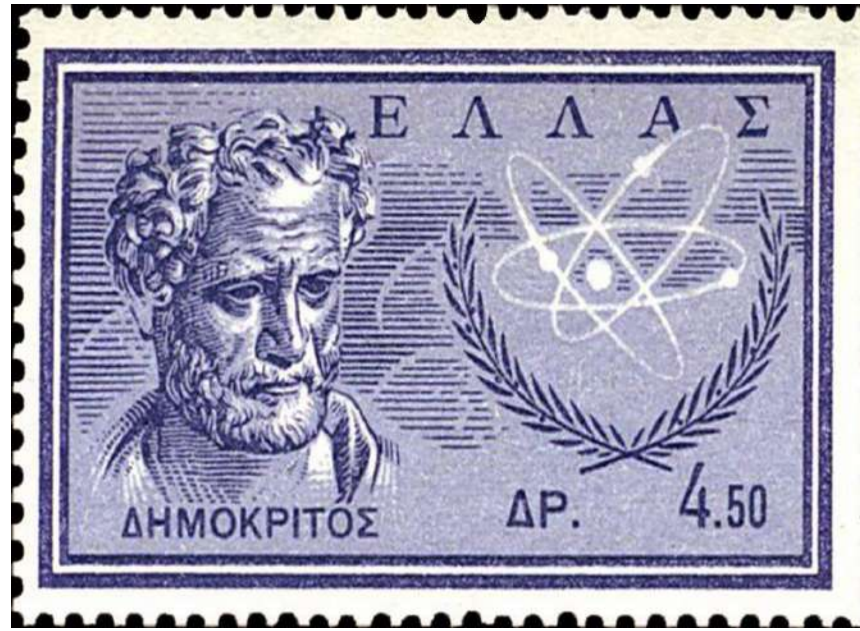

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H. Hassanabadi (Iran)

- Elliott -> shell model

  A. Martinou, D.B., N. Minkov, I.E. Assimakis, S.K. Peroulis, S. Sarantopoulou, J. Cseh, EPJA 56 (2020) 239

- Nilsson -> shell model

  D.B., H. Sobhani, H. Hassanabadi, EPJP 135 (2020) 710

(ambitious) plan

- Proxy-SU(3) in Nilsson model

  0[110] pairs ΔK[ΔN ΔNz ΔΛ]
- Elliott SU(3), cartesian coordinates

  -> shell model, spherical coordinates
- de Shalit – Goldhaber pairs

  |0 1 1 0> pairs |Δn ΔL ΔJ ΔMj>
- Comparison to pseudo-SU(3)
- Shell model calculations with proxy-SU(3)

Proxy-SU(3)

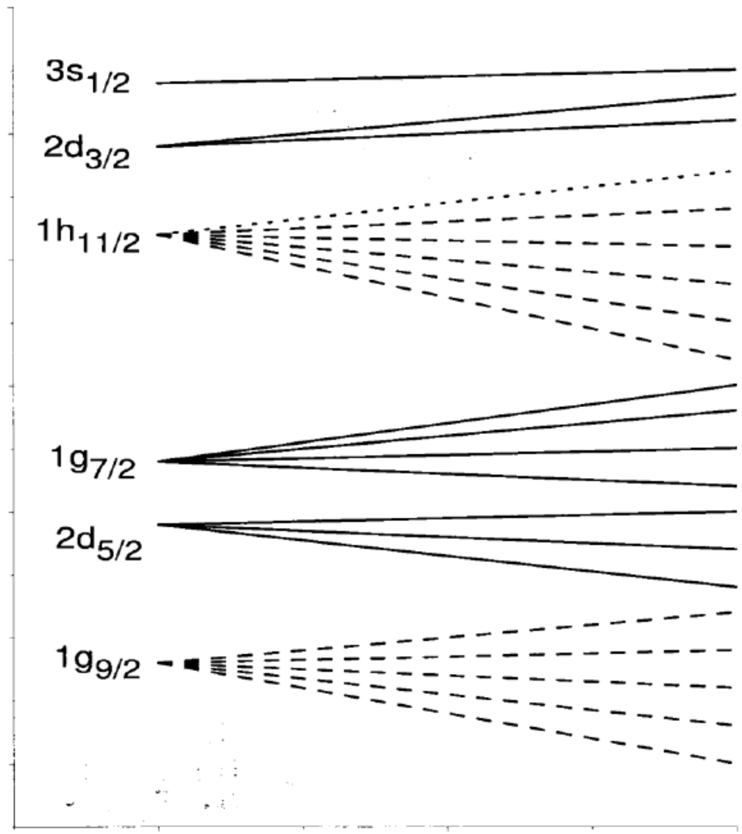
- Proxy-SU(3) symmetry

  D. Bonatsos, I. E. Assimakis, N. Minkov, A. Martinou, R. B. Cakirli, R. F. Casten, and K. Blaum, Phys. Rev. C 95 (2017) 064325

- Nuclear shapes, prolate-oblate shape transition

  D. Bonatsos, I. E. Assimakis, N. Minkov, A. Martinou, S. Sarantopoulou, R. B. Cakirli, R. F. Casten, and K. Blaum, Phys. Rev. C 95 (2017) 064326

Proxy-SU(3) 50-82 shell



proxy-SU(3) replacements

Nilsson 0[110] pairs	ΔK[ΔN ΔNz ΔΛ]
1h11/2	1g9/2
1/2[550]	1/2[440]
3/2[541]	3/2[431]
5/2[532]	5/2[422]
7/2[523]	7/2[413]
9/2[514]	9/2[404]
11/2[505]	

Proxy-SU(3)

Uses Nilsson 0[110] pairs $\Delta K[\Delta N\ \Delta N_z\ \Delta\Lambda]$

First used for proton-neutron interaction

R.B. Cakirli, K. Blaum, and R.F. Casten,
Phys. Rev. C 82 (2010) 061304(R)

Same angular momentum content

Large overlaps

D. B., S. Karampagia, R.B. Cakirli, R.F. Casten, K. Blaum, L. Amon Susam, Phys. Rev. C 88 (2013) 054309

Why does proxy-SU(3) work?

Compare

usual Nilsson calculation

proxy-SU(3) calculation

Few and small extra matrix elements

Nilsson model

$$H = H_{osc} + v_{ls}\hbar\omega_0(\mathbf{l}\cdot\mathbf{s}) + v_{ll}\hbar\omega_0(\mathbf{l}^2 - \langle \mathbf{l}^2\rangle_N)$$

$$H_{osc} = \frac{\mathbf{p}^2}{2M} + \frac{1}{2}M(\omega_z^2 z^2 + \omega_\perp^2(x^2+y^2))$$

$$E_{osc} = \hbar\omega_0\left(N + \frac{3}{2} - \frac{1}{3}\epsilon(3n_z - N)\right)$$

$$\langle \mathbf{l}^2\rangle_N = \frac{1}{2}N(N+3)$$

$$\omega_z = \omega_0\left(1 - \frac{2}{3}\epsilon\right) \qquad \omega_\perp = \omega_0\left(1 + \frac{1}{3}\epsilon\right) \qquad \epsilon = \frac{\omega_\perp - \omega_z}{\omega_0}$$

Matrix of the Nilsson H for the 50-82 proton shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[550]	$\frac{3}{2}$[541]	$\frac{5}{2}$[532]	$\frac{7}{2}$[523]	$\frac{9}{2}$[514]	$\frac{11}{2}$[505]
1/2[400]	6.28	−0.13	0	0.22	0	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		5.74	0	0.18	0	0	0.27	0	0	0	0	0	0	0	0	0
3/2[402]			6.26	0	0.16	0	0	0.19	0	0	0	0	0	0	0	0
1/2[420]				5.30	0	0	−0.16	0	0	0	0	0	0	0	0	0
3/2[411]					5.61	0	0	−0.13	0	0	0	0	0	0	0	0
5/2[402]						6.00	0	0	−0.09	0	0	0	0	0	0	0
1/2[431]							5.06	0	0	0	0	0	0	0	0	0
3/2[422]								5.27	0	0	0	0	0	0	0	0
5/2[413]									5.56	0	0	0	0	0	0	0
7/2[404]										5.93	0	0	0	0	0	0
1/2[550]											5.88	0	0	0	0	0
3/2[541]												5.81	0	0	0	0
5/2[532]													5.82	0	0	0
7/2[523]														5.90	0	0
9/2[514]															6.06	0
11/2[505]																6.30

Matrix of the Nilsson H for the sdg proton shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[440]	$\frac{3}{2}$[431]	$\frac{5}{2}$[422]	$\frac{7}{2}$[413]	$\frac{9}{2}$[404]
1/2[400]	6.28	−0.13	0	0.22	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		5.74	0	0.18	0	0	0.27	0	0	0	0	0	0	0	0
3/2[402]			6.26	0	0.16	0	0	0.19	0	0	0	0	0	0	0
1/2[420]				5.30	0	0	−0.16	0	0	0	**0.27**	0	0	0	0
3/2[411]					5.61	0	0	−0.13	0	0	0	**0.27**	0	0	0
5/2[402]						6.00	0	0	−0.09	0	0	0	**0.19**	0	0
1/2[431]							5.06	0	0	0	**0.18**	0	0	0	0
3/2[422]								5.27	0	0	0	**0.22**	0	0	0
5/2[413]									5.56	0	0	0	**0.22**	0	0
7/2[404]										5.93	0	0	0	**0.18**	0
1/2[440]											**5.73**	0	0	0	0
3/2[431]												**5.74**	0	0	0
5/2[422]													**5.82**	0	0
7/2[413]														**5.98**	0
9/2[404]															**6.22**

Nilsson model	basis K[N Nz Λ]
pairs 0[110]	ΔK[ΔN ΔNz ΔΛ]
shell model	basis |n L J Mj>
pairs ???	|Δn ΔL ΔJ ΔMj>

Elliott SU(3)

sd shell

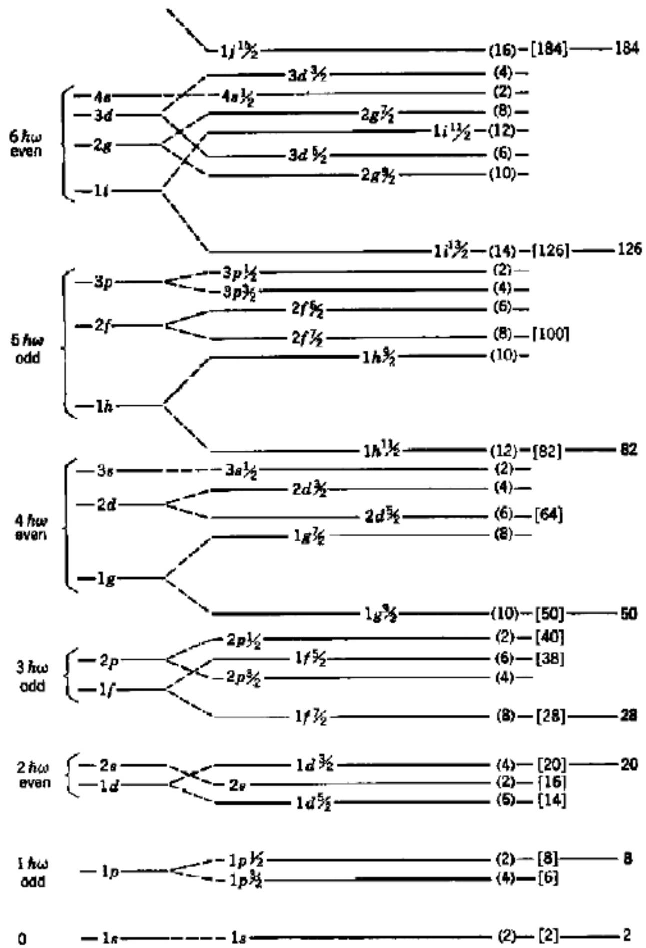
J.P. Elliott, Proc. Roy. Soc. Ser. A 245 (1958) 128, 562

J.P. Elliott and M. Harvey, 272 (1963) 557

classification in terms of SU(3)

cartesian basis [Nz Nx Ny Ms]

Shell model basis |n L J Mj>



Elliott to shell model basis

[Nz Nx Ny Ms] = R [n L M Ms]

R: unitary transformation

Davies and Krieger, Can. J. Phys. 69 (1991) 62

[n L M Ms] = C | n L J Mj>

C: Clebsch Gordan coefficients

[Nz Nx Ny Ms] = R C | n L J Mj>

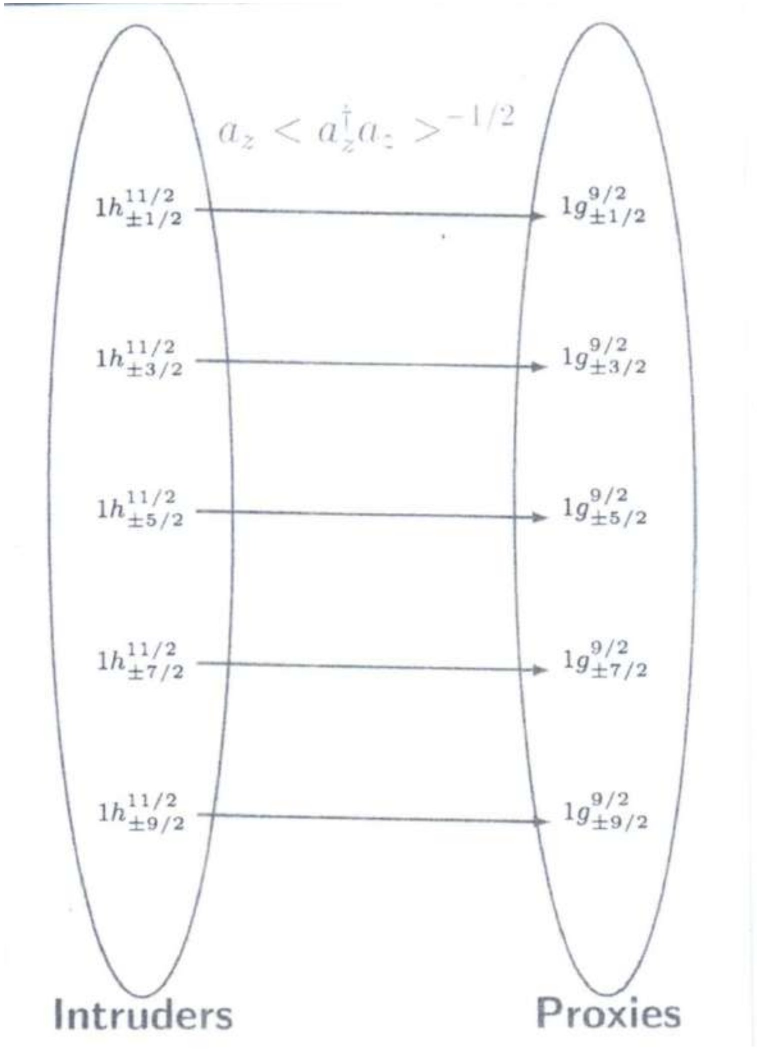
Elliott shell model

Table 3 The same as Table 1, but for $\mathcal{N} = 2$, related the harmonic oscillator shell 8–20 (sd shell), or to the proxy-SU(3) shell 14–26

$\lvert n_z, n_x, n_y, m_s\rangle \lvert n, l, j, m_j\rangle$	$\lvert 2s^{1/2}_{-1/2}\rangle$	$\lvert 2s^{1/2}_{1/2}\rangle$	$\lvert 1d^{3/2}_{-3/2}\rangle$	$\lvert 1d^{3/2}_{-1/2}\rangle$	$\lvert 1d^{3/2}_{1/2}\rangle$	$\lvert 1d^{3/2}_{3/2}\rangle$	$\lvert 1d^{5/2}_{-5/2}\rangle$	$\lvert 1d^{5/2}_{-3/2}\rangle$	$\lvert 1d^{5/2}_{-1/2}\rangle$	$\lvert 1d^{5/2}_{1/2}\rangle$	$\lvert 1d^{5/2}_{3/2}\rangle$	$\lvert 1d^{5/2}_{5/2}\rangle$
$\lvert 0,0,2,-\frac{1}{2}\rangle$	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{15}}$	0	$-\frac{1}{\sqrt{5}}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{10}}$	0	$-\frac{1}{2\sqrt{5}}$	0
$\lvert 0,0,2,\frac{1}{2}\rangle$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{5}}$	0	$\frac{1}{\sqrt{15}}$	0	0	$-\frac{1}{2\sqrt{5}}$	0	$-\frac{1}{\sqrt{10}}$	0	$-\frac{1}{2}$
$\lvert 0,1,1,-\frac{1}{2}\rangle$	0	0	0	0	0	$-i\sqrt{\frac{2}{5}}$	$\frac{i}{\sqrt{2}}$	0	0	0	$-\frac{i}{\sqrt{10}}$	0
$\lvert 0,1,1,\frac{1}{2}\rangle$	0	0	$-i\sqrt{\frac{2}{5}}$	0	0	0	0	$\frac{i}{\sqrt{10}}$	0	0	0	$-\frac{i}{\sqrt{2}}$
$\lvert 0,2,0,-\frac{1}{2}\rangle$	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{15}}$	0	$\frac{1}{\sqrt{5}}$	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{10}}$	0	$\frac{1}{2\sqrt{5}}$	0
$\lvert 0,2,0,\frac{1}{2}\rangle$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{5}}$	0	$\frac{1}{\sqrt{15}}$	0	0	$\frac{1}{2\sqrt{5}}$	0	$-\frac{1}{\sqrt{10}}$	0	$\frac{1}{2}$
$\lvert 1,0,1,-\frac{1}{2}\rangle$	0	0	$\frac{i}{\sqrt{10}}$	0	$i\sqrt{\frac{3}{10}}$	0	0	$i\sqrt{\frac{2}{5}}$	0	$\frac{i}{\sqrt{5}}$	0	0
$\lvert 1,0,1,\frac{1}{2}\rangle$	0	0	0	$-i\sqrt{\frac{3}{10}}$	0	$-\frac{i}{\sqrt{10}}$	0	0	$\frac{i}{\sqrt{5}}$	0	$i\sqrt{\frac{2}{5}}$	0
$\lvert 1,1,0,-\frac{1}{2}\rangle$	0	0	$\frac{1}{\sqrt{10}}$	0	$-\sqrt{\frac{3}{10}}$	0	0	$\sqrt{\frac{2}{5}}$	0	$-\frac{1}{\sqrt{5}}$	0	0
$\lvert 1,1,0,\frac{1}{2}\rangle$	0	0	0	$-\sqrt{\frac{3}{10}}$	0	$\frac{1}{\sqrt{10}}$	0	0	$\frac{1}{\sqrt{5}}$	0	$-\sqrt{\frac{2}{5}}$	0
$\lvert 2,0,0,-\frac{1}{2}\rangle$	$-\frac{1}{\sqrt{3}}$	0	0	$\frac{2}{\sqrt{15}}$	0	0	0	0	$\sqrt{\frac{2}{5}}$	0	0	0
$\lvert 2,0,0,\frac{1}{2}\rangle$	0	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{2}{\sqrt{15}}$	0	0	0	0	$\sqrt{\frac{2}{5}}$	0	0

unitary transformation

|0 1 1 0> pairs |Δn ΔL ΔJ ΔMj>



de Shalit – Goldhaber pairs

A. de Shalit and M. Goldhaber, PR 92 (1953) 1211

β transition probabilities

maximum interaction

neutrons	1i13/2	1h11/2	1g9/2	1f7/2	1d5/2
protons	1h11/2	1g9/2	1f7/2	1d5/2	1p3/2

|0 1 1 0> pairs

|Δn ΔL ΔJ MΔj>

Proxy-SU(3) pairs

Nilsson model basis K[N Nz Λ]

pairs 0[110] ΔK[ΔN ΔNz ΔΛ]

shell model basis |n L J Mj>

pairs |0 1 1 0> |Δn ΔL ΔJ ΔMj>

- Elliott -> shell model

  A. Martinou, D.B., N. Minkov, I.E. Assimakis, S.K. Peroulis, S. Sarantopoulou, J. Cseh, EPJA 56 (2020) 239

- Nilsson -> shell model

  D.B., H. Sobhani, H. Hassanabadi, EPJP 135 (2020) 710

Table 1 Expansions of Nilsson orbitals $\Omega[Nn_z\Lambda]$ in the shell model basis $|Nlj\Omega\rangle$ for three different values of the deformation ϵ

$\frac{3}{2}[541]$

$\lvert Nlj\Omega\rangle$ / ϵ	$\lvert 51\frac{3}{2}\frac{3}{2}\rangle$	$\lvert 53\frac{5}{2}\frac{3}{2}\rangle$	$\lvert 53\frac{7}{2}\frac{3}{2}\rangle$	$\lvert 55\frac{9}{2}\frac{3}{2}\rangle$	$\lvert 55\frac{11}{2}\frac{3}{2}\rangle$
0.05	0.0025	−0.0015	0.0641	−0.0122	0.9979
0.22	0.0371	−0.0286	0.2565	−0.0640	0.9633
0.30	0.0601	−0.0506	0.3287	−0.0922	0.9366

$\frac{3}{2}[651]$

$\lvert Nlj\Omega\rangle$ / ϵ	$\lvert 62\frac{3}{2}\frac{3}{2}\rangle$	$\lvert 62\frac{5}{2}\frac{3}{2}\rangle$	$\lvert 64\frac{7}{2}\frac{3}{2}\rangle$	$\lvert 64\frac{9}{2}\frac{3}{2}\rangle$	$\lvert 66\frac{11}{2}\frac{3}{2}\rangle$	$\lvert 66\frac{13}{2}\frac{3}{2}\rangle$
0.05	−0.0002	0.0046	−0.0013	0.0821	−0.0086	0.9966
0.22	−0.0100	0.0711	−0.0278	0.3240	−0.0469	0.9418
0.30	−0.0207	0.1149	−0.0509	0.4091	−0.0687	0.9010

The Nilsson orbitals shown possess the highest total angular momentum j in their shell. The existence of a leading shell model eigenvector is evident at all deformations. See Sect. 5 for further discussion

Table 3 Expansions of Nilsson orbitals $\Omega[Nn_z\Lambda]$ in the shell model basis $|Nlj\Omega\rangle$ for three different values of the deformation ϵ

$\frac{1}{2}[431]$					
$\lvert Nlj\Omega\rangle$ / ϵ	$\lvert 40\frac{1}{2}\frac{1}{2}\rangle$	$\lvert 42\frac{3}{2}\frac{1}{2}\rangle$	$\lvert 42\frac{5}{2}\frac{1}{2}\rangle$	$\lvert 44\frac{7}{2}\frac{1}{2}\rangle$	$\lvert 44\frac{9}{2}\frac{1}{2}\rangle$
0.05	−0.0213	0.1254	−0.0702	0.9893	0.0127
0.22	−0.2248	0.4393	−0.2791	0.8057	0.1717
0.30	−0.2630	0.5003	−0.2458	0.7447	0.2559

$\frac{1}{2}[541]$						
$\lvert Nlj\Omega\rangle$ / ϵ	$\lvert 51\frac{1}{2}\frac{1}{2}\rangle$	$\lvert 51\frac{3}{2}\frac{1}{2}\rangle$	$\lvert 53\frac{5}{2}\frac{1}{2}\rangle$	$\lvert 53\frac{7}{2}\frac{1}{2}\rangle$	$\lvert 55\frac{9}{2}\frac{1}{2}\rangle$	$\lvert 55\frac{11}{2}\frac{1}{2}\rangle$
0.05	−0.0200	0.1770	−0.0295	0.9780	−0.0446	−0.0944
0.22	−0.2492	0.4619	−0.3768	0.5550	−0.4161	−0.3185
0.30	−0.3121	0.4331	−0.4829	0.3430	−0.4789	−0.3671

The Nilsson orbitals shown do not possess the highest total angular momentum j in their shell. The existence of a leading shell model eigenvector is evident at small deformation, but this is not the case anymore at higher deformations, at which several shell model eigenvectors make considerable contributions. See Sect. 7 for further discussion

Pseudo-SU(3)

R.D. Ratna Raju, J. P. Draayer, and K. T. Hecht, Nucl. Phys. A 202 (1973) 433

J.P. Draayer, K.J. Weeks, and K.T. Hecht, Nucl. Phys. A 381 (1982) 1

Pseudo-SU(3)

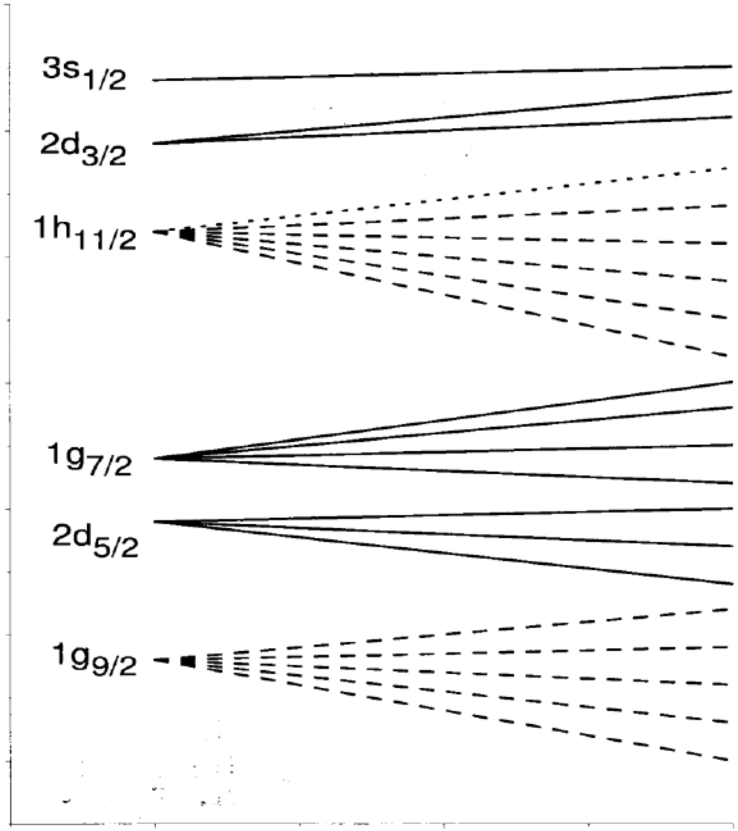
- Map the levels of normal parity through unitary transformation
- Leave levels of intruder parity unchanged

Proxy-SU(3)

- Map the levels of intruder parity through unitary transformation
- Leave levels of normal parity unchanged

proxy-SU(3) 1h11/2->1g9/2

pseudo-SU(3) sdg -> pf



approximation schemes

Shell model	proxy-SU(3)	pseudo-SU(3)
28-50	pf U(10)	sd U6)+1g9/2
50-82	sdg U(15)	pf U(10)+1h11/2
82-126	pfh U(21)	sdg U(15)+1i13/2
126-184	sdgi U(28)	pfh U(21)+1j15/2

50-82 shell

orbitals left out of the symmetry

pseudo-SU(3): 1/2[550], 3/2[541], 5/2[532],
7/2[523], 9/2[514], 11/2[505]

proxy-SU(3): 11/2[505] (at the top)

Nilsson model

H-7

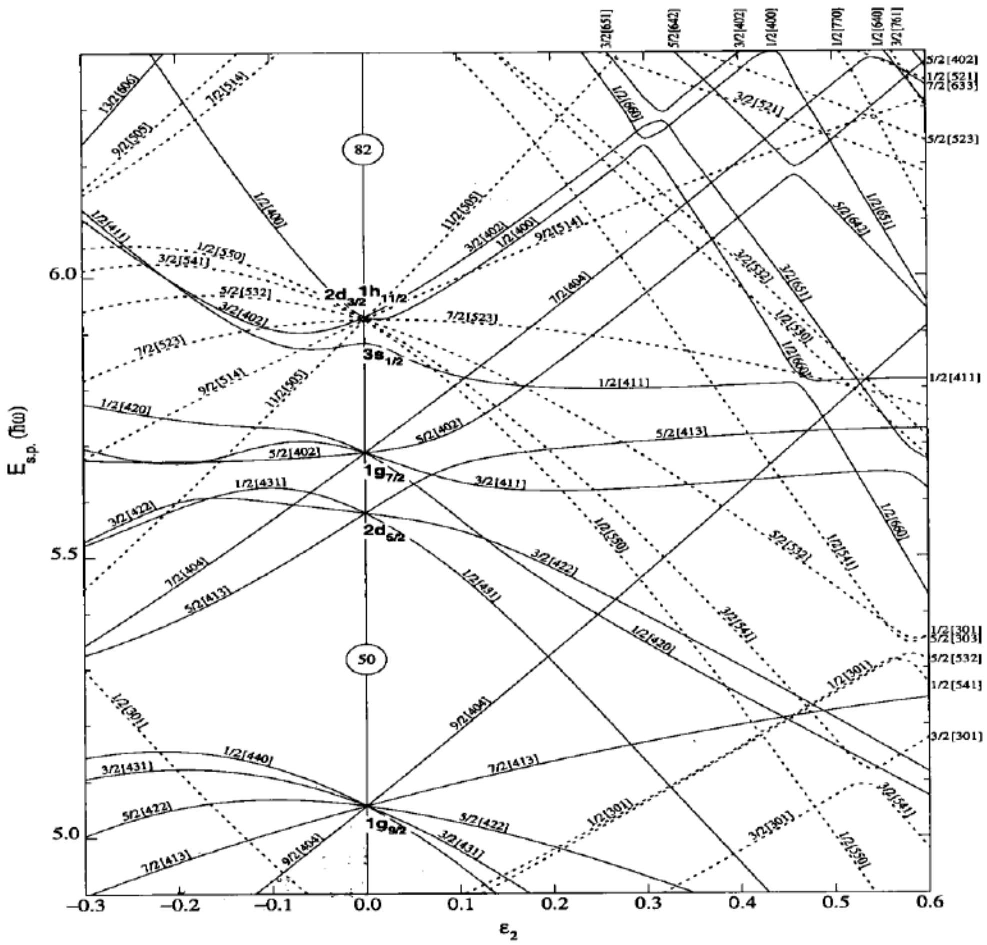


Figure 5. Nilsson diagram for neutrons, $50 \le N \le 82$ ($\varepsilon_4 = \varepsilon_2^2/6$).

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future

shell model calculations

taking advantage of

the proxy-SU(3) symmetry