Constraints on the speed of sound of dense nuclear matter through the tidal deformability of neutron stars (1)

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(1)A. Kanakis-Pegios, P.S. Koliogiannis, and Ch.C. Moustakidis, Phys. Rev. C 102, 055801, 2020.

Outline

- Introduction - Bulk properties
- Mathematical Formalism
- Tidal deformability
- Results
- Conclusions

Introduction

- Neutron Stars (NS) are among the most interesting astrophysical objects in Nature.
- Conditions of extremely high matter densities prevail in their interiors, making them a natural laboratory for extreme physics. The precise form of the equation of state (EoS) of NSs remains unknown.
- Radius: $R \approx 10 - 15km$, Mass: $M \approx 1 - 2M_{\odot}$, Mean density: $\rho \approx 4 \times 10^{14} g/cm^3$, Frequency: $f \approx few\ Hz - 700Hz$, Magnetic field: $B \approx 10^{12} - 10^{18} Gauss$.

Mathematical Formalism

The system of TOV equations of the equilibrium is given by(2)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[m(r) + \frac{4\pi r^3 P(r)}{c^2}\right]\left[1 - \frac{2Gm(r)}{rc^2}^{-1}\right]$$
$$\frac{\mathrm{d}M(r)}{\mathrm{d}r} = 4\pi^2 r^2 \rho(r), \quad \rho(r) = \mathcal{E}(r)/c^2 \qquad (1)$$

The equation of state is a three-piece function given by

$$P(\mathcal{E}) = \begin{cases} P_{\mathrm{crust}}(\mathcal{E}), & \mathcal{E} \leq \mathcal{E}_{\mathrm{c-edge}} \ ^{(3),(4)} \\ P_{\mathrm{NM}}(\mathcal{E}), & \mathcal{E}_{\mathrm{c-edge}} \leq \mathcal{E} \leq \mathcal{E}_{\mathrm{tr}} \ ^{(5)} \\ \left(\frac{v_{\mathrm{s}}}{c}\right)^2 (\mathcal{E} - \mathcal{E}_{\mathrm{tr}}) + P_{\mathrm{NM}}(\mathcal{E}_{\mathrm{tr}}), & \mathcal{E}_{\mathrm{tr}} \leq \mathcal{E}. \end{cases} \qquad (2)$$

For $\mathcal{E}_{\mathrm{tr}} \leq \mathcal{E}$ the EoS is maximally stiff with the speed of sound, defined as $v_s = c\sqrt{(\partial P/\partial \mathcal{E})}_S$, fixed on the two values, $c/\sqrt{3}$ and c.

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(2) K. Ch. Chatzisavvas, V. P. Psonis, C. P. Panos and Ch.C. Moustakidis,Phys. Lett.A 373, 3901 (2009).

(3) R. Feynman, N. Metropolis, and E. Teller, Phys. Rev. 75, 1561 (1949).

(4) G. Baym, C. Pethik, and P. Sutherland, Astrophys. J. 170, 299 (1971).

(5) A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998).

Speed of sound parametrization

In Eq. (2), the continuity in v_s and $n_{\rm tr}$ needs to be defined to ensure the continuity and a smooth phase transition. The following parametrization has been applied(6)

$$\frac{v_{\rm s}}{c} = \left(a - c_1 \exp\left[-\frac{(n-c_2)^2}{w^2}\right]\right)^{1/2}, \quad a = 1, 1/3 \tag{3}$$

where the parameters c_1, c_2, and w are fit to the speed of sound and its derivative at $n_{\rm tr}$, and also to the demands $v_{\rm s}(n_{\rm tr}) = [c, c/\sqrt{3}]$(7). Using Eq. (3), the EoS for $n \geq n_{\rm tr}$ can be constructed with the help of the following recipe(6)

$$\mathcal{E}_{i+1} = \mathcal{E}_i + \Delta\mathcal{E}, \quad P_{i+1} = P_i + \left(\frac{v_s}{c}(n_i)\right)^2 \Delta\mathcal{E}, \tag{4}$$

$$\Delta\mathcal{E} = \Delta n \left(\frac{\mathcal{E}_i + P_i}{n_i}\right), \quad \Delta n = n_{i+1} - n_i. \tag{5}$$

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(6) I. Tews, J. Carlson, S. Gandolfi, and S. Reddy, Astrophys. J. 860, 149 (2018).

(7) Ch. Margaritis, P. S. Koliogiannis, and Ch. C. Moustakidis, Phys. Rev. D 101, 043023 (2020).

Tidal effects during inspiral of a binary neutron star system

- Gravitational-waves (GW) from the late phase of a binary neutron star (BNS) system are among the most significant sources for a GW detector.
- Tidal field E_{ij} induces change of quadrapole moment Q_{ij} of the neutron star.
- The quadrapole moment is given by(8),(9)

$$Q_{ij} = -\lambda E_{ij}, \text{ with } \lambda = \frac{2}{3} k_2 \frac{R^5}{G} \tag{6}$$

- The induced quadrupole moment depends on neutron star structure.
- Tidal deformability λ depends on radius R (the smaller the star, the harder to deform) and k_2. The reaction of the star to the tidal field is described by the parameter k_2, known as tidal love number, which depends on neutron star's structure.

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(8) E. E. Flanagan and T. Hinderer, Phys. Rev. D 77, 021502(R) (2008).

(9) Kip S. Thorne, Phys. Rev. D 58, 124031 (1998).

Binary quantities

One of the binary parameters that is well constrained by the gravitational wave detectors is the chirp mass $\mathcal{M}_c$, which is a combination of the component masses(10),(11)

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = m_1 \frac{q^{3/5}}{(1+q)^{1/5}}, \qquad (7)$$

where m_1 is the mass of the heavier component star and m_2 is the lighter's one. Hence, the binary mass ratio $q = m_2/m_1$ is within $0 \leq q \leq 1$.

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(10) B. P. Abbott et al., Phys. Rev. Lett. 119, 161101 (2017).

(11) B. P. Abbott et al., Phys. Rev. X 9, 011001 (2019).

Binary tidal deformability

- During the last orbits before the neutron stars merging, the orbital phase evolution is affected by the tidal deformation.
- At leading order,the tidal effects are imprinted in the gravitational-wave signal through the effective (binary) tidal deformability(10),(11):

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}, \tag{8}$$

where the key quantity q characterizes the mass asymmetry. Moreover, Λ_i is the dimensionless deformability defined as(10),(11)

$$\Lambda_i = \frac{2}{3} k_2 \left(\frac{R_i c^2}{M_i G} \right)^5 \equiv \frac{2}{3} k_2 \beta_i^{-5}, \quad i = 1, 2. \tag{9}$$

Results: Mass vs radius

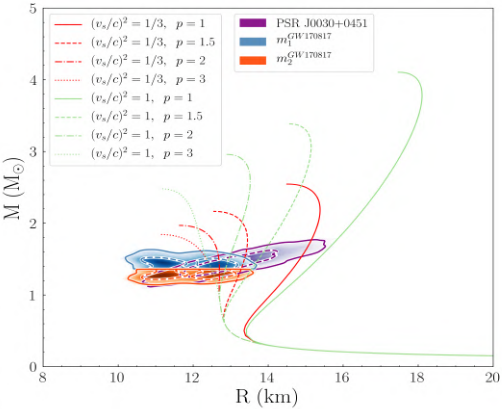

Figure: The green (red) lines correspond to the upper (lower) bound. The purple diagonal shaded region corresponds to NICER's observation$^{(12)}$, while the blue upper (orange lower) shaded region corresponds to the higher (smaller) component of GW170817 event $^{(13)}$.

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(12) M. C. Miller et al., Astrophys. J. Lett. 887, L24 (2019).

(13) B. P. Abbott et al., Phys. Rev. Lett. 121, 161101 (2018).

Results: $\tilde{\Lambda} - q$ for GW170817, and GW190425 events

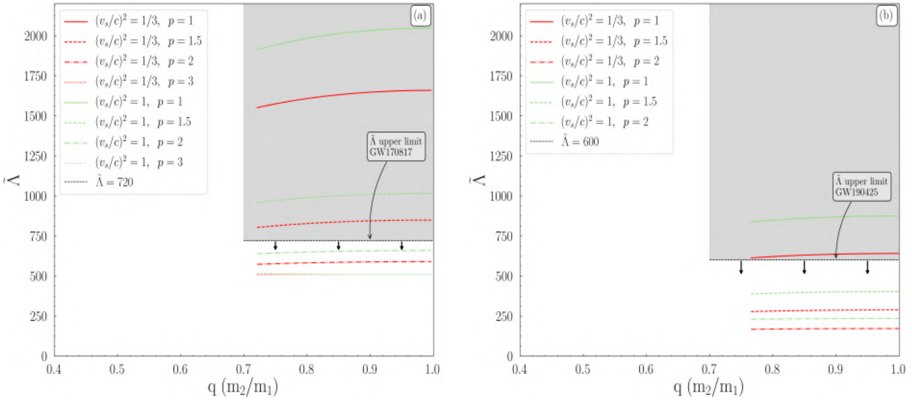

Figure: $\tilde{\Lambda}$ as a function of the binary mass ratio q for the event GW170817 (left panel) and GW190425 (right). The grey shaded regions mark the excluded areas$^{(13),(14)}$. The red (green) lines correspond to the $(v_s/c)^2 = 1/3$ $((v_s/c)^2 = 1)$ limit.

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(14) B. P. Abbott et al., Astrophys. J. Lett. 892, L3 (2020).

Results: $\tilde{\Lambda} - n_{\mathrm{tr}}/n_0$ for GW170817, and GW190425 events

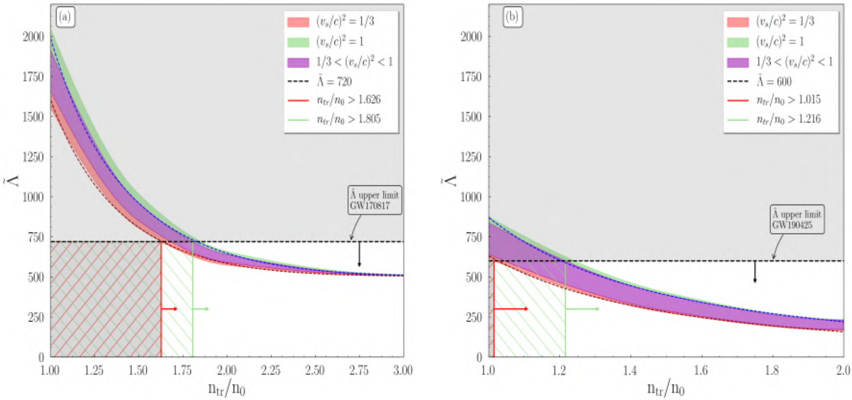

Figure: The corresponding upper observation limits for $\tilde{\Lambda}^{(13),(14)}$ as well as the compatible lower transition density values are also indicated for both events. The red (green) arrow marks the accepted region of transition density for the $v_s = c/\sqrt{3}$ ($v_s = c$) case. The red lower (green upper) curved shaded region corresponds to the $v_s = c/\sqrt{3}$ ($v_s = c$) limit. The purple intermediate shaded region indicates the predictions for middle values of speed of sound bound between the two limits of our study.

Results: $\tilde{\Lambda} - M_{max}$ for GW170817, and GW190425 events

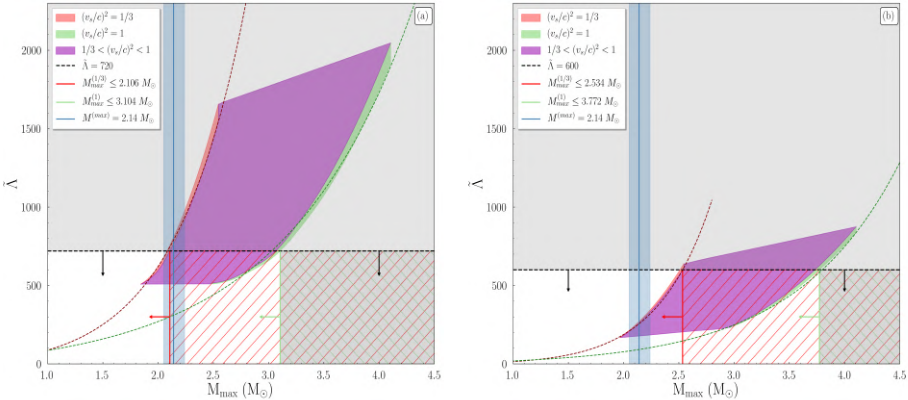

Figure: The corresponding upper observation limits for $\tilde{\Lambda}$ (black dashed(11),(14)), the compatible maximum mass shaded regions, for $v_s = c/\sqrt{3}$ (left red) case, $v_s = c$ case (right green) and the middle cases (purple), as well the current observed maximum neutron star mass $M = 2.14^{+0.10}_{-0.09}\ M_{\odot}$ (blue shaded vertical region(15)) are also indicated. The red left (green right) arrow marks the accepted region of maximum mass M_{max} for $v_s = c/\sqrt{3}$ ($v_s = c$) case.

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(15) H. Cromartie et al., Nat. Astron. 4, 72 (2020).

Remarks

1. We postulate that future observations may offer even more rigorous constraints on the bound of the speed of sound. Further information on the upper limit of $\tilde{\Lambda}$, could lead to more stringent constraints on $n_{\rm tr}$ and the sound speed bounds.
2. The more informative events, for the lower limit of $n_{\rm tr}$, would be those with lighter masses.
3. Lower limit on $\tilde{\Lambda}$ derived from the EM counterpart of the events might be able to lead to the estimation of an upper value on the transition density $n_{\rm tr}$ where the two speed of sound bounds must be reached.
4. Further detection of neutron stars mergers will assist both on the neutron stars maximum mass determination and its link to the speed of sound.

Thank You