

# Shape and electromagnetic properties of the $^{229m}\text{Th}$ isomer

Nikolay Minkov

Institute of Nuclear Research and Nuclear Energy  
Bulgarian Academy of Sciences, Sofia, Bulgaria  
Research Group on Complex Deformed Atomic Nuclei



HINPW6, 14 May 2021

## Collaboration and Support


**Collaborator:** Adriana Pálffy



Max-Planck-Institut für Kernphysik  
Heidelberg

and Friedrich-Alexander-Universität Erlangen-Nürnberg, Erlangen

**Support:**

Deutsche Forschungsgemeinschaft  Deutsche  
Forschungsgemeinschaft

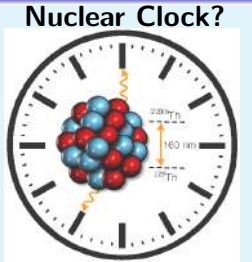
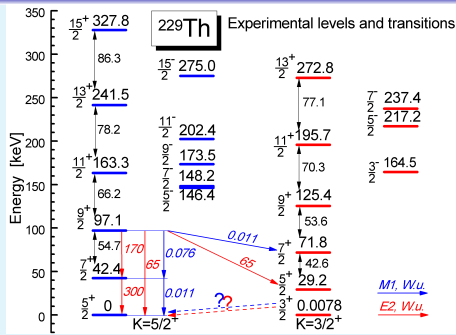
BgNSF (Contract No. KP-06-N48/1)



## Contents

- 1 **The  $^{229m}\text{Th}$  isomer and the idea of nuclear clock**
- 2 **The 8 eV isomer from a nuclear model perspective**
  - Quadrupole-octupole core plus particle model
  - Energy, decay rate and lifetime prediction
  - Magnetic dipole moments
- 3 **Role of the shape in the isomer formation and properties**
  - DSM deformation space and areas of GS-IS quasi-degeneracy
  - Full model fits in the DSM deformation space
- 4 **Concluding remarks and perspective**

$^{229}\text{Th}$  levels and transitions.  $^{229m}\text{Th}$  isomer and nuclear clock.



$E(^{229m}\text{Th}) = E_{IS}(\frac{3}{2}^+) \approx 8 \text{ eV}$   
 $\Leftrightarrow \lambda \sim 160 \text{ nm}, \Delta E/E \simeq 10^{-20}$

**Advantages vs atomic clocks:** better isolation from chem. environment; better frequency stability; higher accuracy: not lag behind / accelerate by more than a second in a period tens of times longer than the age of the Universe.

**Possible applications:** laser and plasma physics, metrology, geodesy, cosmology, gravitation waves, deep space navigation; ultra-precise chemical analysis etc.

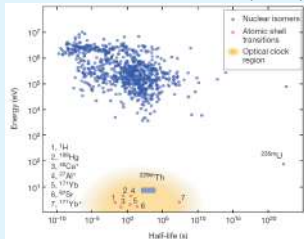
[https://en.wikipedia.org/wiki/Nuclear\\_clock](https://en.wikipedia.org/wiki/Nuclear_clock)

**What do we need to create a  $^{229m}\text{Th}$  clock?**



## Need of high-precision determination of $^{229m}\text{Th}$ lifetime

L. von der Wense, ..., P. Thirolf et al, Nature **533**, 47 (2016)



Lifetime needed 1) to unambiguously identify/characterize the  $^{229m}\text{Th}$  decay; 2) to adjust conditions for the frequency stabilization (laser frequency comb and other methods)

⇒ need of high-precision determination of  $B(M1)$  and  $B(E2)$  rates for nuclear  $3/2^+_I \rightarrow 5/2^+_G$  transitions

**Decay detection and lifetime estimates:**

L. von Wense, ..., P. Thirolf et al, Nature **533**, 47 (2016),  $\tau(\text{IC}) (^{229m}\text{Th}^{2+}) \gtrsim 60\text{s}$

B. Seiferle et al, PRL **118**, 042501 (2017),  $\tau(\text{IC}) (^{229m}\text{Th}) = 7 \pm 1\mu\text{s}$

**Magnetic moment measurement:**

J. Thielking, ..., P. Thirolf et al, Nature **556**, 321 (2018),  $\mu (^{229m}\text{Th}) = -0.37(6)\mu_N$

Knowledge of the magnetic dipole moment needed 1) to reduce the ambiguity in the  $B(M1)$  and  $B(E2)$  transition rates determination; 2) to reveal details of the nuclear microscopic mechanism governing the  $^{229m}\text{Th}$  isomer formation.

# Quadrupole-octupole core plus particle model in $^{229}\text{Th}$

N. Minkov and A. Pálffy, PRL **118**, 212501 (2017)

**Hamiltonian, spectrum, E/M transition rates**

$$H = H_{\text{quad-oct}} + H_{\text{s.p.}} + H_{\text{pair}} + H_{\text{Coriol}}$$

$$E_{nk}(I^\pi, K_b) = \epsilon_{\text{qp}}^{K_b} + \hbar\omega[2n + 1 + \sqrt{k^2 + b\tilde{X}(I^\pi, K_b)}]$$

$$\tilde{\Psi}_{nkIMK_b}^{\pi, \pi^b} = \frac{1}{N_{I^\pi K_b}} [\Psi_{nkIMK_b}^{\pi, \pi^b} + A \sum_{\nu \neq b} C_{K_\nu K_b}^{I^\pi} \Psi_{nkIMK_\nu}^{\pi, \pi^b}]$$

$$\Psi_{nkIMK}^{\pi, \pi^b} = \text{norm} \cdot \Phi_{nkl}^{\pi, \pi^b} [D_{MK}^I \mathcal{F}_K^{(\pi^b)} + \pi \cdot \pi^b (-1)^{l+K} D_{M-K}^I \mathcal{F}_{-K}^{(\pi^b)}]$$

$$B \begin{pmatrix} E\lambda \\ M1 \end{pmatrix} = \frac{1}{2I_i+1} \left\langle \left\| \tilde{\Psi}_{n_f k_f l_f K_f}^{\pi_f, \pi^b f} \right\| \left\| \hat{Q}_\lambda \right\| \left\| \tilde{\Psi}_{n_i k_i l_i K_i}^{\pi_i, \pi^b i} \right\| \right\rangle^2, \quad \lambda = 1, 2, 3$$

$$\hat{M}1 = \sqrt{\frac{3}{4\pi}} \mu_N [g_R(\hat{I} - \hat{J}) + g_S \hat{S} + g_I \hat{I}], \quad \hat{J} = \hat{I} + \hat{S}, \quad \mu_N = \frac{e\hbar}{2mc}$$

N. Minkov and A. Pálffy, PRL **122** 162502 (2019)

**Magnetic dipole moment**

$$\mu = \sqrt{\frac{4\pi}{3}} \langle \tilde{\Psi}_{IIK_b} | \hat{M}1_z | \tilde{\Psi}_{IIK_b} \rangle$$

Application to  $^{229}\text{Th}$  with:

$$g_s = q_s \cdot g_s^{\text{free}}$$

spin gyromagnetic quenching (core polarization effect):  $q_s = 0.6$

$$g_R = q_R \cdot Z/A$$

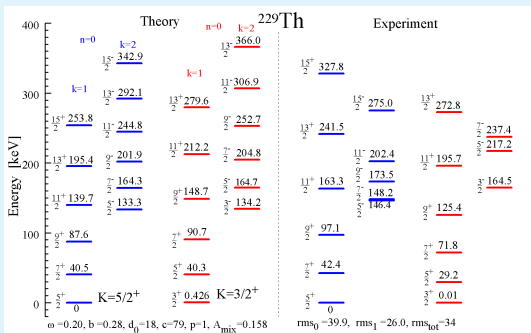
collective gyromagnetic quenching (pairing effect):

$q_R = 1.0, 0.8$  (from exp. M1/E2),  $0.7$  (Nilsson),  $0.6$  (HF+BCS)

Energy, decay rate and lifetime prediction

## $^{229}\text{Th}$ : spectrum description and isomer lifetime prediction

N. Minkov and A. Pálffy, PRL 118, 212501 (2017)



DSM: Ground-state and isomer s.p. orbitals GS(5/2[633]), IS(3/2[631])

Quadrupole and octupole deformations:  $\beta_2 = 0.240, \beta_3 = 0.115$

⇒ Transition probabilities for the  $3/2^+$ -isomer decay in  $^{229}\text{Th}$  predicted in the limits:

$B(E2) = 20-30$  W.u.       $B(M1) = 0.006-0.008$  W.u.

⇒ Predicted  $^{229m}\text{Th}$  lifetime of approx.  $10^4$  sec.



Magnetic moments in the  $3/2_{IS}^+$  and  $5/2_{GS}^+$  states of  $^{229}\text{Th}$ N. Minkov and A. Pálffy, PRL **122** 162502 (2019)Theoretical magnetic moments (in  $\mu_N$ ) for different  $q_R$ s compared to experiment

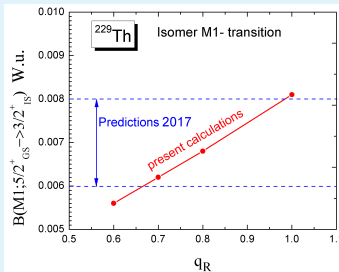
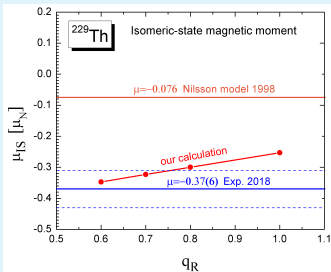
$\mu$	$q_R$ (our work)				other theories		laser spectroscopy			
	1.0	0.8	0.7	0.6	Th77	Th98	Exp74	Exp13	Exp18a	Exp18b
$\mu_{GS}$	0.654	0.591	0.559	0.528	0.54	–	0.46(4)	0.360(7)	–	–
$\mu_{IS}$	–0.253	–0.300	–0.323	–0.347	–	–0.076	–	–	[–0.3, –0.4]	–0.37(6)

Th77: Modified Woods-Saxon Model, R. Chasman et al, RMP **49**, 833 (1977)Th98: Nilsson Model, A. Dykhne and E. Tkalya, JETP Lett. **67**, 251 (1998)Exp74: S. Gerstenkorn et al., J. Phys. **35**, 483 (1974)Exp13: M. Safronova et al, PRA **88**, 060501(R) (2013)Exp18a: R. Müller et al., PRA **98**, 020503(R) (2018)Exp18b: J. Thielking, ..., P.Thirolf, E.Peik, Nature **556**, 321 (2018)Calculated  $B(M1)$  rates (in W.u.) in dependence on  $q_R$  (and  $\mu_{IS}$ ,  $\mu_{GS}$ )

Decay	$q_R$				Experiment [ENDSF]
	1.0	0.8	0.7	0.6	
$3/2_{ex}^+ \rightarrow 5/2_{yr}^+$	0.0081	0.0068	0.0062	0.0056	–
$7/2_{yr}^+ \rightarrow 5/2_{yr}^+$	0.0096	0.0043	0.0025	0.0011	0.0110 (40)
$9/2_{yr}^+ \rightarrow 7/2_{yr}^+$	0.0185	0.0097	0.0065	0.0038	0.0076 (12)
$9/2_{yr}^+ \rightarrow 7/2_{ex}^+$	0.0144	0.0147	0.0149	0.0151	0.0117 (14)

Magnetic dipole moments

# Magnetic moment and B(M1) rate in $^{229m}\text{Th}$ for different $q_R$ values



**Note 1:**  $\mu_{\text{IS}}^{\text{theo}}$  is obtained with model parameters adjusted (PRL2017) to energy levels and transition rates and not to  $\mu_{\text{IS}}^{\text{exp}}$ . Here only  $q_R$  is varied.

**Note 2:** Surprisingly good agreement with  $\mu_{\text{IS}}^{\text{exp}} = -0.37(6) \mu_N$  is found.

**Note 3:** The best agreeing  $\mu_{\text{IS}}^{\text{theo}} = -0.347 \mu_N$  value suggests a smaller  $B(\text{M1})_{\text{IS}} = 0.0056 \text{ W.u.}$  compared to PRL2017 pointing on  $^{229m}\text{Th}$  lifetime possibly larger than  $10^4 \text{ sec.}$

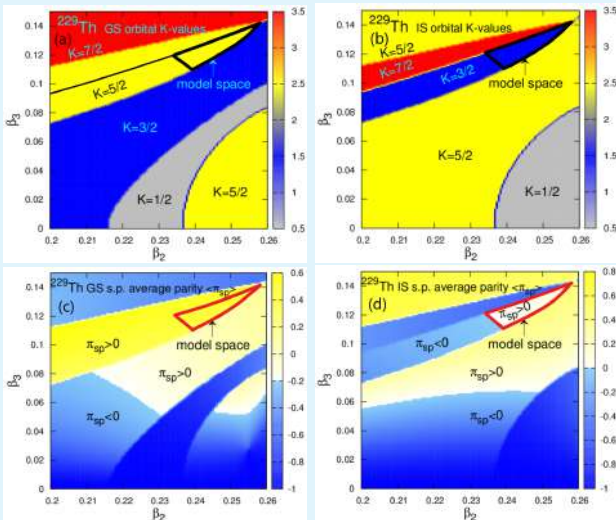
**⇒ Needs to study additional dependencies which may constraint the isomer decay rates and lifetime suggesting further more precise determination of  $^{229m}\text{Th}$  nuclear clock characteristics.**

## Further understanding of the $^{229m}\text{Th}$ isomer formation mechanism and properties

- How does the shape dynamics determine the emergence of such a nuclear structure phenomenon as the tiny energy difference between the IS and the GS?
- What is the degree of arbitrariness in the choice of parameters providing the model predictions?
- In which limits the model predictions for the transition rates and magnetic moments vary and could they be further constrained?
- Is  $^{229m}\text{Th}$  a unique phenomenon appearing by chance, or the considered dynamical mechanism could provide the presence of similar not yet observed phenomena in other nuclei?

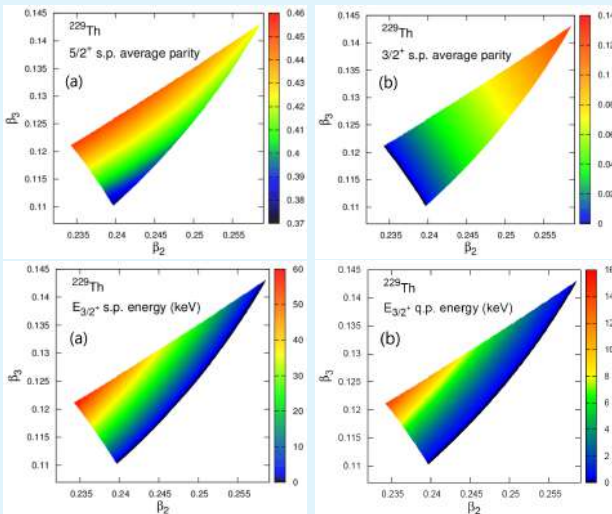
DSM deformation space and areas of GS-IS quasi-degeneracy

## $^{229}\text{Th}$ GS and IS $K$ -values in the DSM ( $\beta_2, \beta_3$ ) space



DSM deformation space and areas of GS-IS quasi-degeneracy

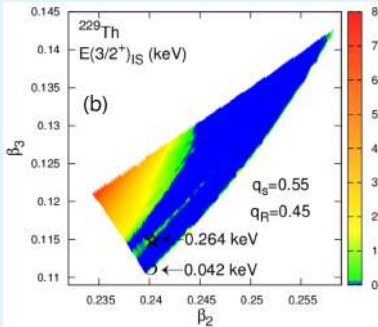
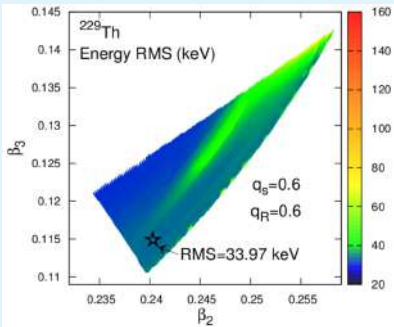
# GS and IS average parity and IS s.p.&q.p. energy in the DSM ( $\beta_2, \beta_3$ ) space



Full model fits in the DSM deformation space

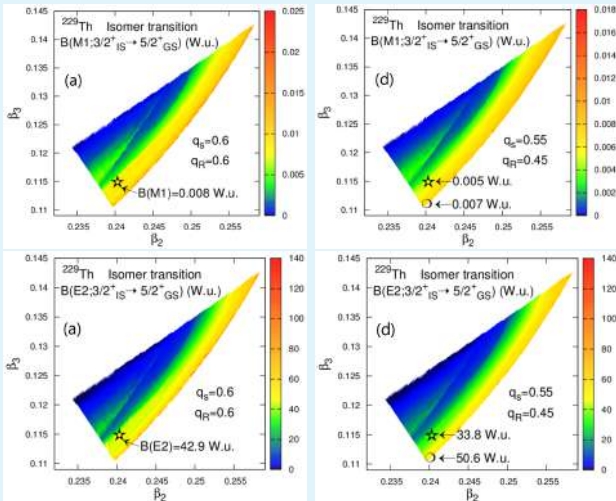
## Model fits in the $(\beta_2, \beta_3)$ space: Energy RMS and IS energy.

Full model fits (CQOM and Coriolis strength) to the energy, transition rates and magnetic moments over a net in the DSM  $(\beta_2, \beta_3)$  space



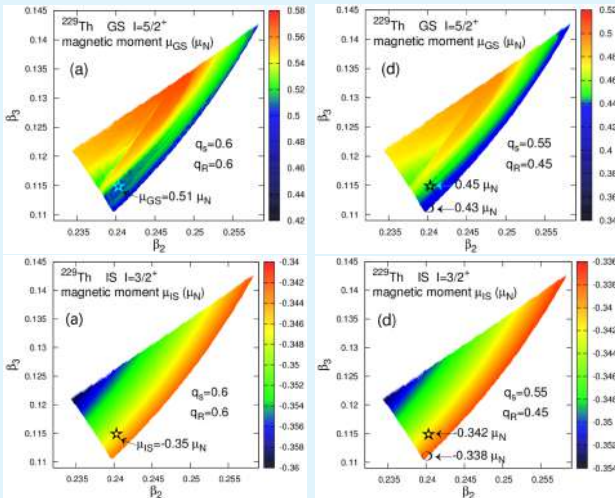
Full model fits in the DSM deformation space

# Model fits in the $(\beta_2, \beta_3)$ space: B(M1) and B(E2) IS decay rates.



Full model fits in the DSM deformation space

## Model fits in the $(\beta_2, \beta_3)$ space: GS and IS magnetic moments.





## Concluding remarks

- **CQOM+DSM+BCS model analysis:** The  $^{229}\text{Th}$  isomer can be formed in rather limited QO deformation space  $0.235 \leq \beta_2 \leq 0.255$  and  $0.11 \leq \beta_3 \leq 0.14$ . **Crucial role of the nonzero octupole deformation.**
- **Model description and predictions:** → Smooth behaviour of the model determined quantities – energy,  $B(M1)$  and  $B(E2)$  transition rates and magnetic moments within the model deformation space. **Rather constrained arbitrariness in the obtained descriptions and predictions.**
- **Slight update of model predictions:** IS  $B(M1) \sim 0.005$  W.u.,  $B(E2) \sim 30 - 50$  W.u.,  $\mu_{GS} \sim 0.43 - 0.48 \mu_N$ ,  $\mu_{IS} \sim (-0.35) - (-0.34) \mu_N$  (**firmly within exp. uncertainty limits**).
- **Model mechanism:** the fine interplay between nuclear collective and intrinsic degrees of freedom may be a plausible reason for the isomer formation. **The same dynamical mechanism may govern also in other nuclei the formation of excitations close to the border of atomic physics energy scale.**