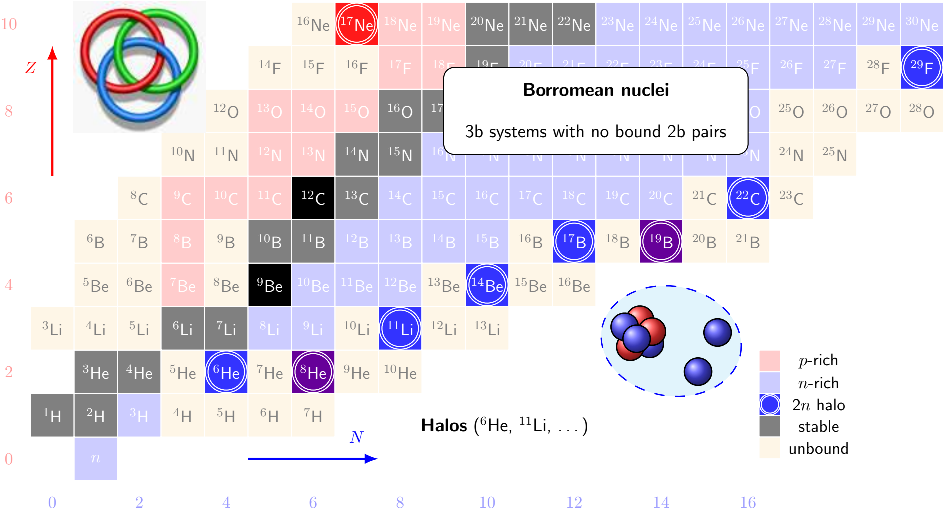
May 14-16, 2021
HINP workshop

Linking structure and dynamics with two-neutron halos

Jesús Casal

Universidad de Sevilla, Spain



Borromean nuclei

3b systems with no bound 2b pairs

Halos (^{6}He, ^{11}Li, ...)

Neutron-knockout from two-neutron halo nuclei

- (p, pn) "quasifree" processes

- Study the properties of the three-body bound-state wave function and the spectrum of the unbound core $+\ n$ subsystem

➥ Transfer to the continuum (TC) formalism with structure overlaps from a full three-body model for the projectile

e.g.: ^{11}Li$(p, pn)^{10}$Li, ^{14}Be$(p, pn)^{13}$Be

- Angular correlations in momentum space

➥ Modified Fourier transform of the structure overlaps to extract momentum and angular distributions

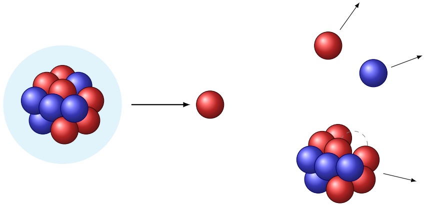
e.g.: recent work on surface localization of the "dineutron" in ^{11}Li

(p, pN) reactions in inverse kinematics

"quasifree" $1N$ removal; p-target knockout

- Fast-moving projectile on a proton target

  One nucleon is removed, leaving the residue in ground or excited state



- High energies to increase mean free path of nucleon inside nucleus
- Structure information inferred from total $1N$ removal cross sections, momentum distributions, γ and particle decay of the residue ...

(p, pN) reactions in inverse kinematics **Borromean**

"quasifree" $1N$ removal; p-target knockout

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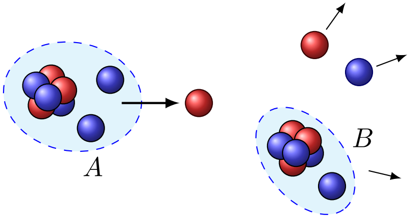


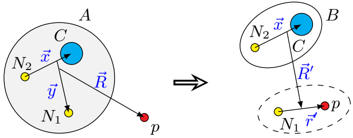
➢ If A is Borromean, the unbound subsystem B will eventually decay

➢ Probe the continuum wave function of the unbound subsystem

[M. Gómez-Ramos, J.C., A.M. Moro, PLB 772 (2017) 115]

Transfer to the Continuum (TC)

- No Impulse Approximation (IA)
- 3-body structure included



➢ Prior-form T-matrix   Participant (N_1) / Spectator (B) approach

$$\mathcal{T}_{if} = \left\langle \phi_B^{2b,q} \Psi_f^{(-)} \middle| V_{pN_1} + U_{pB} - U_{pA} \middle| \Phi_A^{3b} \chi_{pA}^{(+)} \right\rangle$$

$\Phi_A^{3b}(\vec{x},\vec{y}) \equiv$ g.s. wave function of the initial 3-body projectile A
(hyperspherical formalism, e.g. PRC 88 (2013) 014327)

$\chi_{pA}(\vec{R}) \equiv$ distorted p-A wave

$\phi_B^{2b,q}(\vec{x}) \equiv$ continuum wave function of the binary subsystem B

$\Psi_f(\vec{r}',\vec{R}') \equiv$ final $\underbrace{(p+N_1)}_{\text{continuum}} + B$ wave function

[M. Gómez-Ramos, J.C., A.M. Moro, PLB 772 (2017) 115]

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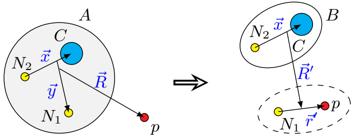

➢ Prior-form T-matrix Participant (N_1) / Spectator (B) approach

$$\mathcal{T}_{if}^{LJJ_T} \equiv \langle \Psi_f^{(-)} | V_{pN_1} + U_{pB} - U_{pA} | \psi^{q}_{LJJ_T} \chi^{(+)}_{pA} \rangle$$

Assume the transition potential does not change the state of B

$|(L,s_n)J, I_c; J_T\rangle$, energy q

$\Rightarrow$ Define **overlaps**:

$$\psi^{q}_{LJJ_T} = \langle \phi_B^{2b,q} | \Phi_A^{3b} \rangle$$

[M. Gómez-Ramos, J.C., A.M. Moro, PLB 772 (2017) 115]

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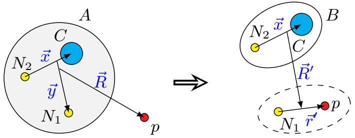

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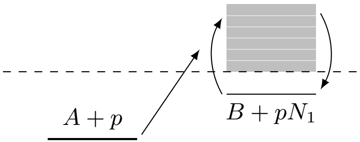
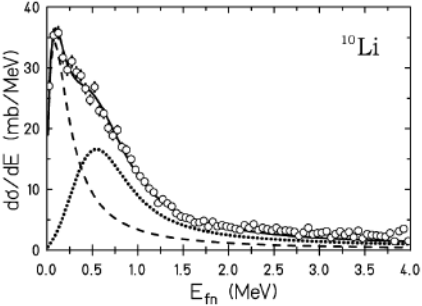


Ψ_f expanded in pN states (discretized *ala* CDCC)

^{11}Li$(p,pn)^{10}$Li in inverse kinematics at 280 MeV/u

[Aksyutina *et al.* PLB 666 (2008) 430]



$l=0$ virtual state $(2s_{1/2})$
$l=1$ resonance $(1p_{1/2})$

extracted through fitting (e.g.: BW)

reaction dynamics not considered

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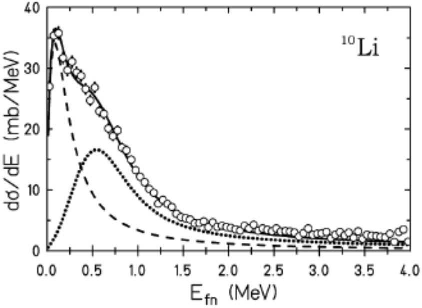

$l=0$ virtual state $(2s_{1/2})$
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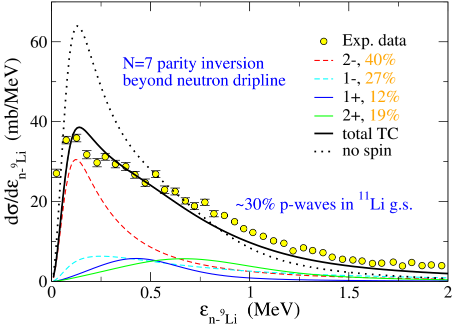
extracted through fitting (e.g.: BW)

reaction dynamics not considered

Our TC calculations:

3b WF: Include spin of ^{9}Li $(3/2^-)$

$s_{1/2} \Rightarrow 1^-, 2^-$
$p_{1/2} \Rightarrow 1^+, 2^+$

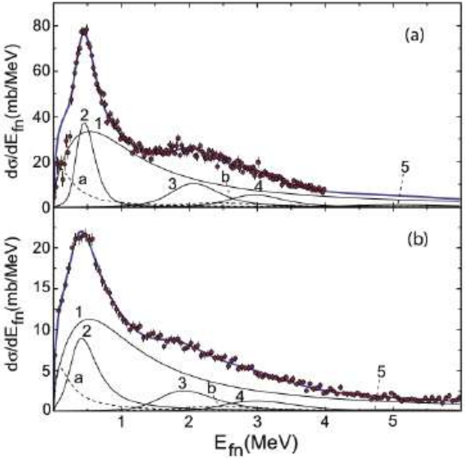


[PLB 772 (2017) 115]

^{14}Be$(p,pn)^{13}$Be

a) Kondo et al. 69 MeV/u
PLB 690 (2010) 245
b) Aksyutina et al. 304 MeV/u
PRC 87 (2013) 064316

1) $l = 0,\ 1/2^+$
2) $l = 1,\ 1/2^-$
3) $l = 2,\ 5/2^+$
4) $l = 1,\ 1/2^-$
5) $l = 2,\ ?$
a) decay $5/2^+ \rightarrow {}^{12}Be(2^+)$
b) decay into ^{12}Be(1^-)



Very difficult to disentangle! Low-lying structure still unclear.

See also ^{13}Be populated in ^{14}B$(p,2p)$ [Ribeiro et al. PRC**98**(2018)024603]

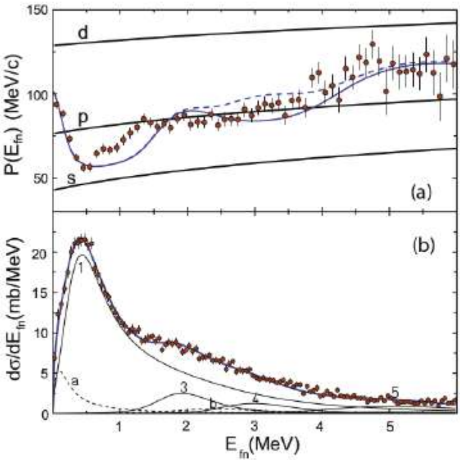
^{14}Be$(p,pn)^{13}$Be

a) Kondo et al. 69 MeV/u
PLB 690 (2010) 245
b) Aksyutina et al. 304 MeV/u
PRC 87 (2013) 064316

1) using two $1/2^+$ states

No low-energy $1/2^-$ resonance required

Momentum profile gives an average: mostly $l=0$, $l=2$



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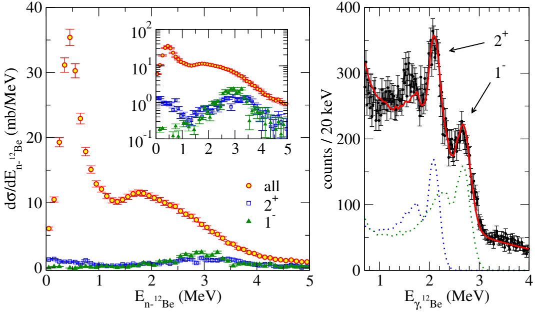
c) Corsi et al. 250 MeV/u
PLB**797**(2019)134843

γ coincidences with ^{12}Be states (2^+, 1^-) $\longrightarrow$



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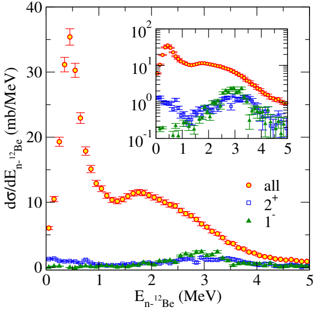
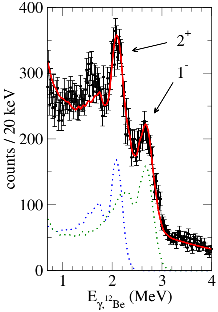
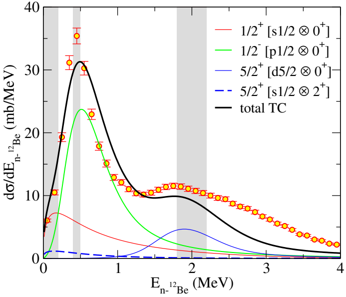
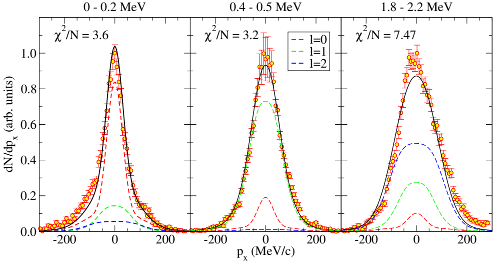




Our analysis with TC:
(includes **2^+ core ex.**, $\sim$ 20% in ^{14}Be wf)





Peak dominated by p-wave resonance

c) Corsi et al. 250 MeV/u
PLB**797**(2019)134843

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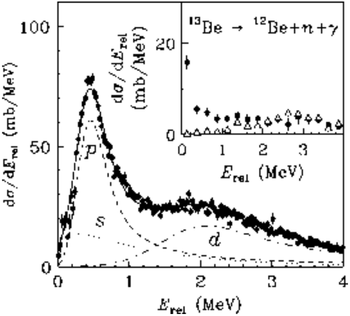
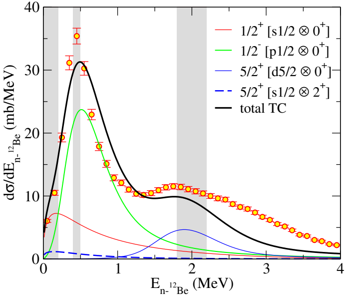
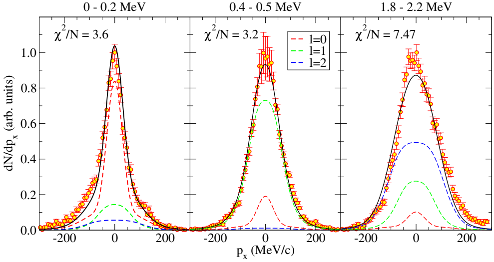
Kondo et al.







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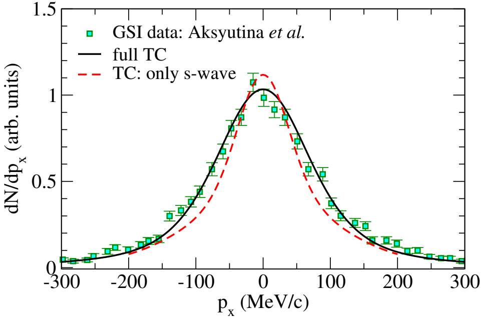

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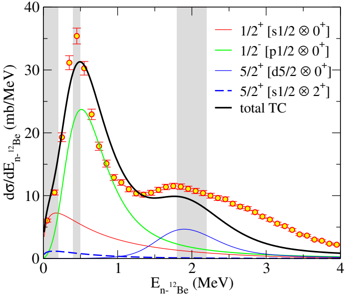
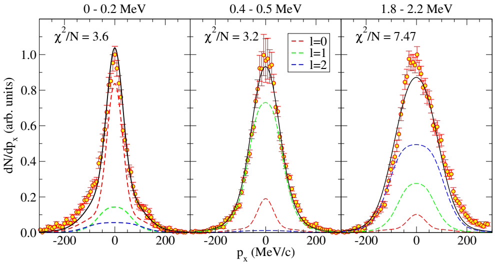




Peak dominated by p-wave resonance

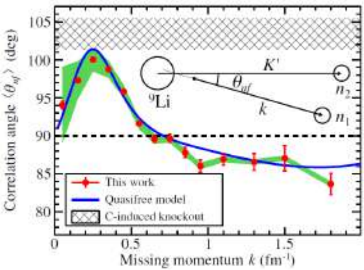
Surface Localization of the Dineutron in ^{11}Li

new $^{11}\mathrm{Li}(p, pn)^{10}\mathrm{Li}$ data - RIKEN [Kubota *et al.*, PRL**125**(2020)252501]

Angular correlations

$\theta_{nf} > \frac{\pi}{2}$ in momentum space

at small $k \Rightarrow$ surface



- ➢ Mixing of different-parity states [Catara *et al.*, PRC **29**(1984)1091]
- ➢ "Quasifree" eikonal model [Kikuchi *et al.*, PTEP **2016**(2016)103D03]

Can we describe this observable with the same structure input?

- eikonal sudden approximation $\Rightarrow$ $\chi_p = S(\boldsymbol{b})e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$

  $\boldsymbol{r}$: distance between proton target and projectile c.m.

  $\boldsymbol{q}$: momentum transferred to the knocked-out neutron

- (core $+\, n$) state not distorted by the proton

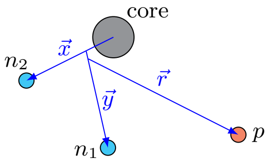
$$\Psi^{3.b.(-)} = \phi_{\text{c-n}}(\boldsymbol{k}_x, \boldsymbol{x})e^{i(\boldsymbol{k}_y+\boldsymbol{q})\cdot\boldsymbol{y}}$$

- contact V_{pn} $\Rightarrow$ $S(\boldsymbol{b}) = S(\boldsymbol{b}_y)$ and approximate $S(\boldsymbol{b}_y) \approx S(y)$

  ➢ p-core eikonal S-matrix evaluated at y

$$\mathcal{T} \propto \left\langle \phi_{\text{c-n}}(\boldsymbol{k}_x, \boldsymbol{x})e^{i\boldsymbol{k}_y\cdot\boldsymbol{y}} \middle| S(y)\Phi^{j\mu}_{\text{g.s.}}(\boldsymbol{x}, \boldsymbol{y})\right\rangle$$

$\Phi^{j\mu}_{\text{g.s.}}$: ground-state WF of the core $+\, n + n$ projectile



➡ "distorted" Fourier transform of the projectile g.s. [Kikuchi et al.]

$$|\Phi^{j\mu}\rangle = \sum_\eta \omega_\eta(k_x, k_y)|\eta; j\rangle \qquad |\eta; j\rangle = |l_x, j_x, j_2, l_y, j_1; j\rangle$$

$$\omega_\eta(k_x, k_y) = (4\pi)^2 \frac{i^{-l_x-l_y}}{k_x} \int \mathrm{d}y \underbrace{\psi_\eta(k_x, y)}_{\langle 2b|3b\rangle \text{ structure overlaps!!}} j_{l_y}(k_y y) S(y) y$$

Differential cross sections (k_x, k_y, θ):

$$\sigma \propto \frac{1}{2j+1} \sum_\mu \Phi^*\Phi = \sum_{\eta\eta'} \omega_\eta(k_x, k_y)\omega_{\eta'}(k_x, k_y) C_{\eta\eta'}$$

$$\times \sum_L D^L_{\eta\eta'} \begin{pmatrix} l_y & l'_y & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_x & l'_x & L \\ 0 & 0 & 0 \end{pmatrix} P_L(\cos\theta)$$

➢ Distributions in $\cos\theta$ will be asymmetric only if $\Phi^{j\mu}_{\text{g.s.}}$ contains different-parity core $+\, n$ components.

Absorption

- estimated via $S(b) = \exp\left[\frac{1}{i\hbar v}\int \mathrm{d}z V(b,z)\right] = |S|e^{i\varphi}$
- Optical p-core potential $V(b,z) = U + iW$

  Use N, M to scale real (V) and imaginary (W) parts, respectively:

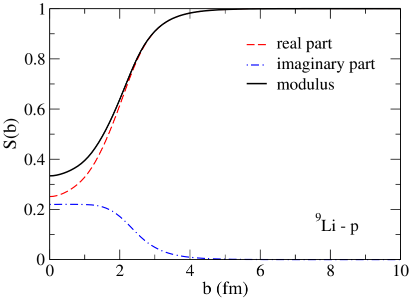
$$S(b) = |S|^M e^{i\varphi N}$$

  If only W is considered $(N = 0, M = 1)$, the S-matrix is real

Ingredients for the calculations

- ^{9}Li density from Hartree-Fock (code OXBASH)
- Folding with effective (complex) NN interaction [Ray, PRC**20**(1979)1857]
- S-matrix computed with FrontHF code (J. Tostevin, unpublished)

Absorption



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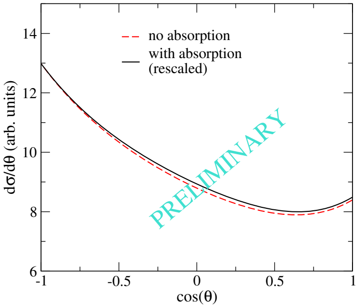
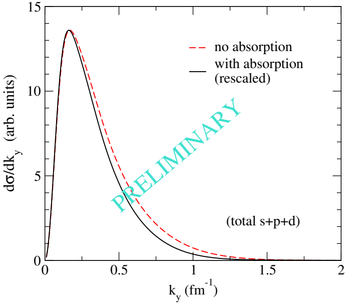
Opening angle distribution

integrate $k_x, k_y \to \theta$ dist.



Missing momentum distribution

integrate $k_x, \theta \to k_y$ dist.



J.C., M. Gómez-Ramos, in preparation

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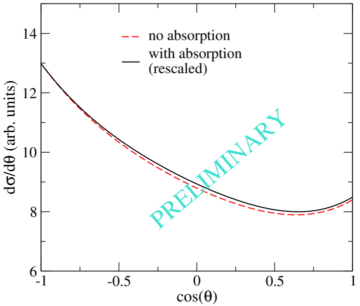

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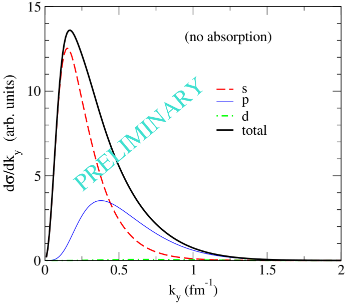
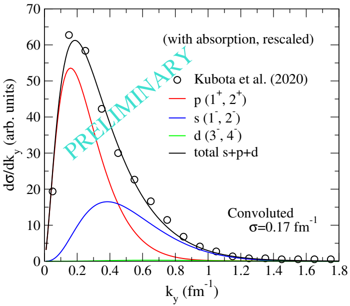


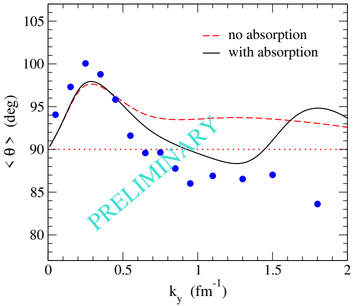
➢ asymmetry in θ-dist. comes from s, p, d mixing

➢ it favors $\theta > 90$ deg. in momentum space

J.C., M. Gómez-Ramos, in preparation

Comparison with Kubota's data and $\langle\theta\rangle$-k_y correlations

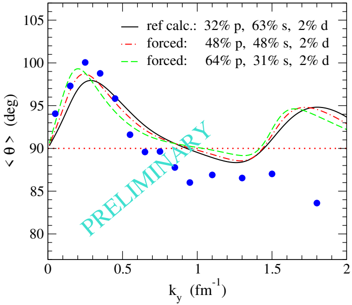




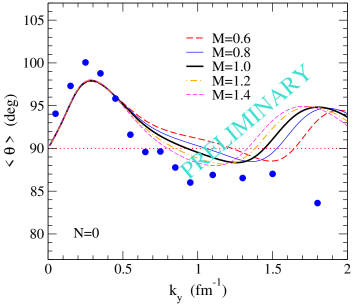
- peak in the correlation plot $\Rightarrow \theta > \pi/2$
- reasonable agreement with the data at low missing momentum

J.C., M. Gómez-Ramos, in preparation

Sensitivity to s.p. weights
(rescale structure overlaps)



Sensitivity to absorption
(scaling $V = N^*U + iM^*W$)



➢ moderate sensitivity to specific s, p, d weights

J.C., M. Gómez-Ramos, in preparation

Sensitivity to s.p. weights
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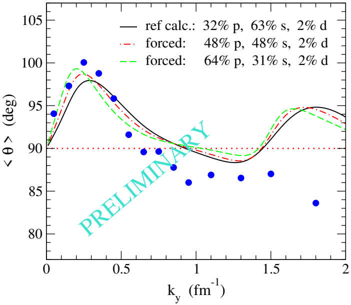

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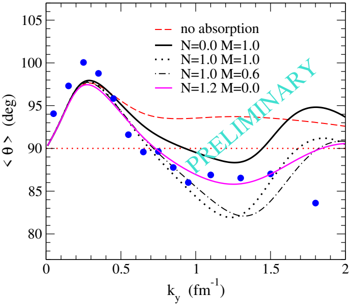
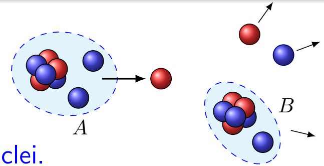


➢ moderate sensitivity to specific s, p, d weights

➢ needs further study of absorption effects!

J.C., M. Gómez-Ramos, in preparation



Summary

- (p, pN) reactions with Borromean nuclei.

  Transfer to the Continuum: participant/spectator approach providing absolute cross sections. The input are 3b/2b structure overlaps.

  We get constraints for partial-wave content in the projectile and the position of states in the unbound residue.

  ^{11}Li$(p, pn)^{10}$Li: parity inversion; splitting due to the spin of the core

  ^{14}Be$(p, pn)^{13}$Be: incl. core ex.; dominance of low-lying p resonance

- Angular correlations

  Use the same structure overlaps, but within a **sudden eikonal model**.

  Asymmetric n-n angular distributions require mixing between states with different parity. Absorption requires further study!

  ^{11}Li$(p, pn)^{10}$Li: "dineutron" peak at low missing momenta

  ^{14}Be$(p, pn)^{13}$Be: in progress

Collaborators:

M. Gómez-Ramos[1], A. M. Moro[1], A. Corsi[2]

[1]: Universidad de Sevilla
[2]: CEA Saclay

External funding:

MSCA - IF - 2020

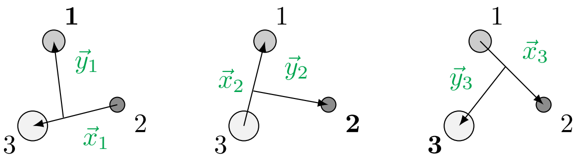
Grant agreement 101023609

Project No.

FIS2017-88410-P

Horizon 2020

Grant agreement 654002



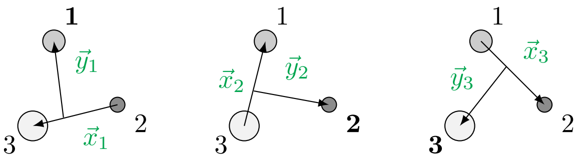
Jacobi coordinates

$\{x, y, \widehat{x}, \widehat{y}\}$

Hyperspherical coordinates

$\{\rho, \alpha, \widehat{x}, \widehat{y}\}$

$\rho = \sqrt{x^2 + y^2}$ $\quad \alpha = \arctan \frac{x}{y}$



Jacobi coordinates $\{x, y, \widehat{x}, \widehat{y}\}$

Hyperspherical coordinates $\{\rho, \alpha, \widehat{x}, \widehat{y}\}$ $\quad \rho = \sqrt{x^2 + y^2} \quad \alpha = \arctan \frac{x}{y}$

$$\Psi^{j\mu}(\rho, \Omega) = \rho^{-5/2} \sum_{\beta} \chi_{\beta}^{j}(\rho) \mathcal{Y}_{\beta}^{j\mu}(\Omega) \qquad \beta \equiv \{K, l_x, l_y, l, S_x, J; I\}$$

Hyperspherical Harmonics (HH) expansion

hypermomentum $\widehat{K}$

$$\mathcal{Y}_{\beta}^{j\mu}(\Omega) = \left[\left(\Upsilon_{Klm_l}^{l_x l_y}(\Omega) \otimes \kappa_{S_x} \right)_J \otimes \phi_I \right]_{j\mu}$$

$$\Upsilon_{Klm_l}^{l_x l_y}(\Omega) = \varphi_K^{l_x l_y}(\alpha) \left[Y_{l_x}(\widehat{x}) \otimes Y_{l_y}(\widehat{y}) \right]_{lm_l}$$

$$\varphi_K^{l_x l_y}(\alpha) = N_K^{l_x l_y} (\sin\alpha)^{l_x} (\cos\alpha)^{l_y} P_n^{l_x + \frac{1}{2}, l_y + \frac{1}{2}} (\cos 2\alpha)$$

Pseudo-State (PS) method

$$\chi_{\beta}^{j}(\rho) = \sum_{i=0}^{N} C_{i\beta}^{j} U_{i\beta}(\rho)$$

expanded in $\mathcal{L}^2$ basis*

N: number of hyperradial excitations included

$$\mathcal{H}\Psi_n^{j\mu} = \varepsilon_n \Psi_n^{j\mu}$$

- $\varepsilon_n < 0$ **bound states** (1)
- $\varepsilon_n > 0$ **discretized continuum** (2)

*We use a THO basis [PRC**88**(2013)014327, PRC**90**(2014)044304]

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$$V_{\beta'\beta}^{j\mu}(\rho) = \left\langle \mathcal{Y}_{\beta}^{j\mu}(\Omega) \middle| V_{12} + V_{13} + V_{23} \middle| \mathcal{Y}_{\beta'}^{j\mu}(\Omega) \right\rangle + \delta_{\beta\beta'} V_{\mathrm{3b}}(\rho)$$

➢ V_{ij} interaction between pairs

central, spin-orbit, spin-spin, tensor. To reproduce binary subsystem

➢ V_{3b} phenomenological three-body force

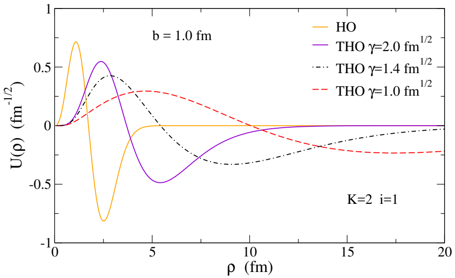
diagonal term. Fixed to fine-tune the three-body energies

*We use a THO basis [PRC**88**(2013)014327, PRC**90**(2014)044304]

Analytical **T**ransformed **H**armonic **O**scillator **(THO)** basis

$$U_{i\beta}^{\text{THO}}(\rho) = \sqrt{\frac{ds}{d\rho}} U_{iK}^{\text{HO}}[s(\rho)]$$

$$s(\rho) = \frac{1}{\sqrt{2}\boldsymbol{b}} \left[\frac{1}{\left(\frac{1}{\rho}\right)^4 + \left(\frac{1}{\gamma\sqrt{\rho}}\right)^4} \right]^{\frac{1}{4}}$$



[PRC**88**(2013)014327, PRC**90**(2014)044304]

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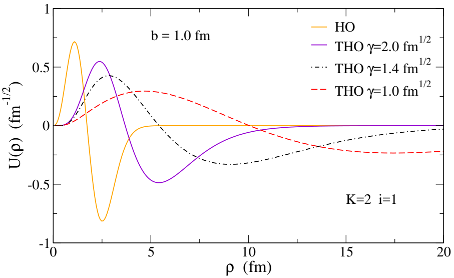
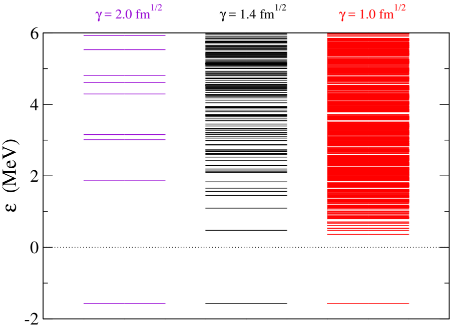
Example:

$\Psi_n^{j\mu}(\rho,\Omega)$ PS spectra, ε_n
$b = 0.7$ fm

The ratio γ/b controls the density of PS as a function of the energy.



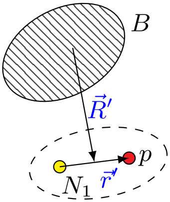


[PRC**88**(2013)014327, PRC**90**(2014)044304]

Final wave function

Expanded in proton-nucleon states (CDCC)

$$\Psi_f(\vec{r}', \vec{R}') \simeq \sum_{n\mathcal{J}\Pi} \phi_n^{\mathcal{J}\Pi}(k_n, \vec{r}')\chi_n^{\mathcal{J}\Pi}(\vec{K}', \vec{R}')$$



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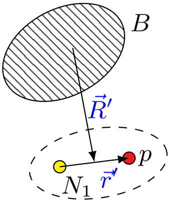

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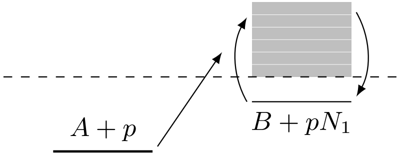
Basis of N discretized bins

$$\phi_n^{\mathcal{J}\Pi}(k_n, \vec{r}') = \sqrt{\frac{2}{\pi N}} \int_{k_{n-1}}^{k_n} \phi_{pN_1}^{\mathcal{J}\Pi}(k, \vec{r}')dk.$$



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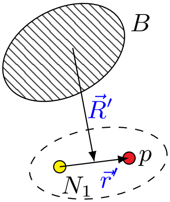

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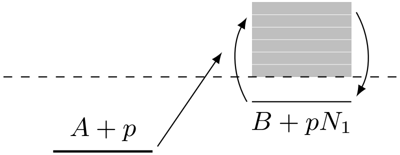
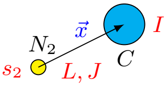


If we select the (p, d) channel TC reduces to DWBA

$$\Psi_f(\vec{r}', \vec{R}') \simeq \phi_d(\vec{r}')\chi_{d\text{-}B}(\vec{R}')$$

2b continuum state of fragment B



$$\varphi^{(+)}_{\vec{q},\sigma_2,\iota}(\vec{x}) = \frac{4\pi}{qx} \sum_{LJJ_TM_T} i^L Y^*_{LM}(\widehat{q}) \langle LM s_2\sigma_2 | JM_J \rangle \times \langle JM_J I\iota | J_T M_T \rangle f^{J_T}_{LJ}(qx) \left[\mathcal{Y}_{Ls_2J}(\widehat{x}) \otimes \kappa_I \right]_{J_T M_T}$$

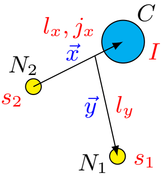
Coupling order: $\vec{L} + \vec{s}_2 = \vec{J}, \qquad \vec{J} + \vec{I} = \vec{J}_T$

Obtain solution for each $(L, J)J_T$:

$$f^{J_T}_{LJ}(qx) \longrightarrow \frac{i}{2} e^{i\sigma_L} \left[H^{(-)}_L(qx) - S^{J_T}_{LJ} H^{(+)}_L(qx) \right]$$

3b g.s. wave function of A

Analytical THO method



$$\beta \equiv \{K, l_x, j_x, j_1, l_y, j_2\}$$

$$\vec{l}_x + \vec{s}_2 = \vec{j}_x, \qquad \vec{j}_x + \vec{I} = \vec{j}_1$$

$$\vec{l}_y + \vec{s}_1 = \vec{j}_2, \qquad \vec{j}_1 + \vec{j}_2 = \vec{j}$$

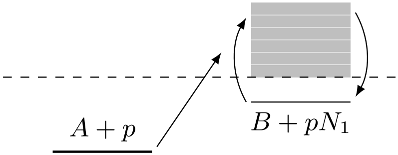
Diagonalize $\mathcal{H}_{3b}$ using: [*PRC 88 (2013) 014327*]

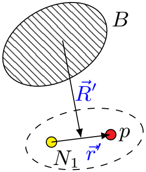
- Binary interactions C-N_i, N_1-N_2
- Three-body force to fine-tune g.s. energy

$$\Phi_A^{j\mu}(\vec{x},\vec{y}) = \sum_\beta w_\beta^j(x,y)\left\{\left[\mathcal{Y}_{l_x s_2 j_x}(\widehat{x}) \otimes \kappa_I\right]_{j_1} \otimes \left[Y_{l_y}(\widehat{y}) \otimes \kappa_{s_1}\right]_{j_2}\right\}_{j\mu}$$

2-body continuum φ_B^{2b} computed with the same C-N potential

$\Psi_f(\vec{r}', \vec{R}')$ expanded in discretized pN states (CDCC-like)





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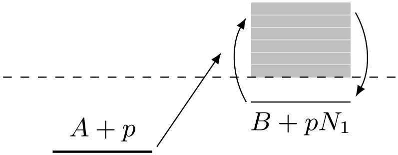
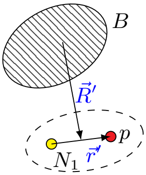




➠ Assume $(V_{pN_1} + U_{pB} - U_{pA})$ does not change the state of B

Define **overlaps** $\psi_{LJJ_TM_T}(q, \vec{y}) = \int \varphi^{2b}_{B,LJJ_TM_T}(q, \vec{x})\Phi^{3b}_A(\vec{x}, \vec{y})d\vec{x}$

Cross section from auxiliary amplitudes:

$$\boldsymbol{\mathcal{T}}_{if}^{LJJ_TM_T} \equiv \langle \Psi_f^{(-)}(\vec{r}', \vec{R}')|V_{pN_1} + U_{pB} - U_{pA}|\psi_{LJJ_TM_T}(q, \vec{y})\chi^{(+)}_{pA}\rangle$$

$$\frac{d\sigma^2}{d\Omega_B dq} \propto \sum \left|\mathcal{T}_{if}^{LJJ_TM_T}\right|^2$$

TC calculations [spin of ^{9}Li ignored, $I^\pi = 0^+$]

- ^{10}Li (^{9}Li $+ n$)

  $2s_{1/2}$ virtual state: $a = -20.9$ fm

  $1p_{1/2}$ resonance at $\sim$ 0.5 MeV

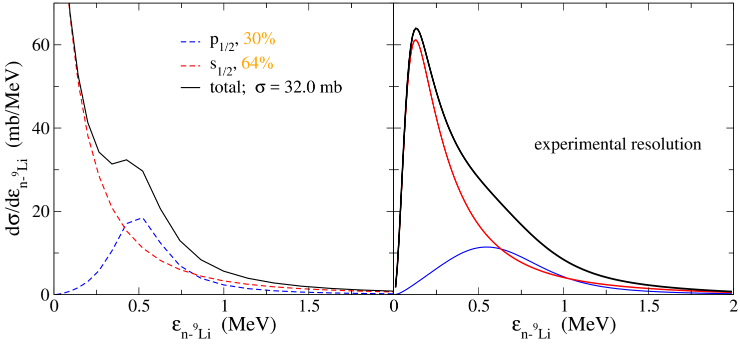
  $1d_{5/2}$ state around 4.5 MeV

- ^{11}Li (^{9}Li $+ n + n$)

  0^+ g.s. at -0.37 MeV

  $r_{mat} = 3.55$ fm, $r_{ch} = 2.48$ fm

  64% $s_{1/2}$, 30% $p_{1/2}$, 3% $d_{5/2}$



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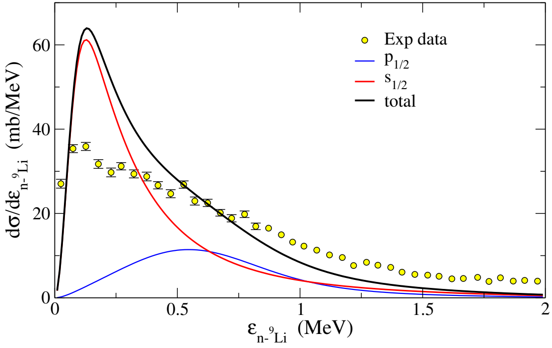
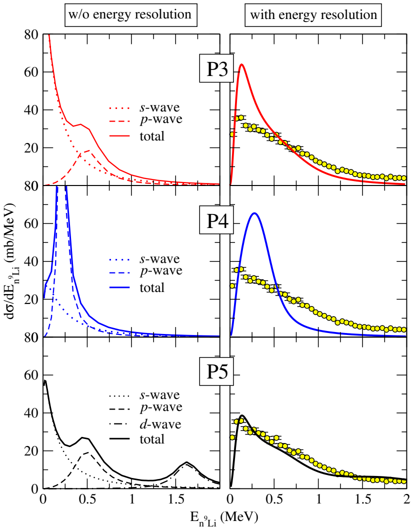




Sensitivity to the structrure model

- P3: reference model
- P4: virtual state at higher E
  p resonance at lower E
- P5: with d resonance ∼1.5 MeV

	a	$E_r[p_{1/2}]$	$E_r[d_{5/2}]$
P3	-29.8	0.50	4.3
P4	-16.2	0.23	4.3
P5	-29.8	0.50	1.5
	(fm)	(MeV)	(MeV)

	$\%s_{1/2}$	$\%p_{1/2}$	$\%d_{5/2}$
P3	64	30	3
P4	27	67	3
P5	39	35	23

Not sufficient to confirm $d_{5/2}$ resonance

Include spin of ^{9}Li; $I^\pi = 3/2^-$

[PLB 772 (2017) 115]

spin-spin splitting:

$s_{1/2} \Rightarrow 1^-, 2^-$

$p_{1/2} \Rightarrow 1^+, 2^+$

Model: P1I:

- ^{10}Li:

$a = -37.9$ fm (2^-)

res. at 0.37, 0.61 MeV

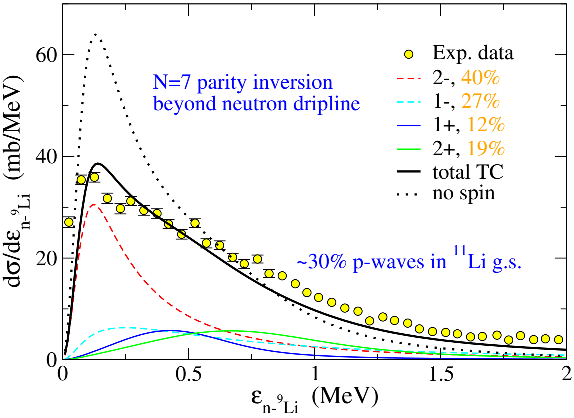
- ^{11}Li:

$3/2^-$ g.s. at -0.37 MeV

$r_{mat} = 3.2$ fm

$r_{ch} = 2.41$ fm

67% s, 31% p



Data from Aksyutina *et al.* [PLB 666 (2008) 430]

Factorization of the cross section

$$\frac{d\sigma^{LJJ_T}}{d\varepsilon_x} \simeq \mathcal{C}^{LJJ_T} K(E) \eta^{LJJ_T}(E)$$

Structure form factors (SF):

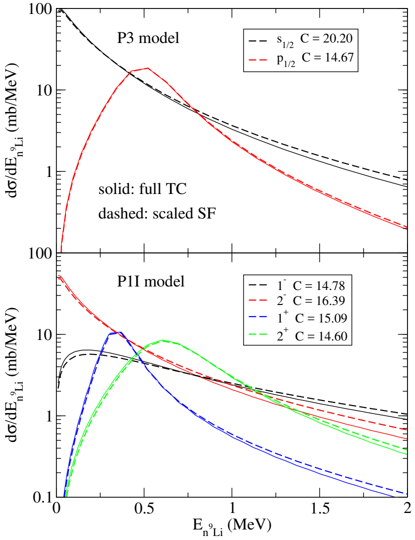
$$\eta^{LJJ_T}(E) \equiv \int d\vec{y} |\psi_{LJJ_TM_T}(E,\vec{y})|^2$$

Correct up to certain extent

BUT!!

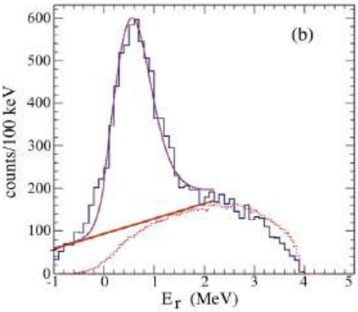
ratio depends on L, J, J_T

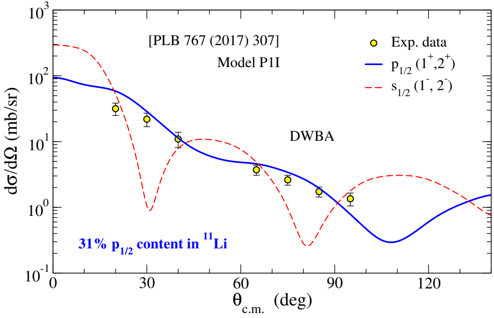
- reaction calc. to obtain relative weights
- less ambiguous than fitting



Transfer reaction ^{11}Li$(p,d)^{10}$Li

IRIS at TRIUMF, 5.7 AMeV   Sanetullaev *et al.* [PLB 755 (2016) 481]

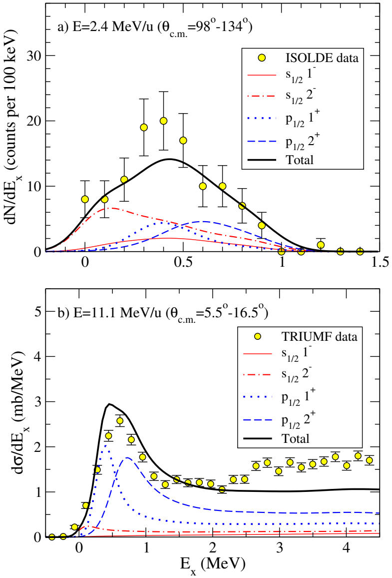


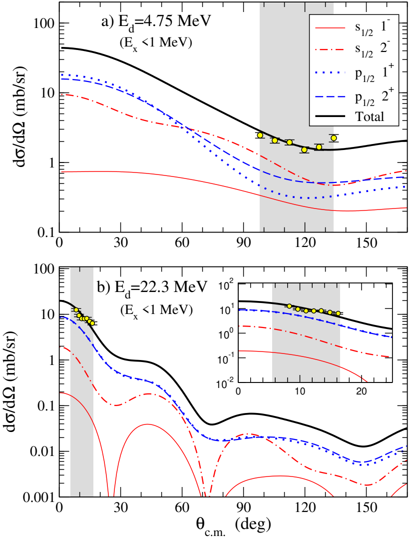


➢ Same model gives good agreement on (p,pn) and (p,d) reactions

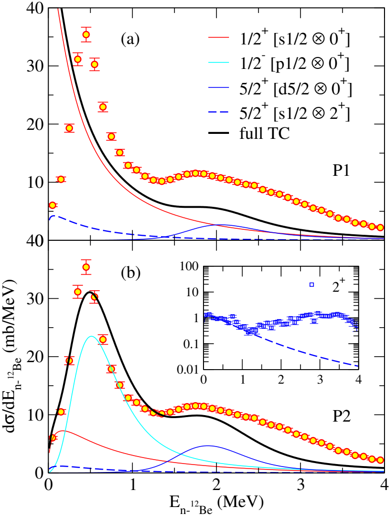
[Phys. Lett. **B767** (2017) 307]

Transfer reaction ^{9}Li$(d,p)^{10}$Li



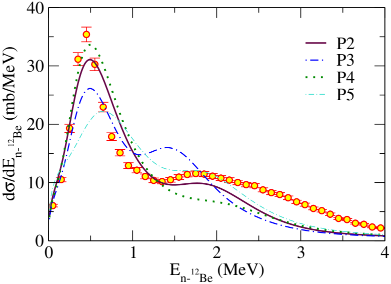


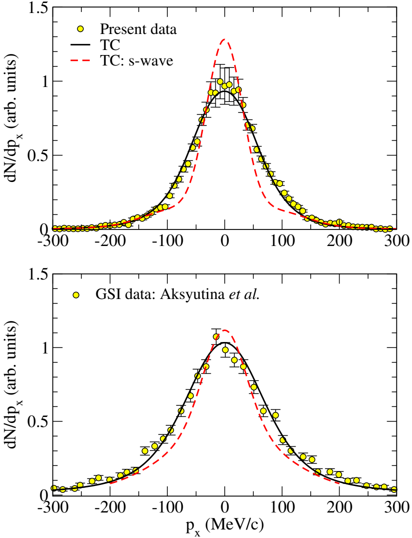
[Phys. Lett. **B793** (2019) 13]



^{14}Be$(p,pn)^{13}$Be

diff. models




1.5
1
0.5
0
-300
-200
-100
0
100
200
300
dN/dp$_x$ (arb. units)
Present data
TC
TC: s-wave
GSI data: Aksyutina *et al.*
p$_x$ (MeV/c)