

Thermal properties of hot and dense matter

Influence of rapid rotation on protoneutron stars, hot neutron stars, and neutron star merger remnants

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Hot equation of state

Protoneutron stars

- Shortly after neutron stars born
- Trapped neutrinos
- Protons + Neutrons + Leptons
- Isentropic EOSs

T (MeV)	Y_l	S (k_B)
0 - 50	0.01 - 0.4	0 - 10

Hot equation of state

Hot neutron stars

- Heat up by mass accretion due to a companion (Neutron stars merger)
- Protons + Neutrons + Leptons
- Isothermal/Isentropic EOSs

T (MeV)	Y_l	S (k_B)
0 - 100	0.01 - 0.6	0 - 100

Momentum dependent interaction model

The energy of asymmetric nuclear matter is given by the relation

$$\mathcal{E}(n_n, n_p, T) = \mathcal{E}_{\text{kin}}^n(n_n, T) + \mathcal{E}_{\text{kin}}^p(n_p, T) + V_{\text{int}}(n_n, n_p, T), \quad (1)$$

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- kinetic part: $\mathcal{E}_{\text{kin}}^n(n_n, T) + \mathcal{E}_{\text{kin}}^p(n_p, T)$
- interaction part: $V_{\text{int}}(n_n, n_p, T)$

Momentum dependent interaction model

- $\mathcal{E}_{\text{kin}}^n(n_n, T) + \mathcal{E}_{\text{kin}}^p(n_p, T)$

$$\mathcal{E}_{\text{kin}}^\tau(n_\tau, T) = 2 \int \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} f_\tau(n_\tau, k, T), \quad (2)$$

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where the Fermi-Dirac distribution is

$$f_\tau(n_\tau, k, T) = \left[1 + \exp \left(\frac{e_\tau(n_\tau, k, T) - \mu_\tau(n_\tau, T)}{T} \right) \right]^{-1}, \quad (3)$$

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with nucleon density and single particle energy evaluated through

$$n_\tau = 2 \int \frac{d^3k}{(2\pi)^3} f_\tau(n_\tau, k, T), \quad \text{and} \quad e_\tau(n_\tau, k, T) = \frac{\hbar^2 k^2}{2m} + U_\tau(n_\tau, k, T). \quad (4)$$

Momentum dependent interaction model

- $V_{\text{int}}(n_n, n_p, T)$

$$V_{\text{int}}(n_n, n_p, T) = V_A + V_B + V_C, \quad (5)$$

Momentum dependent interaction model

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where

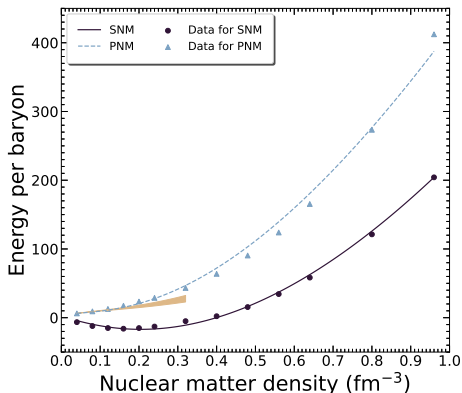
$$V_A = \frac{1}{3} A n_s \left[\frac{3}{2} - \left(\frac{1}{2} + x_0 \right) I^2 \right] u^2, \quad (6)$$

$$V_B = \frac{\frac{2}{3} B n_s \left[\frac{3}{2} - \left(\frac{1}{2} + x_3 \right) I^2 \right] u^{\sigma+1}}{1 + \frac{2}{3} B' \left[\frac{3}{2} - \left(\frac{1}{2} + x_3 \right) I^2 \right] u^{\sigma-1}}, \quad (7)$$

$$V_C = u \sum_{i=1,2} \left[C_i \left(\mathcal{J}_n^i + \mathcal{J}_p^i \right) + I \frac{(C_i - 8Z_i)}{5} \left(\mathcal{J}_n^i - \mathcal{J}_p^i \right) \right], \quad (8)$$

- n_s denotes the saturation density and $u = n/n_s$
- $I = 1 - 2Y_p$ is the asymmetry parameter and Y_p is the proton fraction
- $[A, B, B', C_i]$ are the parameters for SNM, $[x_0, x_3, Z_i]$ are the parameters for ANM
- $\mathcal{J}_\tau^i = 2 \int \frac{d^3k}{(2\pi)^3} g(k, \Lambda_i) f_\tau(n_\tau, k, T)$ with $g(k, \Lambda_i) = \left[1 + \left(\frac{k}{\Lambda_i} \right)^2 \right]^{-1}$

Momentum dependent interaction model



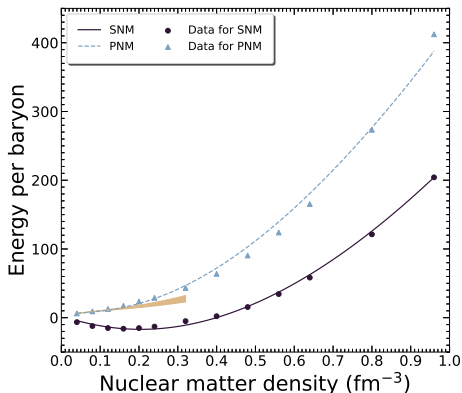
Properties of NM	MDI+APR1	Units
L	77.696	MeV
Q_{sym}	223.061	MeV
K_{sym}	0.016	MeV
E_{sym}	31.071	MeV
Q_s	-25.687	MeV
K_s	220.671	MeV
m_{τ}^*/m_{τ}	0.822	

- MDI + data from Akmal *et al*¹
- 1 Cold EOS + 10 hot EOSs based on various temperatures in the range [1, 60] MeV + nine hot EOSs based on various lepton fractions and entropies per baryon in the ranges [0.2, 0.4] and [1, 3] k_B , respectively.

¹ A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998).

² M. Piarulli, I. Bombaci, D. Logoteta, A. Lovato, and R. B. Wiringa, Phys. Rev. C **101**, 045801 (2020).

Momentum dependent interaction model



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- reproduces with high accuracy the properties of SNM
- reproduces correctly the microscopic calculations of the Chiral model and the results of state-of-the-art calculations of Akmal *et al*¹
- predicts M_{max} at least higher than the observed ones

¹ A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998).

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Bulk thermodynamic quantities

- Helmholtz free energy

$$F(n, T, I) = E(n, T, I) - TS(n, T, I), \quad (9)$$

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- Pressure and chemical potentials

$$P = - \left. \frac{\partial E}{\partial V} \right|_{S, N_i} = n^2 \left. \frac{\partial (\mathcal{E}/n)}{\partial n} \right|_{S, N_i}, \quad (11)$$

$$\mu_i = \left. \frac{\partial E}{\partial N_i} \right|_{S, V, N_{j \neq i}} = \left. \frac{\partial \mathcal{E}}{\partial n_i} \right|_{S, V, n_{j \neq i}}. \quad (12)$$

Bulk thermodynamic quantities

Pressure and chemical potentials are connected with the free energy as

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The entropy per particle is given through the relation

$$S(n, T) = -\left.\frac{\partial (f/n)}{\partial T}\right|_{V, N_i} = -\left.\frac{\partial F}{\partial T}\right|_n. \quad (14)$$

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The chemical potentials take the form

$$\mu_n = F + u \left.\frac{\partial F}{\partial u}\right|_{Y_p, T} - Y_p \left.\frac{\partial F}{\partial Y_p}\right|_{n, T}, \quad \mu_p = \mu_n + \left.\frac{\partial F}{\partial Y_p}\right|_{n, T}, \quad \hat{\mu} = \mu_n - \mu_p = -\left.\frac{\partial F}{\partial Y_p}\right|_{n, T}. \quad (15)$$

Bulk thermodynamic quantities

The free energy $F(n, T, I)$ and the internal energy $E(n, T, I)$ can be expressed by the following parabolic approximations

$$F(n, T, I) = F(n, T, I = 0) + I^2 F_{\text{sym}}(n, T), \quad (16a)$$

$$E(n, T, I) = E(n, T, I = 0) + I^2 E_{\text{sym}}(n, T), \quad (16b)$$

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The key quantity of Eq. (15) can be obtained by using Eq. (16a) as

$$\hat{\mu} = \mu_n - \mu_p = 4(1 - 2Y_p)F_{\text{sym}}(n, T). \quad (17)$$

Bulk thermodynamic quantities

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$$\hat{\mu} = \mu_n - \mu_p = 4(1 - 2Y_p)F_{\text{sym}}(n, T). \quad (17)$$

It is intuitive to assume, based mainly on Eqs. (16a) and (16b), that the entropy must also exhibit a quadratic dependence on asymmetry parameter I , that is according to the parabolic law

$$S(n, T, I) = S(n, T, I = 0) + I^2 S_{\text{sym}}(n, T), \quad (18)$$

In general: $Q_{\text{sym}} = Q(n, T, I = 1) - Q(n, T, I = 0)$, where $Q = F, E, S$

Leptons contribution

β decay and electron capture would take place simultaneously as

$$n \longrightarrow p + e^{-} + \bar{\nu}_e, \quad \text{and} \quad p + e^{-} \longrightarrow n + \nu_e. \quad (19)$$

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$$n \longrightarrow p + e^- + \bar{\nu}_e, \quad \text{and} \quad p + e^- \longrightarrow n + \nu_e. \quad (19)$$

► Isothermal process

- $n + p + e$
- $\mu_n = \mu_p + \mu_e$
- $Y_p = Y_p(n)$

► Isentropic process

- $n + p + e + \nu_e$
- $\mu_n + \mu_{\nu_e} = \mu_p + \mu_e$
- $Y_p = Y_e$ and $Y_l = Y_e + Y_{\nu_e}$

key relation

$$\hat{\mu} = \mu_n - \mu_p = 4(1 - 2Y_p)F_{\text{sym}}(n, T)$$

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$$n \longrightarrow p + e^- + \bar{\nu}_e, \quad \text{and} \quad p + e^- \longrightarrow n + \nu_e. \quad (19)$$

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- $n + p + e + \nu_e$
- $\mu_n + \mu_{\nu_e} = \mu_p + \mu_e$
- $Y_p = Y_e$ and $Y_l = Y_e + Y_{\nu_e}$

key relation

$$\hat{\mu} = \mu_n - \mu_p = 4(1 - 2Y_p)F_{\text{sym}}(n, T)$$

- In isentropic case we consider that: $Y_p = 2/3Y_l + 0.05$ within 3% accuracy ³

³ T. Takatsuka, PThPh **95**, 901-912 (1996).

Leptons contribution

The energy density and pressure of leptons are calculated through the following formulas

$$\mathcal{E}_l(n_l, T) = \frac{2}{(2\pi)^3} \int \frac{d^3 k \sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4}}{1 + \exp\left[\frac{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4} - \mu_l}{T}\right]}, \quad (20)$$

$$P_l(n_l, T) = \frac{1}{3} \frac{2(\hbar c)^2}{(2\pi)^3} \int \frac{1}{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4}} \times \frac{d^3 k k^2}{1 + \exp\left[\frac{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4} - \mu_l}{T}\right]}. \quad (21)$$

Equation of state

Total energy density

$$\mathcal{E}_t(n, T, I) = \mathcal{E}_b(n, T, I) + \sum_l \mathcal{E}_l(n, T, I), \quad (22)$$

where

$$\mathcal{E}_b(n, T, Y_p) = nF_{\text{PA}} + nTS_{\text{PA}}. \quad (23)$$

Equation of state

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where

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Total pressure

$$P_t(n, T, I) = P_b(n, T, I) + \sum_l P_l(n, T, I), \quad (24)$$

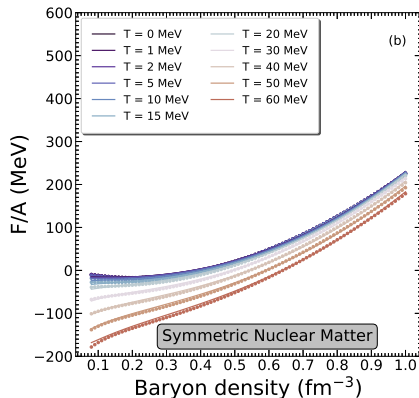
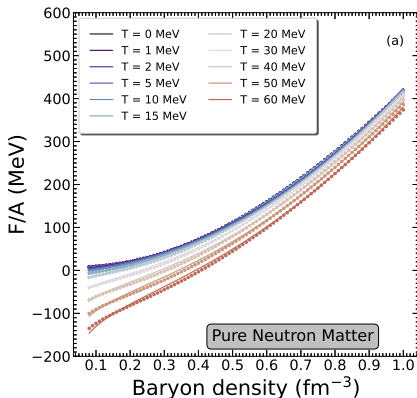
where

$$P_b(n, T, Y_p) = n^2 \left. \frac{\partial F_{\text{PA}}(n, T, Y_p)}{\partial n} \right|_{T, n_i}. \quad (25)$$

Free energy and proton fraction

- key quantity for the calculation of proton fraction
- Thermal effects are more pronounced at low densities while at high densities there is a tendency for convergence

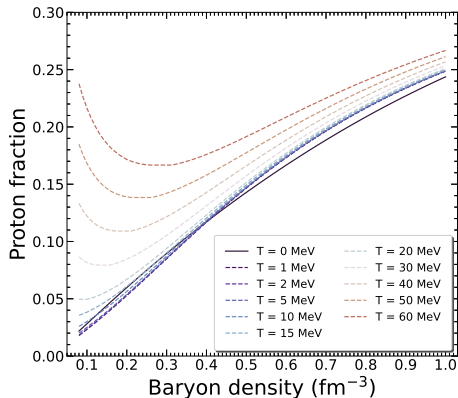
$$\bullet \frac{F}{A}(n, T) = a_0 + (a_1 + a_2 t^2) n + a_3 n^{a_4} + a_5 t^2 \ln(n) + (a_6 t^2 + a_7 t^{a_8}) / n^4$$



⁴ J.J. Lu et al, Phys. Rev. C **100**, 054335 (2019).

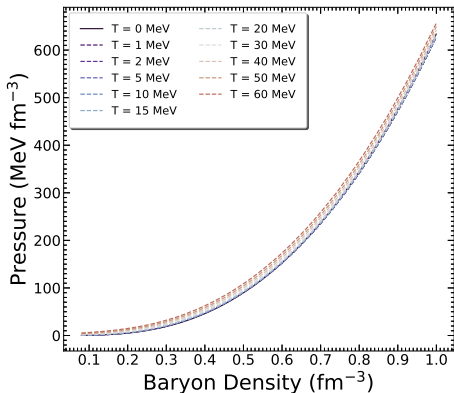
Free energy and proton fraction

- It is related not only to the specific structure of a neutron star but also to some dynamical processes, including mainly the cooling of the star through various forms of the dURCA process
- While in the low density region the proton fraction is very sensitive to the temperature, in the high density region thermal effects are very mild. This is a direct consequence of the similar sensitivity of the free energy per particle to temperature

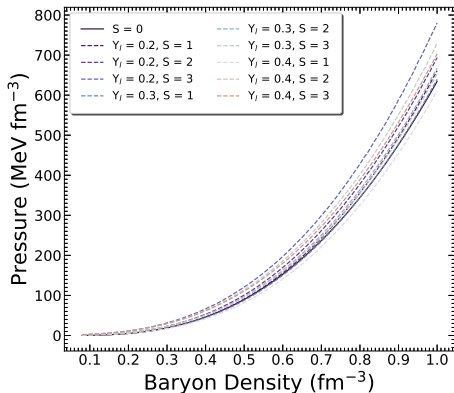


Equation of state, adiabatic index and speed of sound

► Isothermal Equations of state



► Isentropic Equations of state

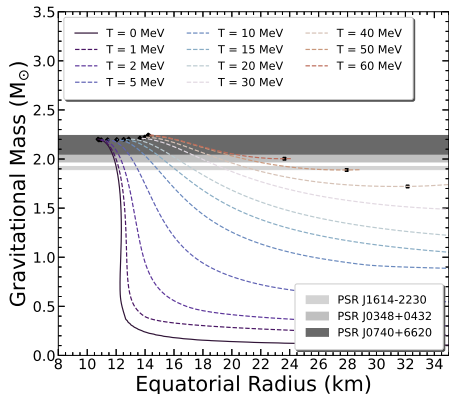


Equation of state, adiabatic index and speed of sound

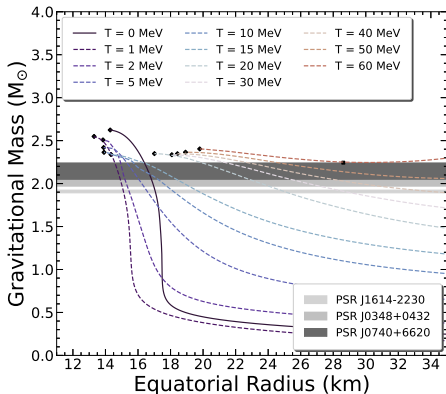
► Mass-radius diagram for isothermal equations of state

- $T \sim 15$ MeV \rightarrow transition from non-homogeneous to homogeneous matter ⁵
- Employ the MDI+APR1 equation of state for the core and the LS220 for the low density region

Nonrotating



Maximally rotating

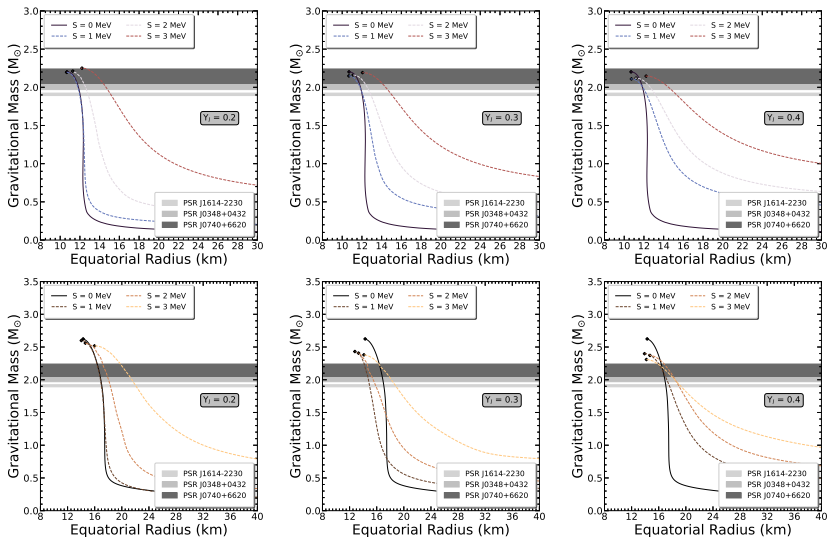


⁵ H. Shen *et al.*, Nuc. Phys. A **637**, 435-450 (1998).

Equation of state, adiabatic index and speed of sound

► Mass-radius diagram for isentropic equations of state

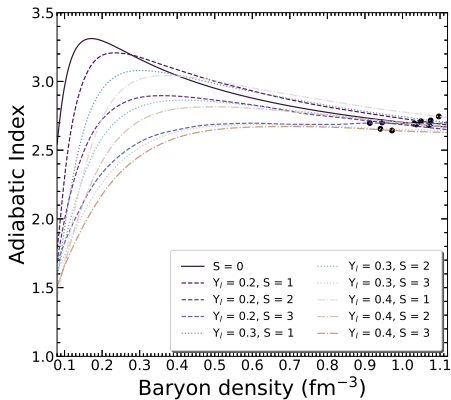
Top panel: Nonrotating and Bottom panel: Maximally rotating



Equation of state, adiabatic index and speed of sound

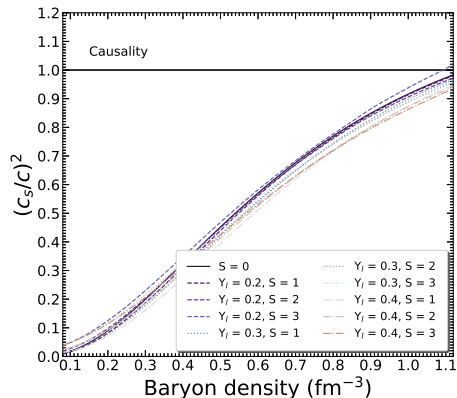
► Adiabatic index

$$\Gamma = \frac{n}{P} \frac{\partial P}{\partial n} \Big|_S \quad (26)$$



► Speed of sound

$$\frac{c_s}{c} = \sqrt{\frac{\partial P}{\partial \mathcal{E}}} \Big|_S \quad (27)$$



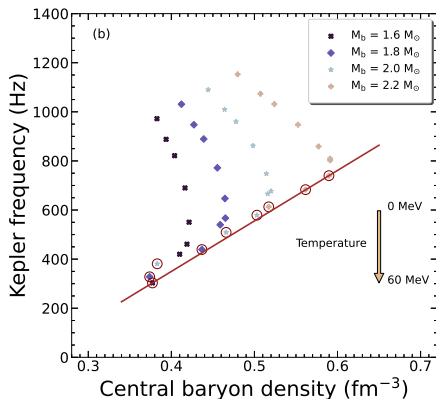
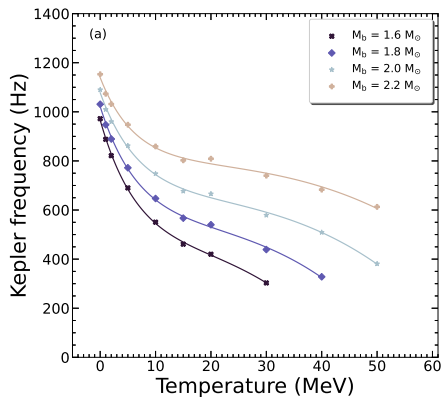
- MDI model prevents the EOS from reaching the causality point.

Sequences of constant baryon mass on rotating neutron stars

- Sequences of constant baryon mass are a very useful way to study thermal effects on the evolution, as well as on the instability conditions of hot neutron stars
- Using isothermal EOSs, we have constructed a sequence related to the cooling of a neutron star
- The quantities under consideration are the Kepler frequency, the central baryon density, and the temperature of each EOS

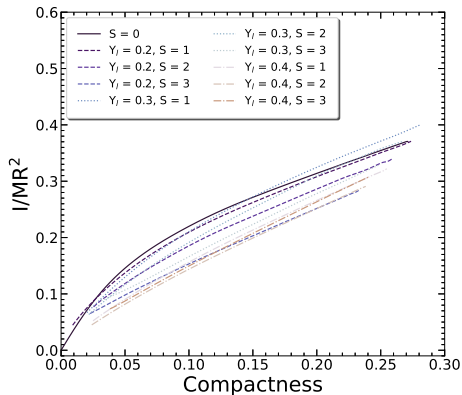
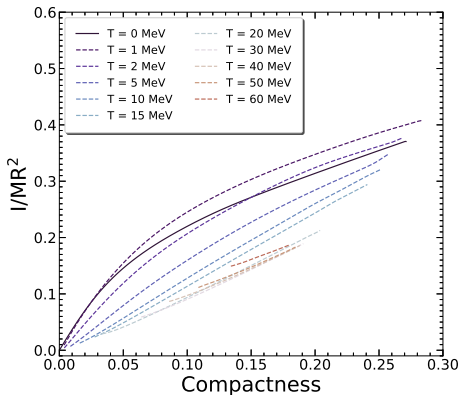
Sequences of constant baryon mass on rotating neutron stars

- $f(T) = a_0 + a_1 T^3 + a_2 \exp[a_3 T]$ (Hz)
- For $T \geq 30$ MeV: $f(n_b^c) = -473.144 + 2057.271 n_b^c$ (Hz)



Moment of inertia, kerr parameter and ratio T/W on rotating neutron stars

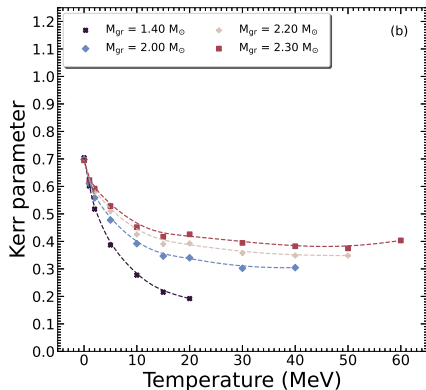
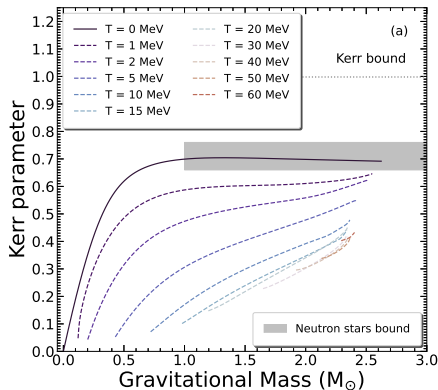
- The dimensionless moment of inertia provides an important constraint for the interior structure of neutron stars
- The increase of temperature/entropy per baryon, except for some specific cases ($T < 2 \text{ MeV}/Y_l = 0.2, 0.3$ and $S = 1$), leads to lesser compact objects than the cold EOS



Moment of inertia, kerr parameter and ratio T/W on rotating neutron stars

- $\mathcal{K} \equiv cJ/(GM^2)$
- $\mathcal{K}_k \simeq 1.34\sqrt{\beta_{\max}}^6$
- $0.24 \leq \beta_{\max} \leq 0.32 \Rightarrow 0.66 \leq \mathcal{K}_k \leq 0.76$

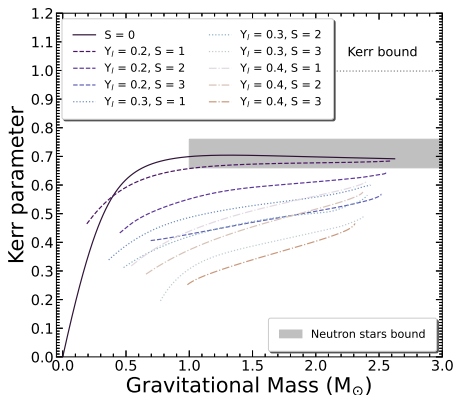
$$\mathcal{K}_{\text{B.H.}} \approx 0.998^7$$



⁶ P.S. Koliogiannis and Ch.C. Moustakidis, Phys. Rev. C **101**, 015805 (2020).

⁷ K.S. Thorne, ApJ **191**, 507-520 (1974).

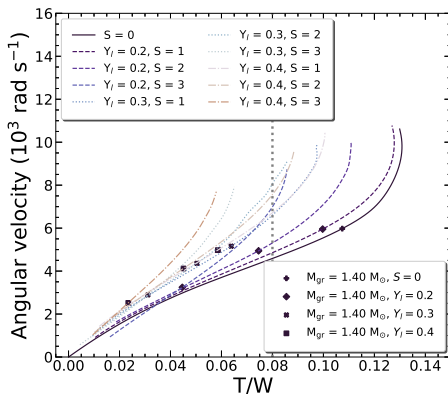
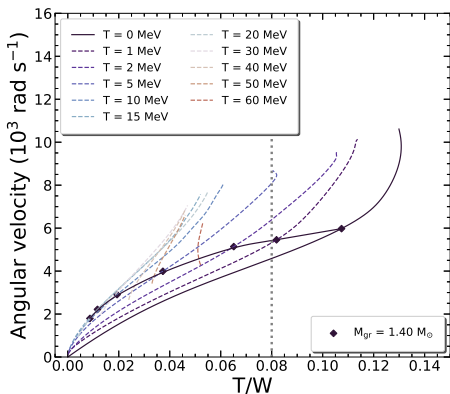
Moment of inertia, kerr parameter and ratio T/W on rotating neutron stars



- $\mathcal{K}_{N.S.}$ and $\mathcal{K}_{B.H.}$ cannot be exceeded as the temperature in neutron stars increasing
- The gravitational collapse of a hot, uniformly rotating neutron star, cannot lead to a maximally rotating Kerr black hole

Moment of inertia, kerr parameter and ratio T/W on rotating neutron stars

- Instabilities driven by gravitational radiation would set in at $T/W \sim 0.08$ for models with $M_{\text{gr}} = 1.4 M_{\odot}$ ⁸
- For sufficiently compact neutron stars the non-axisymmetric instability will set in before the mass-shedding limit is reached



⁸ S.M. Morsink et al, ApJ 510, 854-861 (1999).

Hot rapidly rotating remnant

- Hot, rapidly rotating remnant: at least $T \geq 30$ MeV for isothermal EOSs, $S = 1$ and $Y_l = 0.2$ for isentropic ones
- $\beta_{\text{rem}}^{\text{iso}} \leq 0.19$ and $\beta_{\text{rem}}^{\text{ise}} \leq 0.27$
- $\mathcal{K}_{\text{rem}}^{\text{iso}} = 0.42$ and $\mathcal{K}_{\text{rem}}^{\text{ise}} = 0.68$
- $(T/W)_{\text{rem}}^{\text{iso}} = 0.05$ and $(T/W)_{\text{rem}}^{\text{ise}} = 0.127$

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Isentropic EOS

In isentropic case it creates an object comparable to the one of cold EOS, and unstable toward the dynamical instabilities

Concluding remarks

► Baryon Mass

The dominant quantity that manifests the thermal effects in neutron stars, is the baryon mass

Supramassive limit: In the cold case the baryon mass is $3.085 M_{\odot}$ while for a hot one at $T = 30$ MeV is $2.427 M_{\odot}$ and at $S = 1$ is $3.05 M_{\odot}$

Merger components (assuming equal masses of components): $\sim 1.5425 M_{\odot}$, $\sim 1.2135 M_{\odot}$, and $\sim 1.525 M_{\odot}$ baryon masses respectively

GW170817 $\rightarrow 2.7 M_{\odot}$ remnant

GW190425 $\rightarrow 3.7 M_{\odot}$ remnant

✓ Cold EOS

× Cold EOS

× Isothermal EOS

× Isothermal EOS

✓ Isentropic EOS

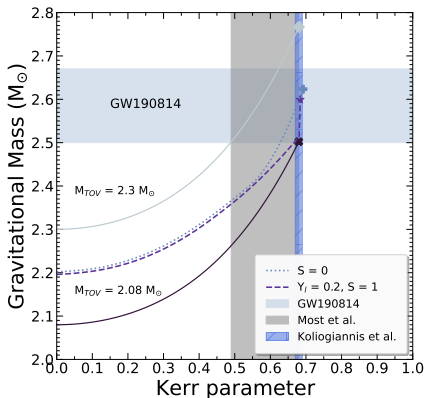
× Isentropic EOS

Rotation

- Uniform rotation
- Differential rotation?

Concluding remarks

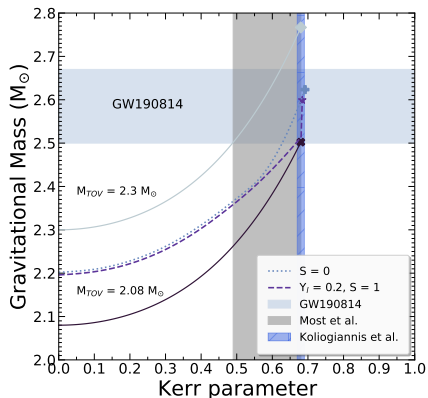
A very recent event, the GW190814, had one of the components with mass $\sim 2.6 M_{\odot}$. It is believed to be either the lightest black hole or the most massive neutron star. However, an approach in Ref. *Most et al., MNRAS 499, L82-L86 (2020)* suggests that this star was rapidly spinning with \mathcal{K} in the range [0.49, 0.68]



Concluding remarks

MDI+APR1 EOS

This approach coincide with the supramassive limit of the MDI+APR1 EOS, both in cold catalysed matter and isentropic one with $S = 1$ and $Y_i = 0.2$.



Concluding remarks

- As the entropy per baryon and temperature (until $T = 15$ MeV) increases for isentropic and isothermal EOSs respectively, moment of inertia decreases leading to lower values of torques that the neutron star needs in order to change its rate of rotation than the cold case.
- The endpoint from kerr parameter is that thermal support cannot lead a star to collapse into a maximally rotating Kerr black hole. On the other hand, it is fascinating the effect on the star. Although in the cold case, after $\sim 1 M_{\odot}$, kerr parameter is stabilized at a constant value, when temperature is added, kerr parameter becomes an increasing function of the gravitational mass leading to a maximum value.
- Instabilities driven by gravitational radiation never occur in a hot, rapidly rotating neutron star.
- For temperatures $T \geq 30$ MeV, a linear relation holds on between Kepler frequency and central baryon density, leading to a universal behavior and description for the central baryon density at the mass-shedding limit. This relation defines the allowed region on both the central baryon density and Kepler frequency, for a rotating hot neutron star at its mass-shedding limit.

Concluding remarks

Future work...

- Rotating configurations based on differential laws
- The threshold mass and the hot, rapidly rotating remnant

Thank you for your attention!