

# Connecting the NEoS to the interplay between fusion and quasi-fission processes in low-energy nuclear reactions

HINPw6 Workshop

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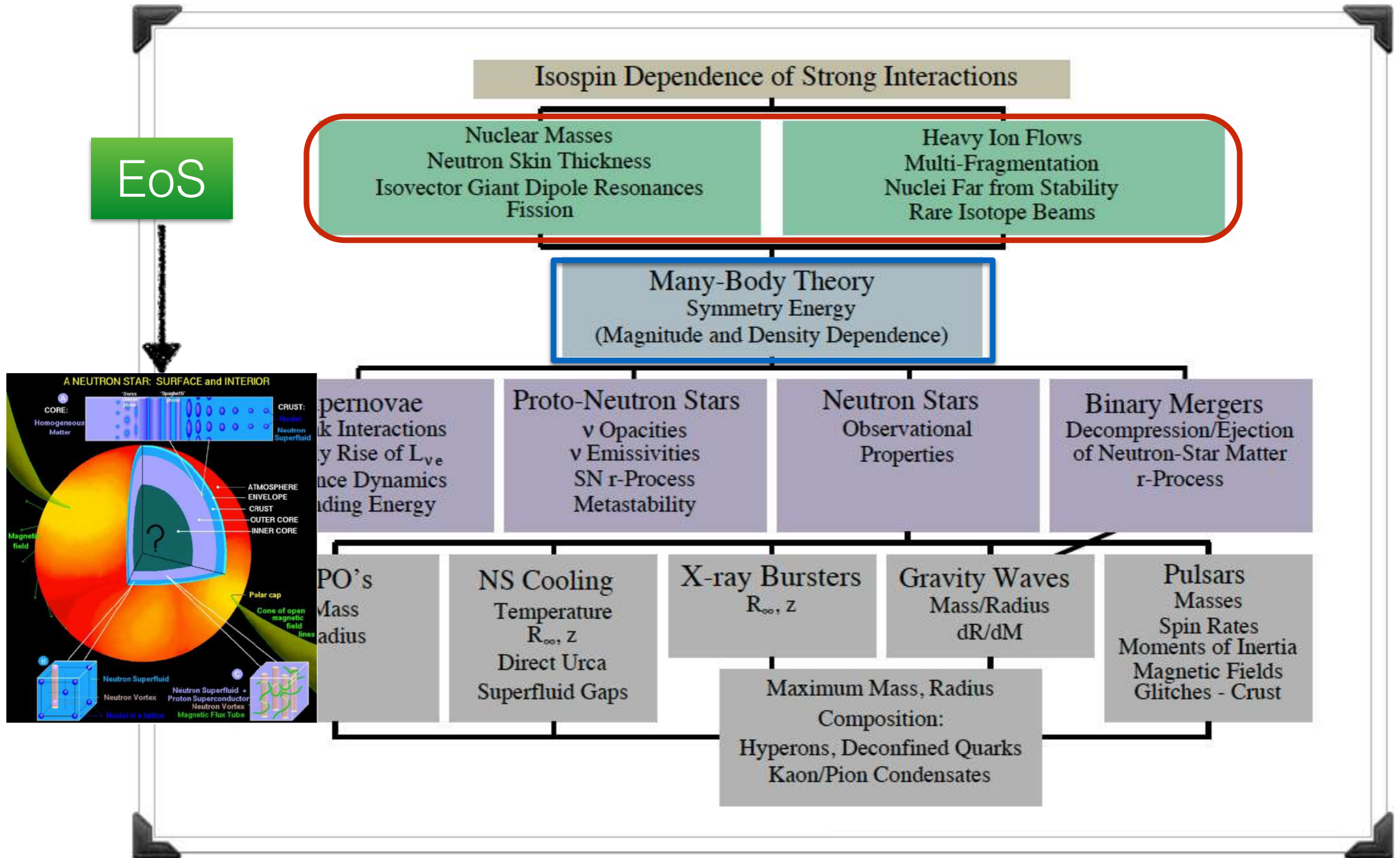


**Collaborators: S. Burrello, M. Colonna, D. Lacroix, G. Scamps**

# Outline

- Equation of state (EoS) of nuclear matter
- Low-energy ( $E/A \sim 5-10$  MeV/A), reaction mechanisms: from fusion to quasi-fission and deep-inelastic
- The tool: mean-field models (TDHF, Vlasov) and effective interactions
- Sensitivity of selected observables to specific ingredients of the effective interaction
- Conclusions

# Equation of State (EoS) is important



# Nuclear Density functional theory

- Nuclear DFT has been introduced by **effective Hamiltonians**: by [Vautherin and Brink PRC 5, 626 \(1972\)](#), using the Skyrme model as a vehicle

$$E = \langle \Psi | H | \Psi \rangle \approx \langle \Phi | \hat{H}_{eff}(\hat{\rho}) | \Phi \rangle = E[\hat{\rho}]$$

Based on the philosophy of Bethe, Goldstone, and Brueckner, one has a density dependent interaction in the nuclear interior  $E(\rho)$

At present, the ansatz for  $E(\rho)$  is phenomenological:

- **Skyrme:** non-relativistic, zero range
- **Gogny:** non-relativistic, finite range (Gaussian)
- **CDFT:** Covariant density functional theory



# Skyrme EoS (standard form)

- Effective interaction in standard form

$$\begin{aligned}
 V(\mathbf{r}_1, \mathbf{r}_2) = & t_0 (1 + x_0 P_\sigma) \delta(\mathbf{r}) && \text{central term} \\
 & + \frac{1}{2} t_1 (1 + x_1 P_\sigma) \left[ \mathbf{P}'^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{P}^2 \right] \\
 & + t_2 (1 + x_2 P_\sigma) \mathbf{P}' \cdot \delta(\mathbf{r}) \mathbf{P} && \text{non-local terms} \\
 & + \frac{1}{6} t_3 (1 + x_3 P_\sigma) [\rho(\mathbf{R})]^\sigma \delta(\mathbf{r}) && \text{density-dependent term} \\
 & + iW_0 \boldsymbol{\sigma} \cdot \left[ \mathbf{P}' \times \delta(\mathbf{r}) \mathbf{P} \right] && \text{spin-orbit term.}
 \end{aligned}$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = \frac{1}{2} (\mathbf{r}_1 + \mathbf{r}_2),$$

$$\mathbf{P} = \frac{1}{2i} (\nabla_1 - \nabla_2), \quad \mathbf{P}' \text{ cc of } \mathbf{P} \text{ acting on the left}$$

and

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2, \quad P_\sigma = (1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) / 2.$$

Modifications can be introduced and are referred as non-standard form.

# Skyrme EoS (standard form)

- Within the standard form, the total energy density is

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}} + \mathcal{H}_{\text{so}} + \mathcal{H}_{\text{sg}} + \mathcal{H}_{\text{Coul}}$$

where  $\mathcal{K} = \frac{\hbar^2}{2m} \tau$  is the kinetic-energy term,  $\mathcal{H}_0$  a zero-range term,  $\mathcal{H}_3$  the density-dependent term,  $\mathcal{H}_{\text{eff}}$  an effective-mass term,  $\mathcal{H}_{\text{fin}}$  a finite-range term,  $\mathcal{H}_{\text{so}}$  a spin-orbit term and  $\mathcal{H}_{\text{sg}}$  a term due to the tensor coupling with spin and gradient.

$$\mathcal{H}_0 = \frac{1}{4} t_0 [(2 + x_0) \rho^2 - (2x_0 + 1) (\rho_p^2 + \rho_n^2)] ,$$

$$\mathcal{H}_3 = \frac{1}{24} t_3 \rho^\sigma [(2 + x_3) \rho^2 - (2x_3 + 1) (\rho_p^2 + \rho_n^2)] ,$$

$$\mathcal{H}_{\text{eff}} = \frac{1}{8} [t_1 (2 + x_1) + t_2 (2 + x_2)] \tau \rho$$

$$+ \frac{1}{8} [t_2 (2x_2 + 1) - t_1 (2x_1 + 1)] (\tau_p \rho_p + \tau_n \rho_n) ,$$

$$\mathcal{H}_{\text{fin}} = \frac{1}{32} [3t_1 (2 + x_1) - t_2 (2 + x_2)] (\nabla \rho)^2$$

$$- \frac{1}{32} [3t_1 (2x_1 + 1) + t_2 (2x_2 + 1)] [(\nabla \rho_p)^2 + (\nabla \rho_n)^2] ,$$

$$\mathcal{H}_{\text{so}} = \frac{1}{2} W_0 [\mathbf{J} \cdot \nabla \rho + \mathbf{J}_p \cdot \nabla \rho_p + \mathbf{J}_n \cdot \nabla \rho_n] ,$$

$$\mathcal{H}_{\text{sg}} = -\frac{1}{16} (t_1 x_1 + t_2 x_2) \mathbf{J}^2 + \frac{1}{16} (t_1 - t_2) [\mathbf{J}_p^2 + \mathbf{J}_n^2] .$$

Nine parameters

$(t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \sigma)$

# Skyrme EoS

- The total energy density in another form is

$$\mathcal{E}(\rho) = \frac{\hbar^2}{2m}\tau + C_0\rho^2 + D_0\rho_3^2 + C_3\rho^{\sigma+2} + D_3\rho^\sigma\rho_3^2 + C_{eff}\rho\tau + D_{eff}\rho_3\tau_3 + C_{surf}(\nabla\rho)^2 + D_{surf}(\nabla\rho_3)^2, \quad (2)$$

$$\begin{aligned} \rho &= \rho_n + \rho_p & \rho_3 &= \rho_n - \rho_p \\ \tau &= \tau_n + \tau_p & \tau_3 &= \tau_n - \tau_p, \end{aligned}$$

with  $\rho_i$  and  $\tau_i$  ( $i = p, n$ , for protons and neutrons) particles and kinetic energy density, respectively.

$$C_3 = \frac{1}{16}t_3,$$

$$D_3 = -\frac{1}{48}t_3(2x_3 + 1),$$

$$C_0 = \frac{3}{8}t_0,$$

$$D_0 = -\frac{1}{8}t_0(2x_0 + 1),$$

$$C_{eff} = \frac{1}{16}[3t_1 + t_2(4x_2 + 5)],$$

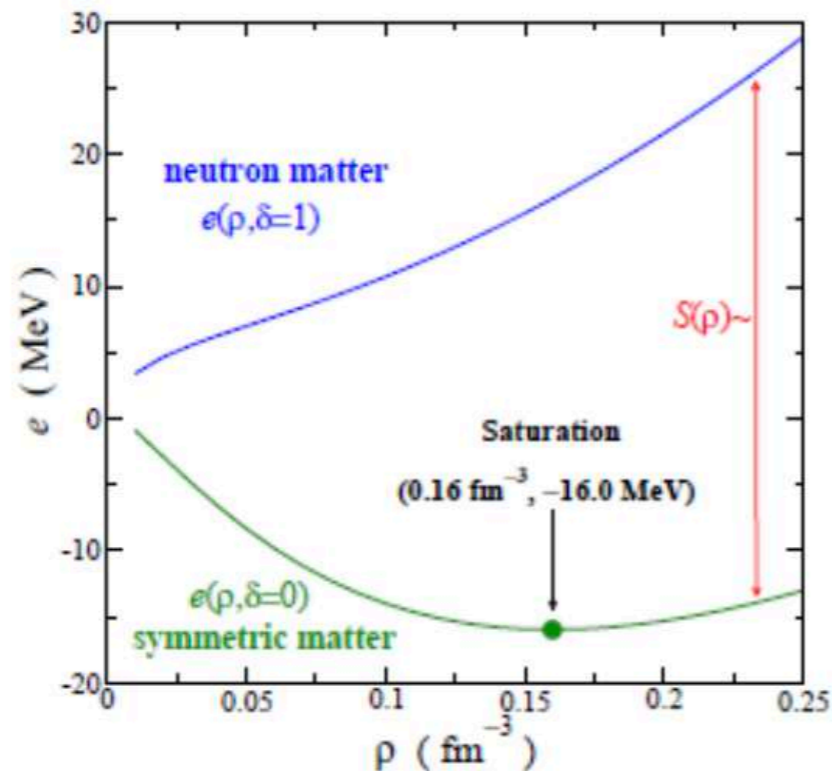
$$D_{surf} = -\frac{1}{64}[3t_1(2x_1 + 1) + t_2(2x_2 + 1)].$$

$$D_{eff} = -\frac{1}{16}[t_1(2x_1 + 1) - t_2(2x_2 + 1)],$$

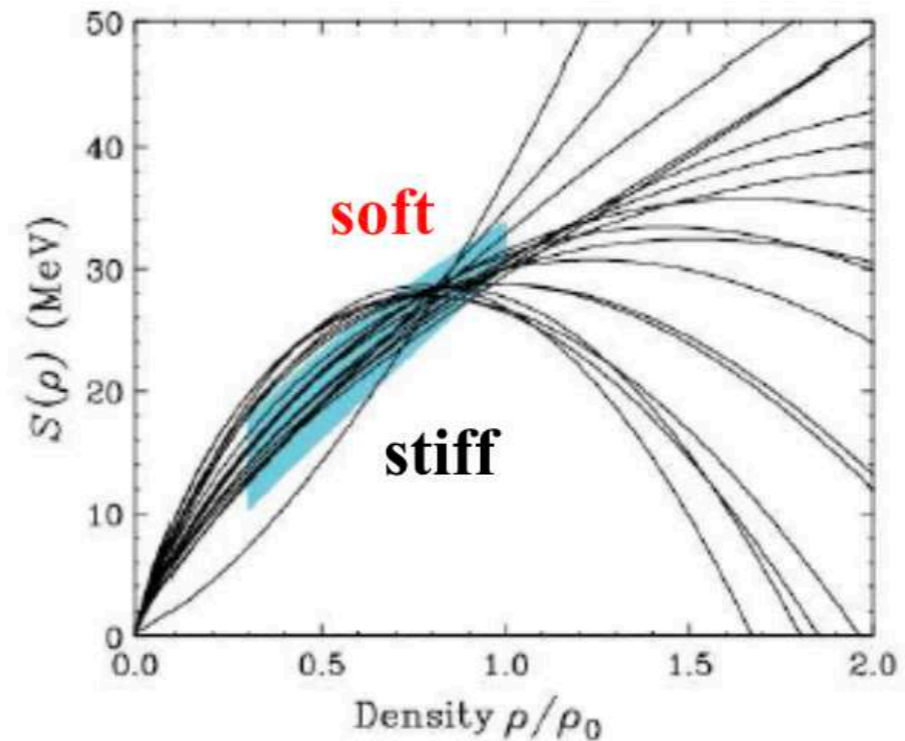
$$C_{surf} = \frac{1}{64}[9t_1 - t_2(4x_2 + 5)],$$

# EoS(T=0) and symmetry energy

Energy per nucleon  $E/A$  (MeV)



Symmetry energy  $E_{sym}$  (MeV)



poorly known ...

*predictions of several effective interactions*

$$E/A(\rho, \delta) = E/A(\rho, \delta = 0) + E_{sym}(\rho)\delta^2 + O(\delta^4)$$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

➤ analogy with **Weizsacker mass formula** for nuclei (symmetry term) !

$$E_{sym}(\rho) = S_0(\text{or } J) + L \frac{\rho - \rho_0}{3\rho_0} + \dots$$

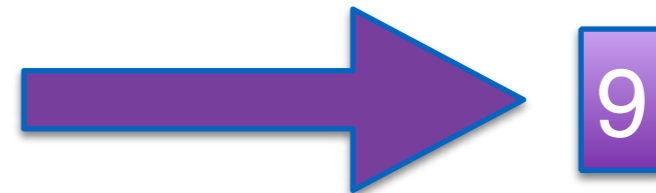
$$25 \leq J \leq 35 \text{ MeV}$$

$$20 \leq L \leq 120 \text{ MeV}$$



# Associate the nuclear properties with Skyrme EoS

1. Saturation density  $\rho_0$
2. Energy per nucleon  $E/A(\rho_0)$
3. Incompressibility  $K_0$
4. Isoscalar effective mass  $m_S^*$
5. Isovector effective mass  $m_V^*$
6. Symmetry energy  $J$
7. Slope of the symmetry energy  $L$
8. isoscalar surface term  $G_S$
9. Isovector surface term  $G_V$

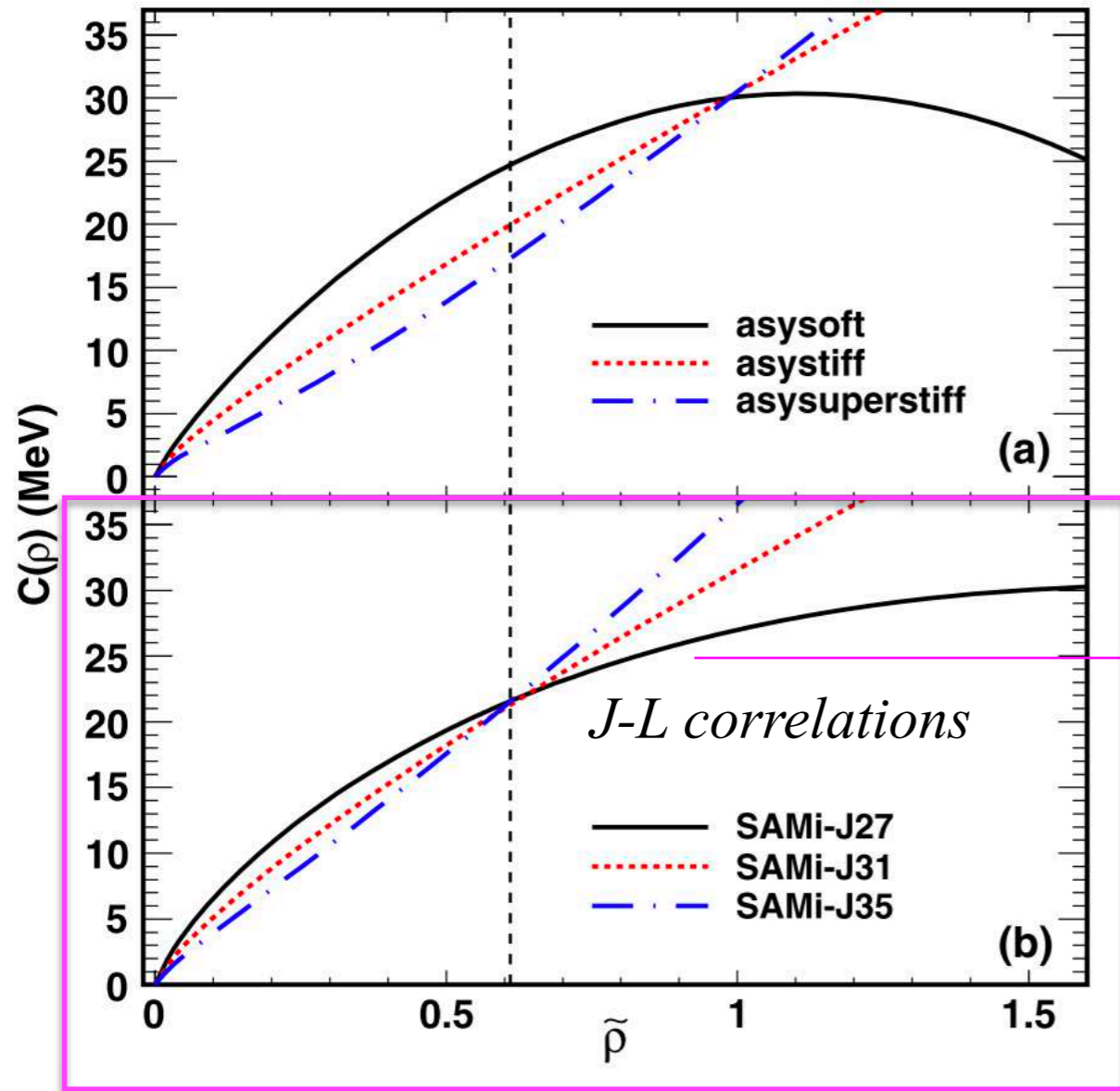


Vs

9

The parameters can be determined  $(t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \sigma)$

# Skyrme EoS adopted



Effective interaction	J [MeV]	L [MeV]	Effective interaction	J [MeV]	L [MeV]
asy-soft	30	14.8	SAMi-J27	27	29.9
asy-stiff	30.5	79	SAMi-J31	31	74.5
asy-superstiff	30.5	106	SAMi-J35	35	115.2

**SAMi-J:**  
changing the **symmetry energy** slope

Taking SAMi-J31 as a reference:  
consider interactions with different

- symmetry energy
- **incompressibility**
- **effective mass**
- n/p effective mass splitting
- **surface terms**

## SAMi-J:

X. Roca-Maza, G. Colò, H. Sagawa, Phys. Rev. C 86, 031306(R) (2012); X. Roca-Maza *et al.*, Phys. Rev. C 87, 034301 (2013).

# Skyrme EoS adopted

No.	EOS	$\rho_0$ (fm <sup>-3</sup> )	$E_0$ (MeV)	$K_0$ (MeV)	$J$ (MeV)	$L$ (MeV)	$m_s^*/m$	$m_v^*/m$	$f_I$	$G_S$	$G_V$	Result
S1	SAMi- <i>J27</i>	0.160	-15.93	245	27	30	0.675	0.664	-0.0251	149.2	-8.6	Fusion
	SAMi- <i>J31</i>	0.156	-15.83	245	31	74	0.675	0.664	-0.0251	140.9	3.1	Fusion
	SAMi- <i>J35</i>	0.154	-15.69	245	35	115	0.675	0.664	-0.0251	131.1	15.4	Fission
S2	<i>J27</i>	0.156	-15.83	245	27	30	0.675	0.664	-0.0251	140.9	3.1	Fusion
S3	<i>J35</i>	0.156	-15.83	245	35	115	0.675	0.664	-0.0251	140.9	3.1	Fusion
	Gs35	0.156	-15.83	245	31	74	0.675	0.664	-0.0251	131.1	3.1	Fission
	<i>J35_Gs35</i>	0.156	-15.83	245	35	115	0.675	0.664	-0.0251	131.1	3.1	Fission
S4	<i>J35_Gv35</i>	0.156	-15.83	245	35	115	0.675	0.664	-0.0251	140.9	15.4	Fusion
	<i>J35_Gs35Gv35</i>	0.156	-15.83	245	35	115	0.675	0.664	-0.0251	131.1	15.4	Fission
	K200	0.156	-15.83	200	31	74	0.675	0.664	-0.0251	140.9	3.1	Fission
S5	K290	0.156	-15.83	290	31	74	0.675	0.664	-0.0251	140.9	3.1	Fusion
S6	ms085	0.156	-15.83	245	31	74	0.85	0.832	-0.0251	140.9	3.1	Fusion
S7	ms100	0.156	-15.83	245	31	74	1.0	0.976	-0.0251	140.9	3.1	Fusion
	Gs35_ms085	0.156	-15.83	245	31	74	0.85	0.832	-0.0251	131.1	3.1	Fusion
	Gs35_ms100	0.156	-15.83	245	31	74	1.0	0.976	-0.0251	131.1	3.1	Fusion
S8	fI020	0.156	-15.83	245	31	74	0.675	0.781	0.20	140.9	3.1	Fusion
S9	fIn024	0.156	-15.83	245	31	74	0.675	0.581	-0.24	140.9	3.1	Fusion
	Gs35_fI020	0.156	-15.83	245	31	74	0.675	0.781	0.2	131.1	3.1	Fission
	Gs35_fIn024	0.156	-15.83	245	31	74	0.675	0.581	-0.24	131.1	3.1	Fission

The units of  $G_S$  and  $G_V$  are  $MeV fm^5$

The EoS name follows the convention that we only label the terms which are different with respect to the ingredients of the SAMi-J31 parametrization.



# Skyrme EoS adopted

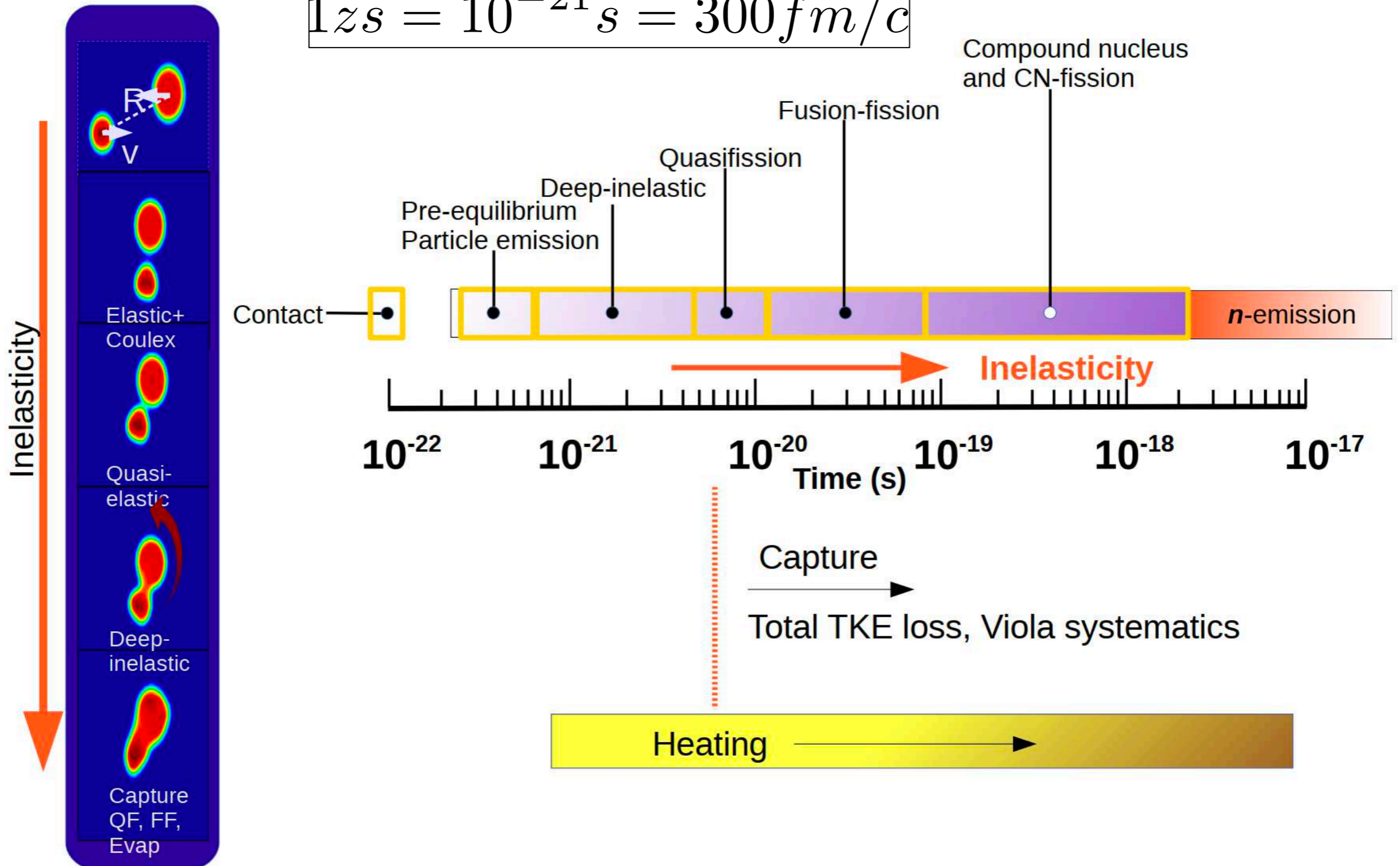
No.	EOS	$t_0$	$t_1$	$t_2$	$t_3$	$x_0$	$x_1$	$x_2$	$x_3$	$\sigma$
S1	SAMi- <i>J27</i>	-1876.09	481.087	-75.7069	10184.6	0.482235	-0.557967	0.213066	1.00219	0.254634
	SAMi- <i>J31</i>	-1844.28	460.727	-110.200	10112.4	-0.0237088	-0.458608	-0.431251	0.00764843	0.268372
	SAMi- <i>J35</i>	-1799.53	436.229	-144.972	9955.45	-0.443908	-0.343557	-0.783861	-0.882427	0.284323
S2	<i>J27</i>	-1844.27	460.727	-110.200	10112.4	0.478794	-0.458608	-0.431252	1.012559	0.268374
S3	<i>J35</i>	-1844.27	460.727	-110.200	10112.4	-0.461008	-0.458608	-0.431252	-0.879839	0.268374
	Gs35	-1844.28	434.803	-84.2766	10112.4	-0.0237087	-0.456140	-0.410106	0.00764882	0.268374
	<i>J35_Gs35</i>	-1844.27	434.803	-84.2767	10112.4	-0.461008	-0.456140	-0.410106	-0.879839	0.268374
	<i>J35_Gv35</i>	-1844.27	460.727	-175.617	10112.4	-0.461008	-0.352118	-0.736234	-0.879839	0.268374
	<i>J35_Gs35Gv35</i>	-1844.27	434.803	-149.694	10112.4	-0.461008	-0.343301	-0.777144	-0.879839	0.268374
S4	K200	5698.04	460.727	-110.200	-36164.8	0.0177978	-0.458608	-0.431251	0.00764843	-0.0421665
S5	K290	-1295.07	460.727	-110.200	8342.72	-0.0370067	-0.458608	-0.431251	0.00764843	0.578726
S6	ms085	-1696.12	406.841	-271.859	11451.3	-0.105374	-0.453125	-0.472133	-0.281046	0.354121
S7	ms100	-1654.78	375.621	-365.519	12510.9	-0.130771	-0.449229	-0.479273	-0.397744	0.388782
	Gs35_ms085	-1696.12	380.917	-245.935	11451.3	-0.105374	-0.449935	-0.469195	-0.281046	0.354121
	Gs35_ms100	-1654.78	349.697	-339.596	12511.0	-0.130771	-0.445466	-0.477691	-0.397744	0.388782
S8	fI020	-1844.27	460.727	-349.145	10112.4	0.144457	-0.588264	-0.991579	0.514540	0.268374
S9	fIn024	-1844.27	460.727	117.911	10112.4	-0.184250	-0.334830	-2.015208	-0.476261	0.268374
	Gs35_fI020	-1844.27	434.803	-323.221	10112.4	0.144457	-0.593527	-1.031006	0.514540	0.268374
	Gs35_fIn024	-1844.27	434.803	143.834	10112.4	-0.184250	-0.324982	-1.742118	-0.476261	0.268374

The EoS name follows the convention that we only label the terms which are different, with respect to the ingredients of the SAMi-J31 parametrization.



# Inelasticity and time scales at low-energy nuclear reactions

$$1z s = 10^{-21} s = 300 fm/c$$

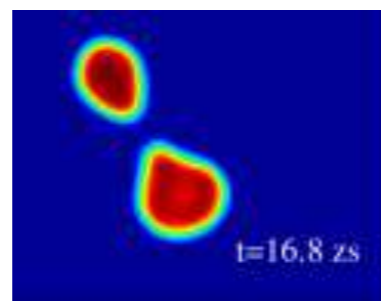
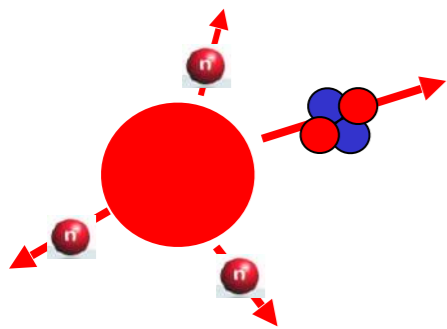


Courtesy of Yu. Ts. Oganessian

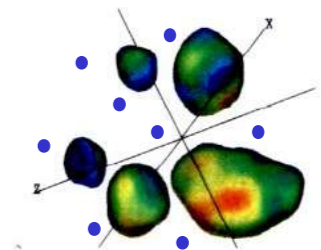
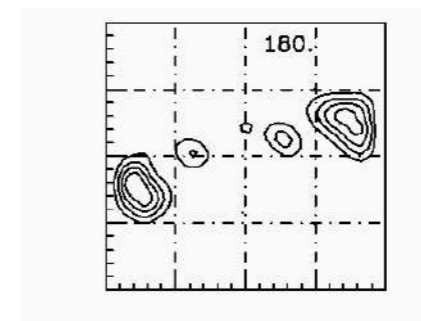
# Low-energy reaction mechanisms: a study within mean-field models

(Fermi energies)

- Fusion vs Quasi-fission or Deep Inelastic
- Charge equilibration



- Fragmentation
- Fragment isotopic composition
- Phase transition



# (Beyond) Mean-field models and effective interactions

$$i\hbar \frac{\partial}{\partial t} \hat{\rho}(t) = [H_{eff}[\hat{\rho}], \hat{\rho}(t)] + K(\hat{\rho}) + \delta K(\hat{\rho}, \delta\sigma)$$

TDHF

ETDHF

One-body description:

$\hat{\rho}$  : the one-body density matrix

Semi-classical approximation

$$\frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} + \{f, H_{eff}\} = k[f] + \delta k$$

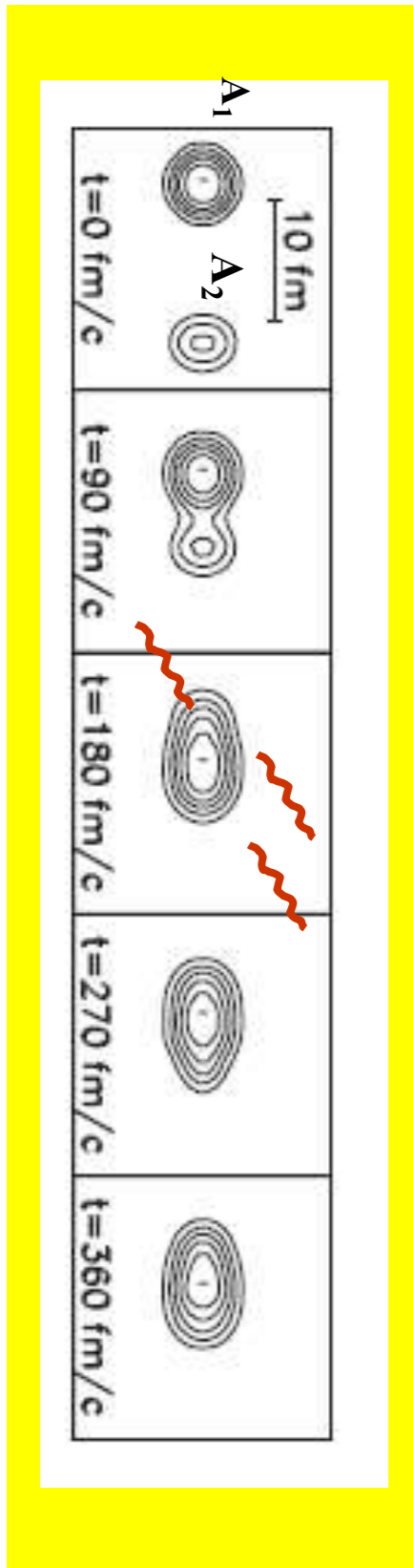
$H_{eff}$  : effective Hamiltonian

Vlasov

BUU,  
Boltzmann-Langevin

Residual interaction:  
In-medium NN cross-section  
2-body correlations, fluctuations

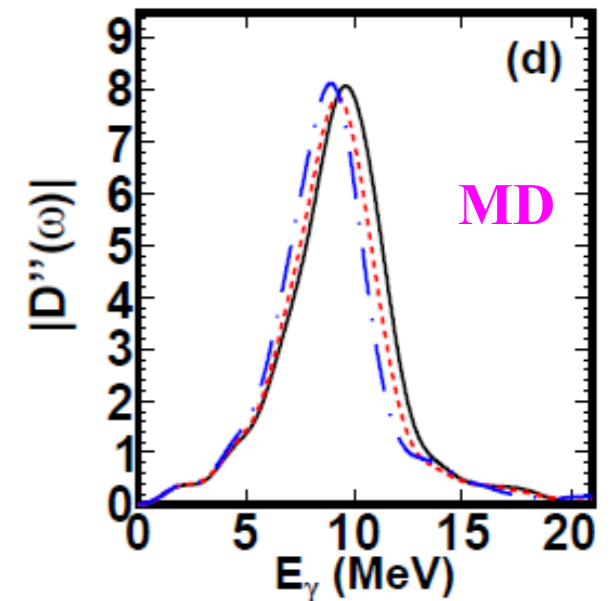
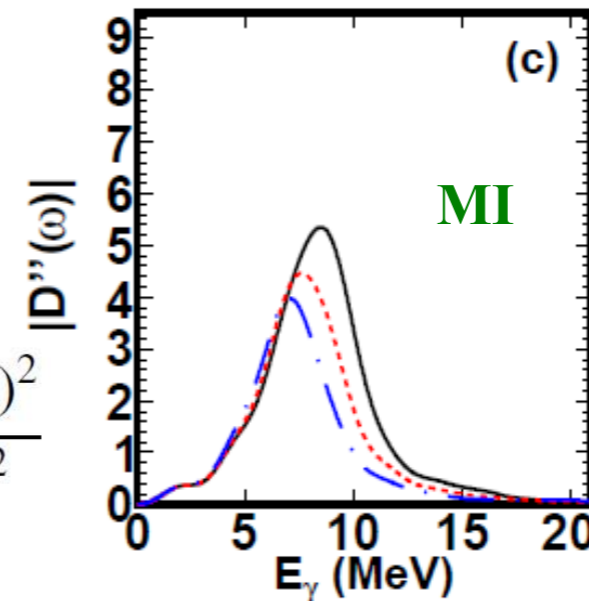
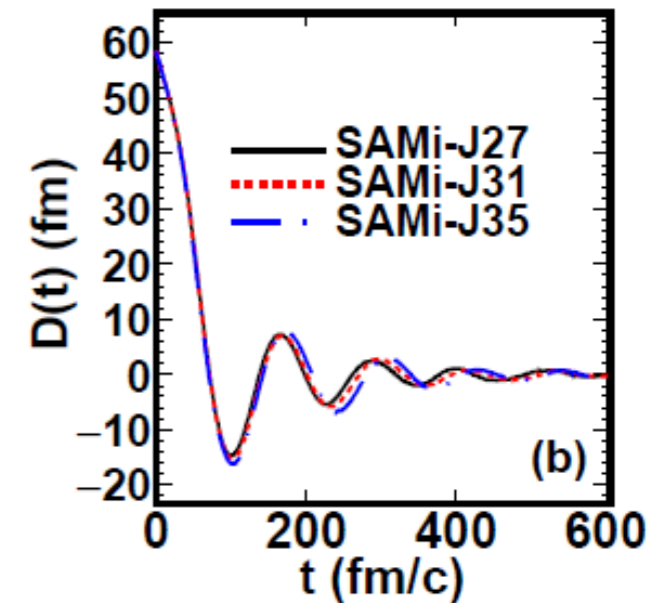
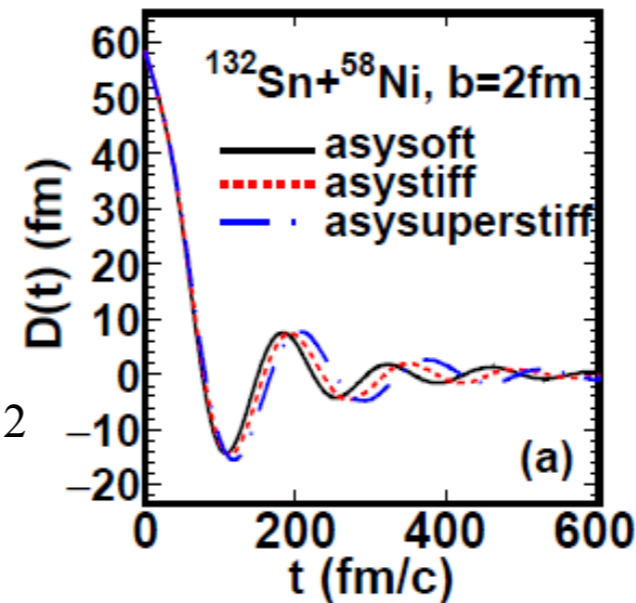
# Charge equilibration and dipole oscillations: dependence on the effective interaction



$$\frac{dP}{dE_\gamma} = \frac{2e^2}{3\pi\hbar c^3 E_\gamma} |D''(\omega)|^2$$

$$|D''(\omega)|^2 = \frac{(\omega_0^2 + 1/\tau^2)^2 D(t_0)^2}{(\omega - \omega_0)^2 + 1/\tau^2}$$

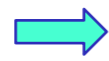
$^{132}\text{Sn} + ^{58}\text{Ni}$ ,  $E/A = 10$  MeV



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$$P_\gamma \approx D_0^2 E_{centr}^3 \tau_{coll}$$

(damped harmonic oscillator)

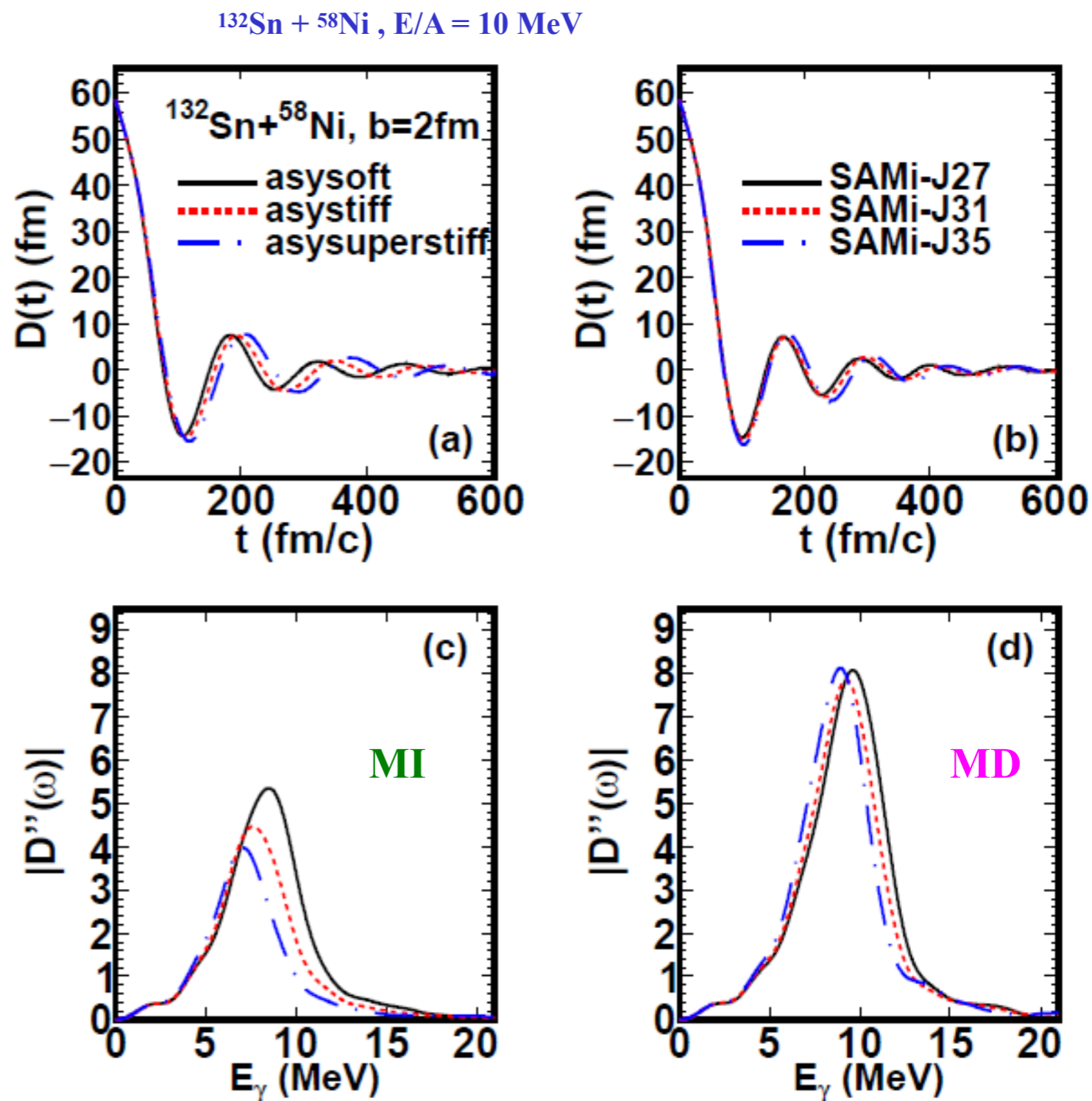
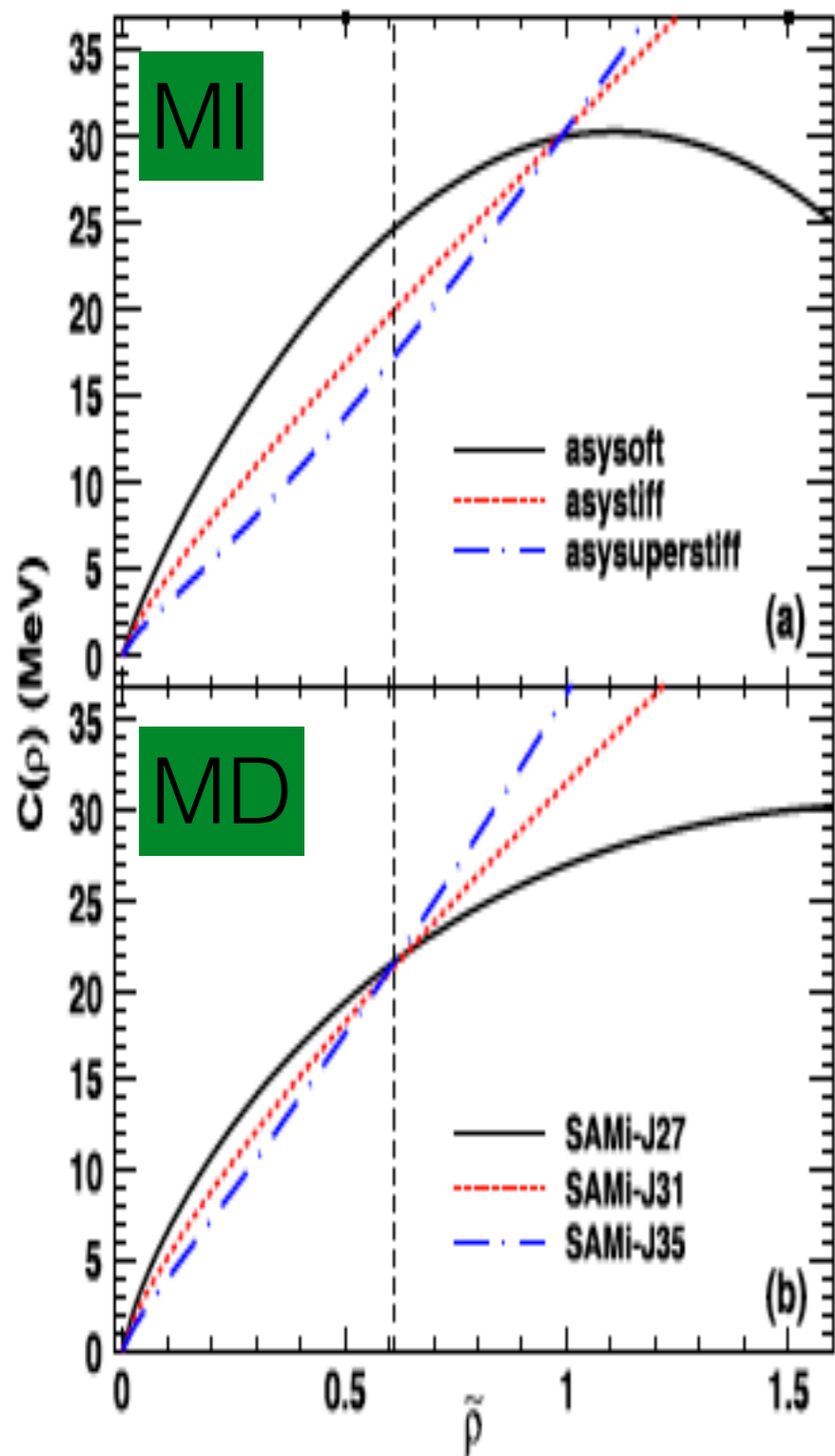


- The DD emission looks sensitive to  $E_{sym}$  at  $\rho = 0.6 \rho_{sat}$
- Larger strength seen in the MD case
- damping connected to n-n collision time ( $\tau_{coll}$ )



# The pre-equilibrium dipole strength in $^{132}\text{Sn}+^{58}\text{Ni}$ , 10 MeV/A

H. Zheng et al. PLB 769, 424, 2017



# TDHF simulations process

**D.Lacroix**  
**IPN-Orsay**

## Important parameters

Mass/Charge:

Projectile  $(N_P, Z_P)$

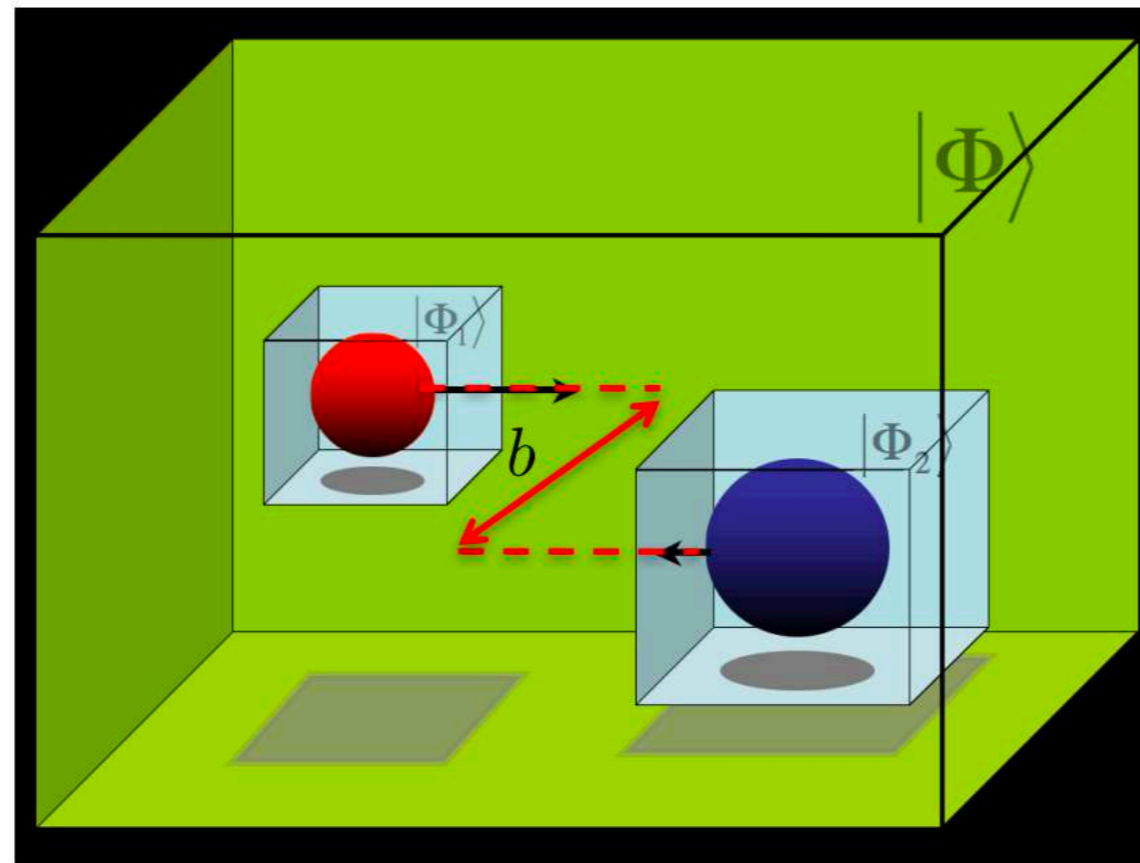
Target  $(N_T, Z_T)$

Impact parameter:  $b$

➔  $L = r \wedge p = b p_{ini}$

Beam Energy:  $E_B/A$

$$E_B^{Fus} \simeq 5 \text{ MeV}.A$$



Step 1

Initialize both nuclei separately

Step 2

Put both in a 3D Mesh (in the c.m.)

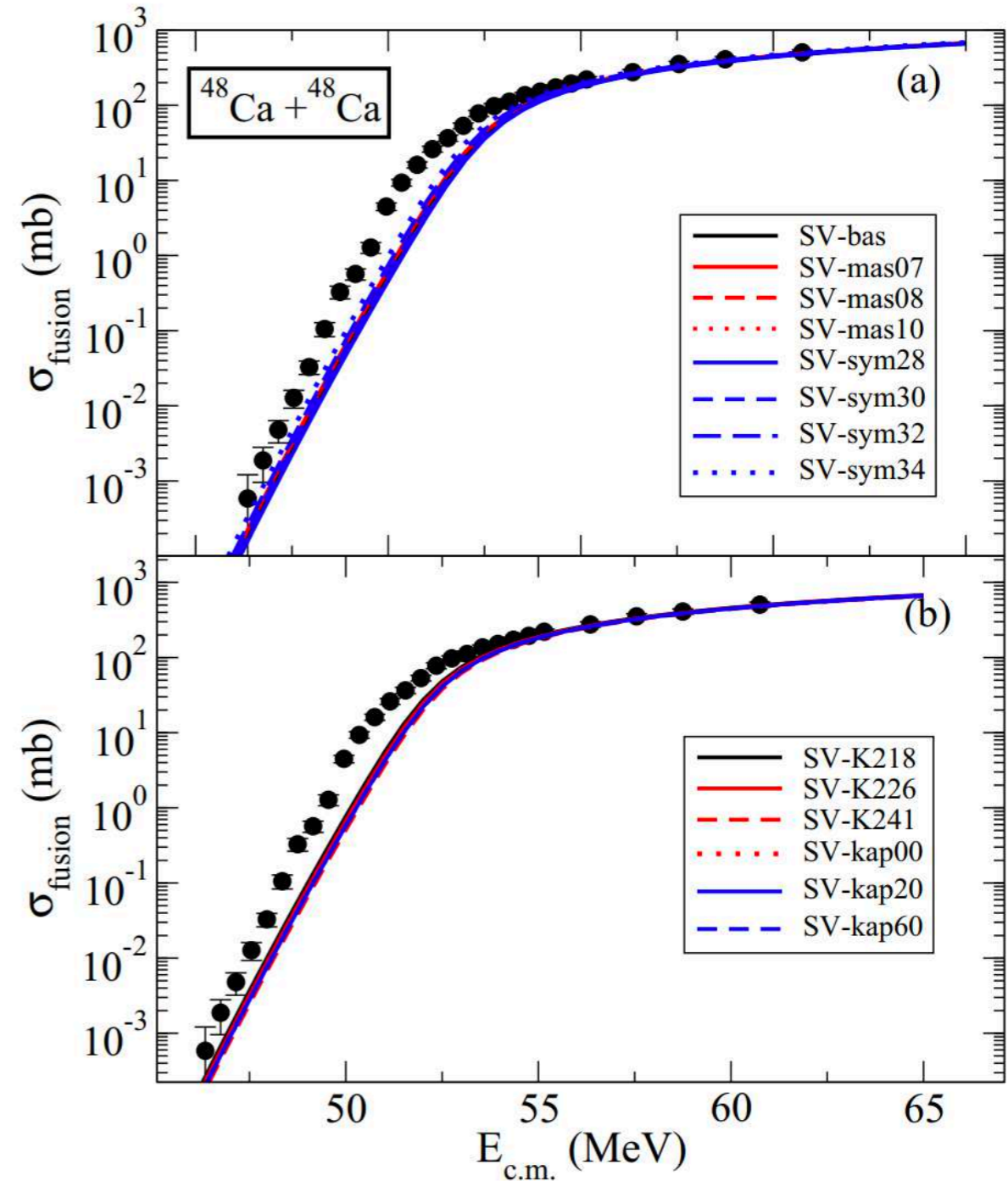
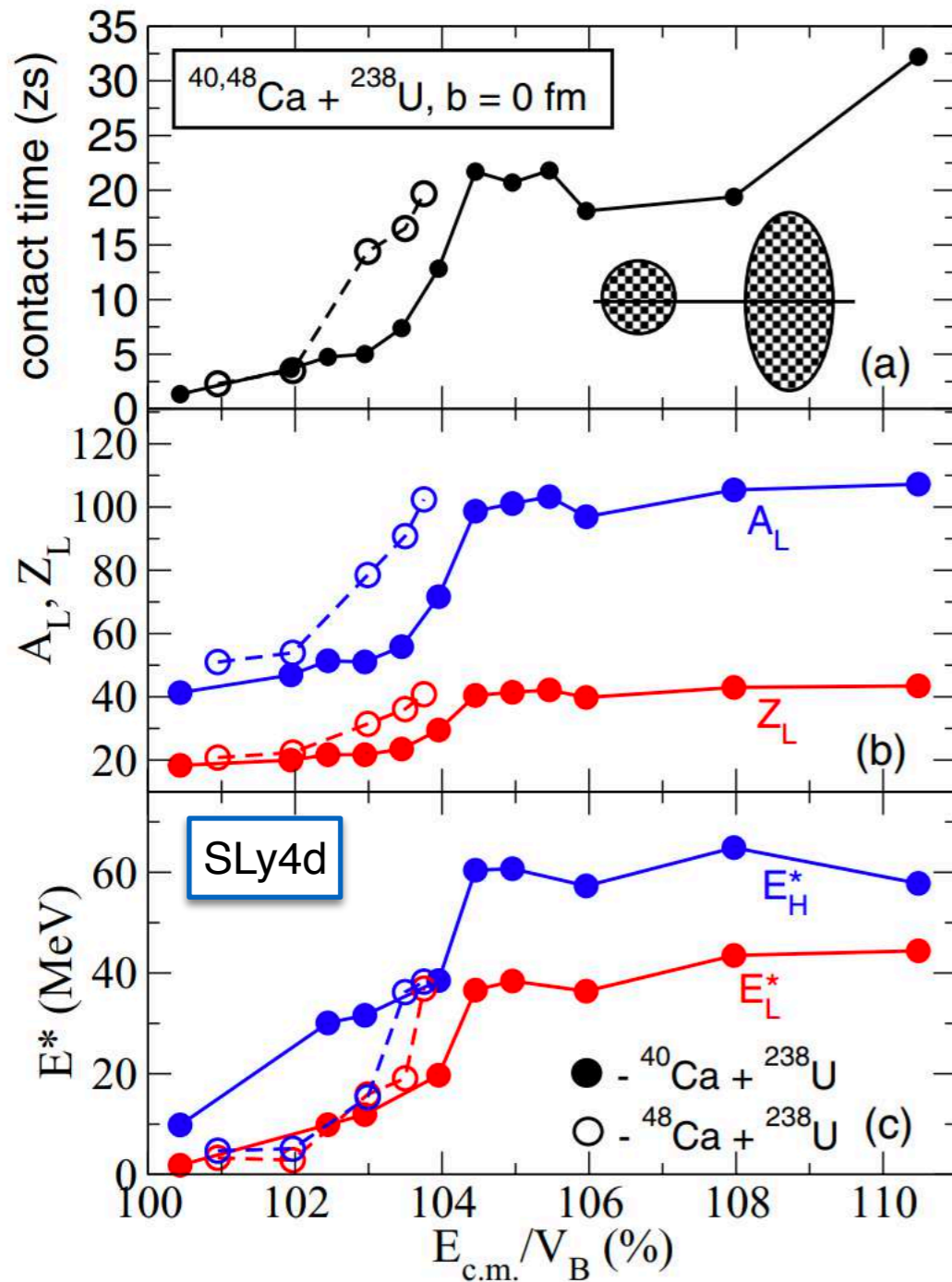
Step 3

Boost wave-packets  
 $|\phi_\alpha^{T/P}\rangle \rightarrow e^{-ip_{T/P}r/\hbar} |\phi_\alpha^{T/P}\rangle$

Step 4

Perform evolution

# Fusion vs Quasi-fission: TDHF simulations

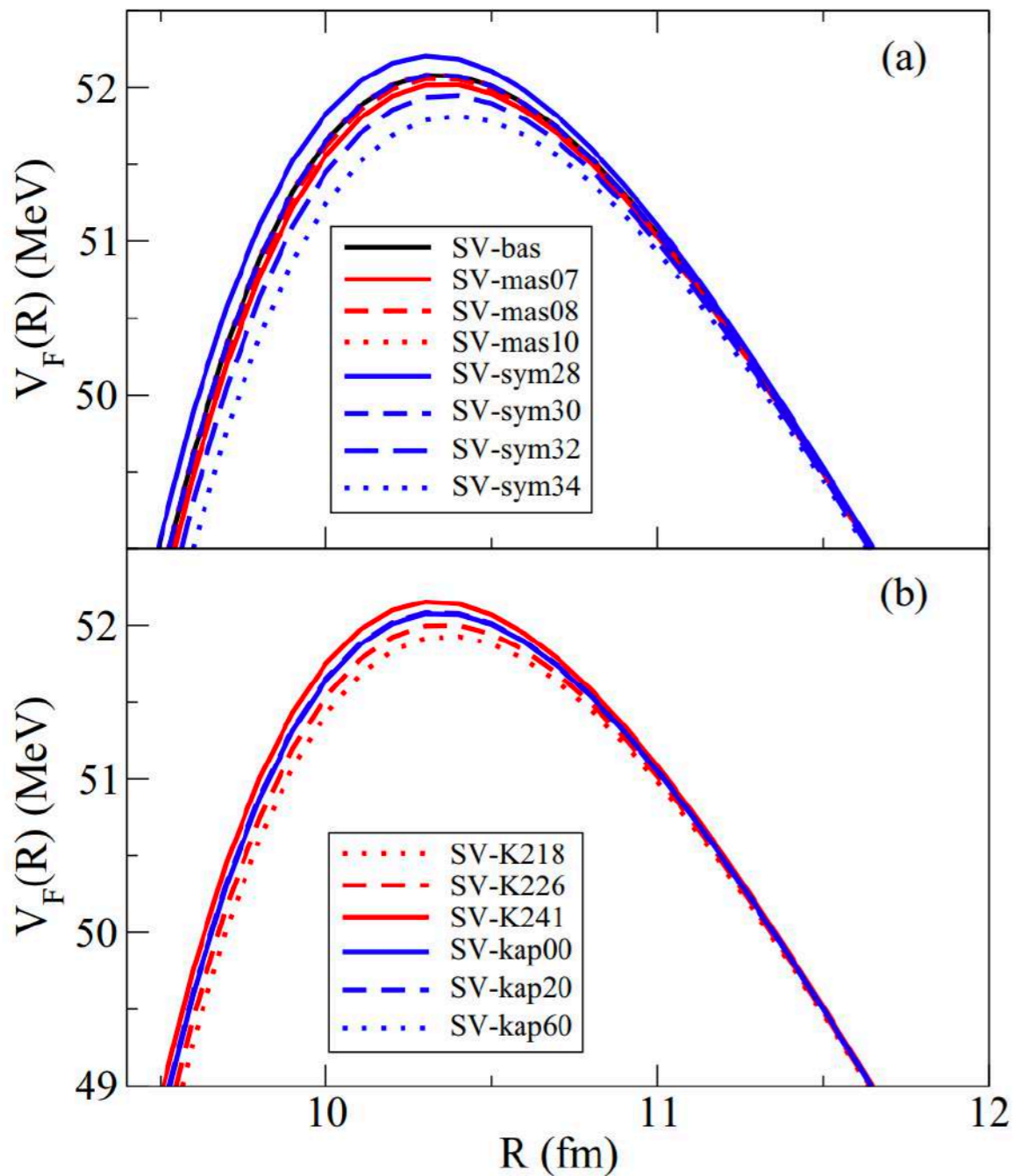


The frozen HF barrier for  $^{40}\text{Ca}$  is  $V_B = 199.13 \text{ MeV}$

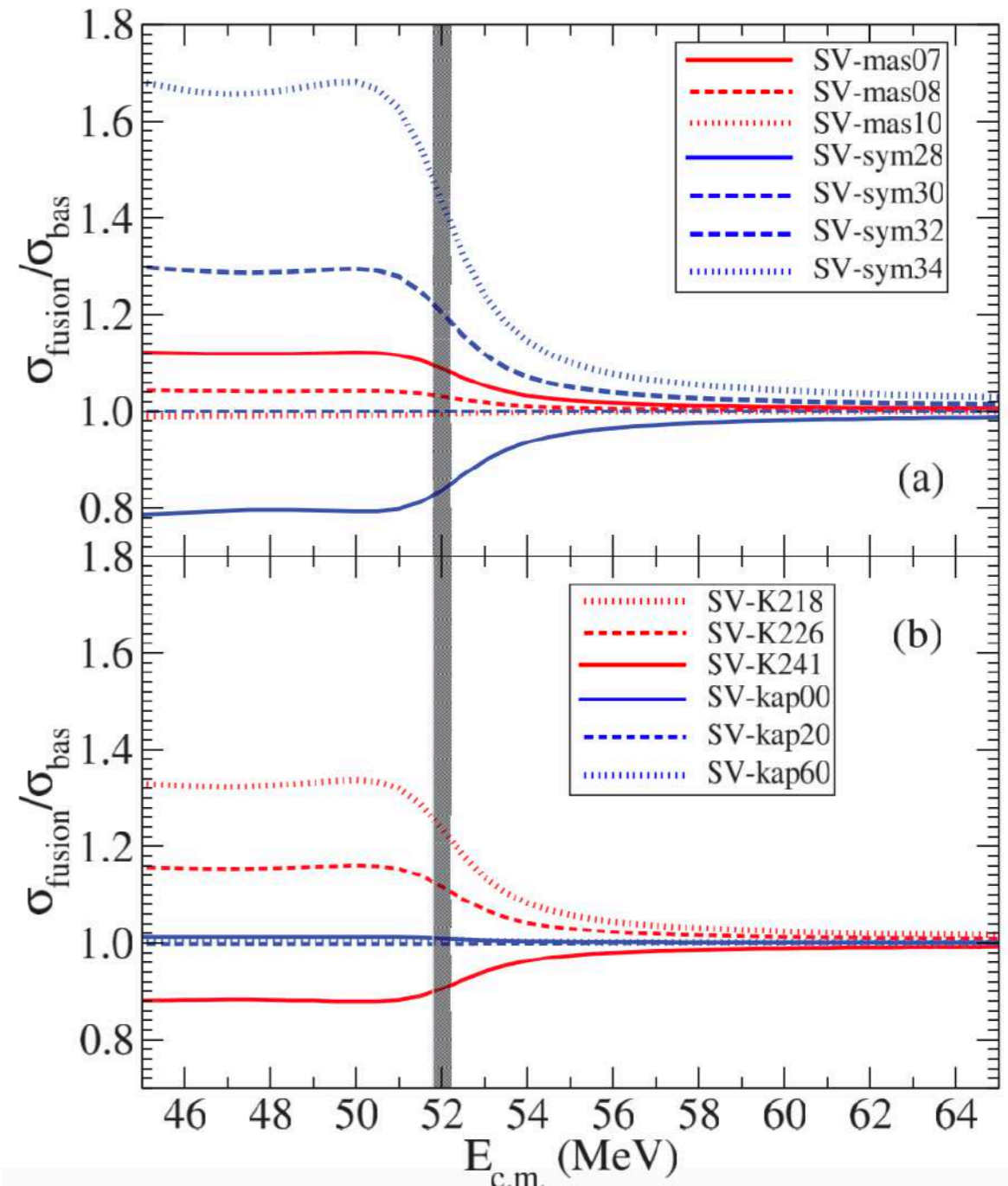
Sensitivity of sub-barrier fusion cross-section to EoS ingredients



# Fusion vs Quasi-fission: TDHF simulations



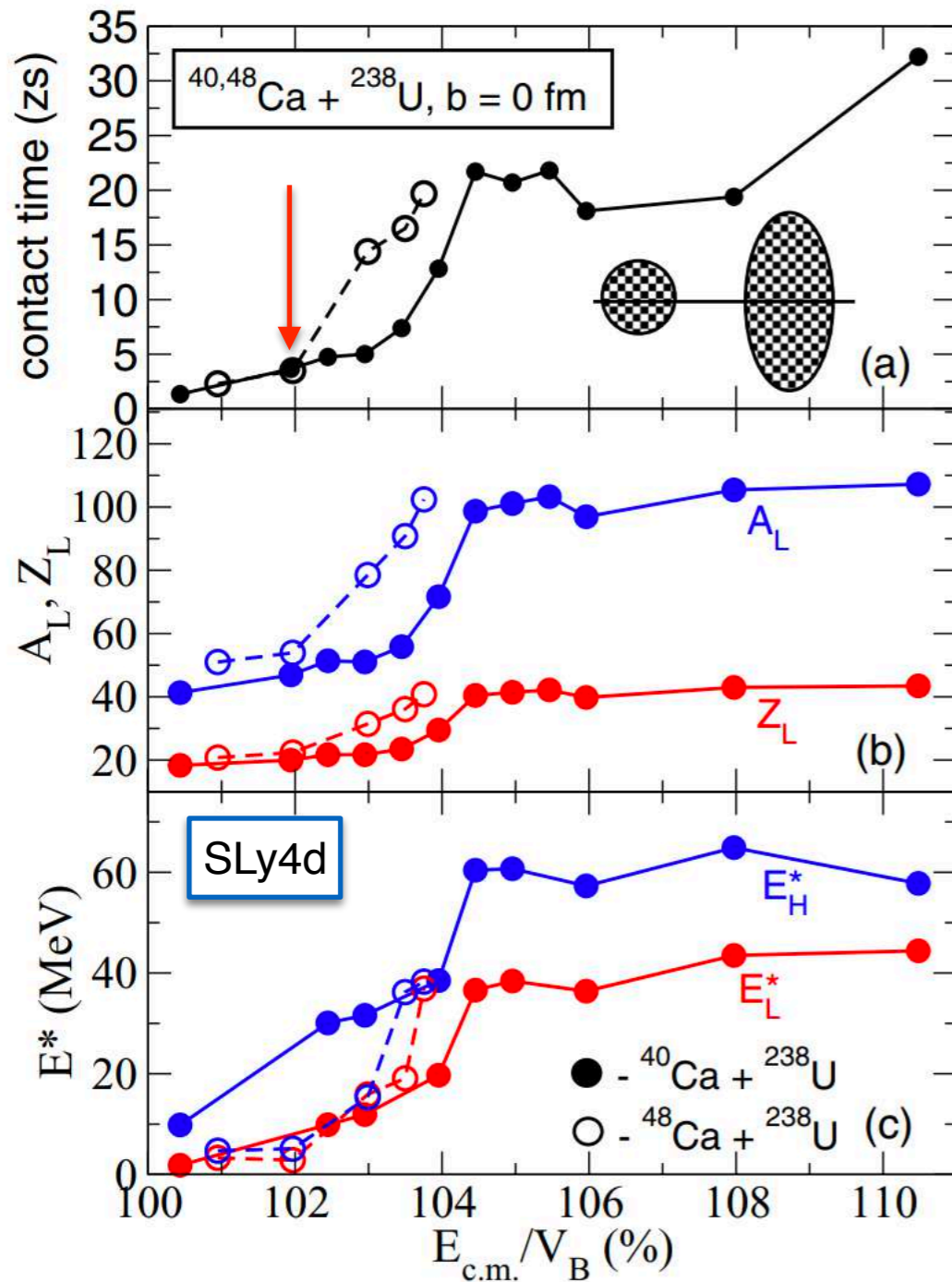
Sensitivity of ion-ion interaction potential to EoS ingredients



Sensitivity of sub-barrier fusion cross-section to EoS ingredients





# Fusion vs Quasi-fission: TDHF simulations



$$^{238}\text{U} + ^{40}\text{Ca} \text{ at } E_{cm} = 203\text{MeV}$$

At the threshold between fusion and quasi-fission

$^{238}\text{U}$  is deformed:

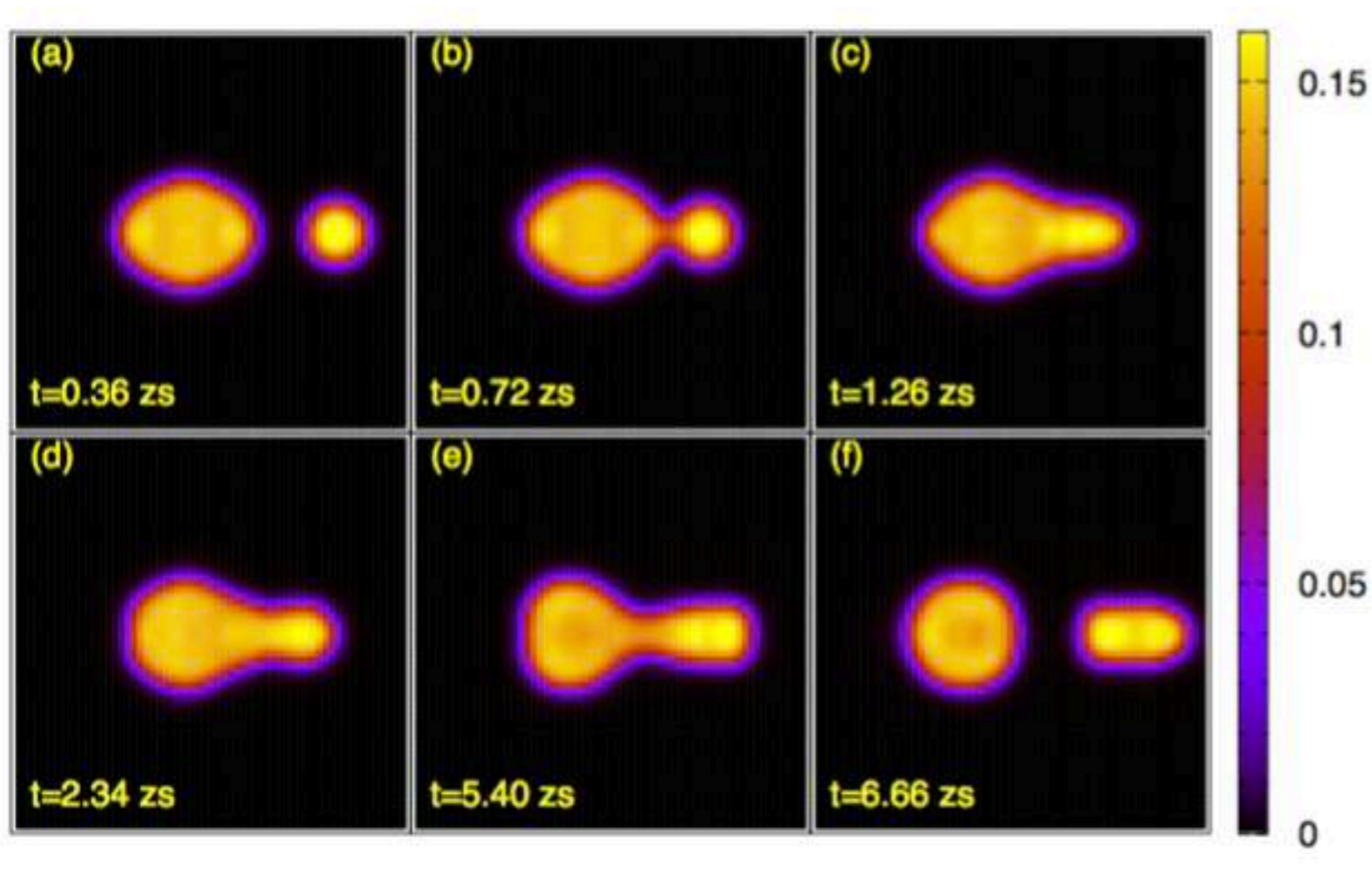
- 1)  Side
- 2)  Tip

Quasi-fission is observed for the tip configuration

The frozen HF barrier for  $^{40}\text{Ca}$  is  $V_B = 199.13\text{MeV}$

# TDHF simulations

$^{238}\text{U} + ^{40}\text{Ca}$  at  $E_{cm} = 203\text{MeV}$



tip collisions: SAMi-J31

Quadrupole moment evolution

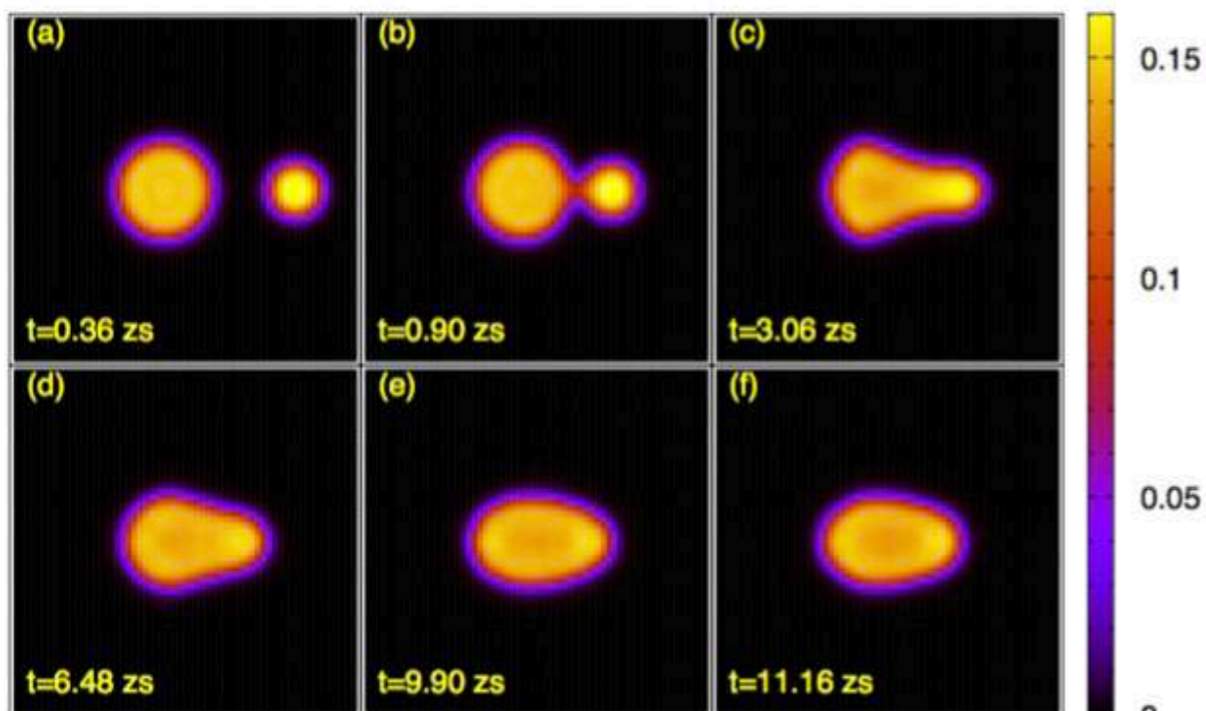
$$Q_2(t) = \langle 2x^2 - y^2 - z^2 \rangle$$

x axis: beam direction

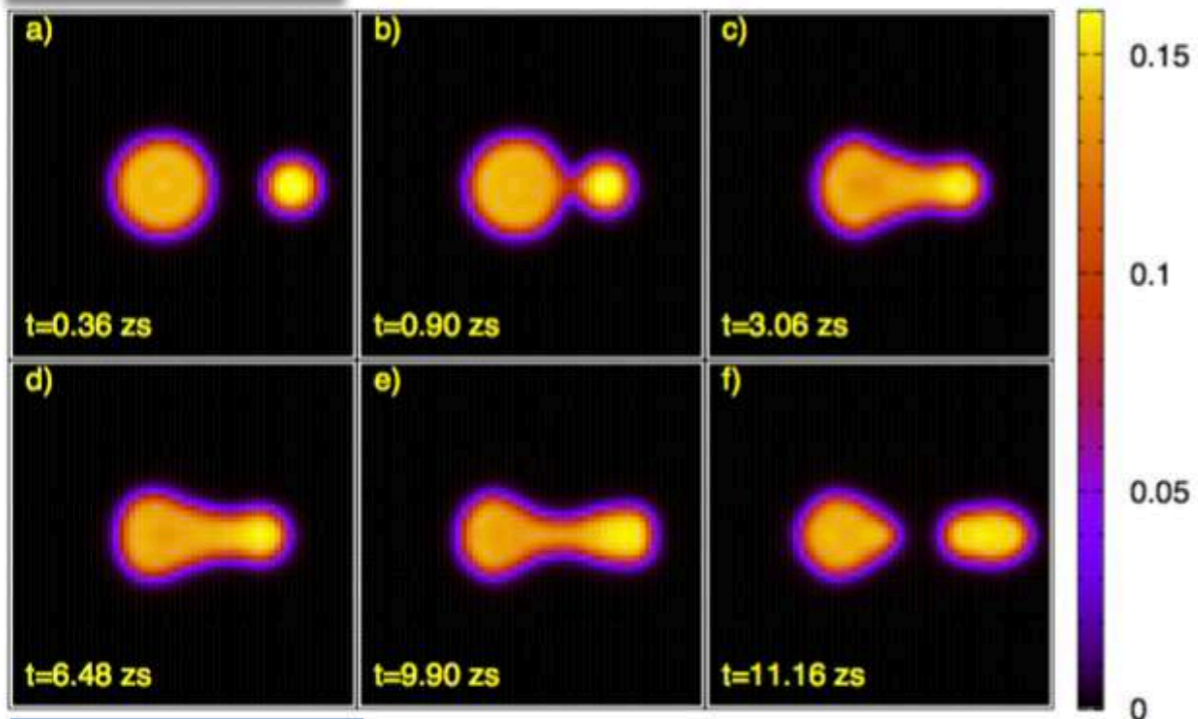
# TDHF simulations

$$^{238}\text{U} + ^{40}\text{Ca} \text{ at } E_{cm} = 203\text{MeV}$$

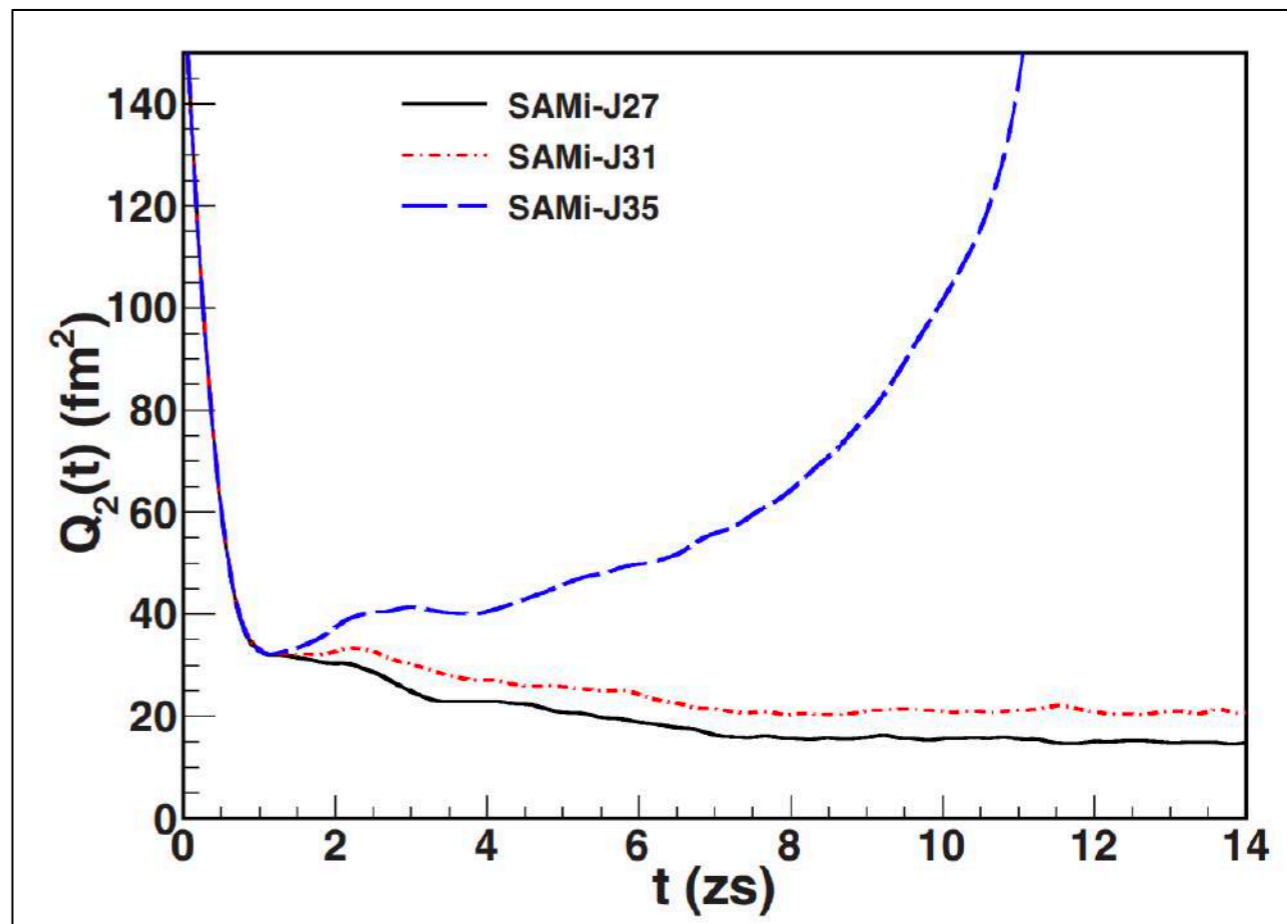
Side collisions,  $b=0$  fm



SAMi-J31



SAMi-J35



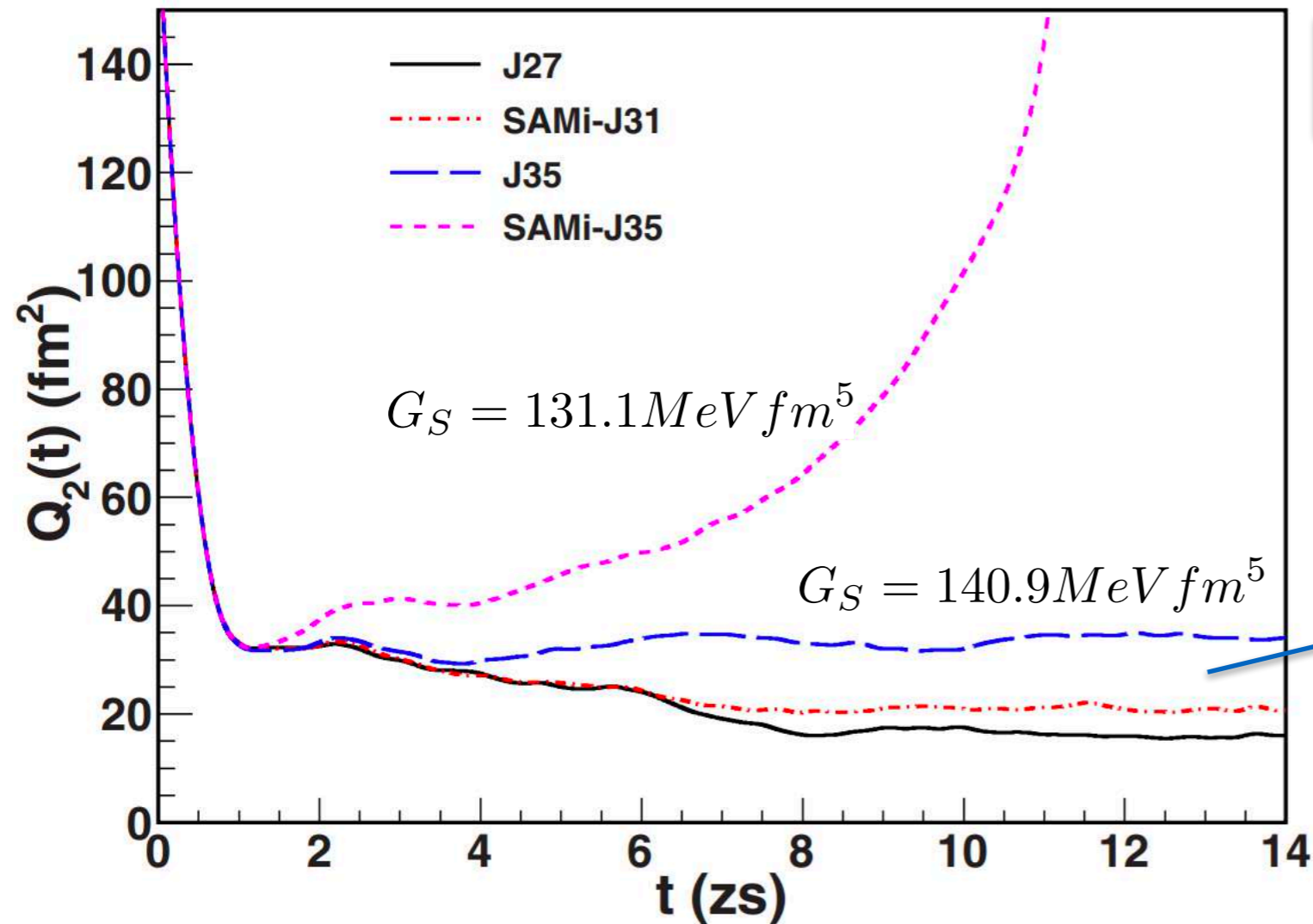
Intuitively, larger L, thicker neutron skin results in quasi-fission



# TDHF simulations

Side collisions,  $b=0$  fm

$^{238}\text{U} + ^{40}\text{Ca}$  at  $E_{cm} = 203\text{MeV}$



## Symmetry energy effects

- J27, SAMi-J31, J35 have the same surface term
- Ordered by the symmetry energy  
(Symmetry energy effects: associated with neutron skin)

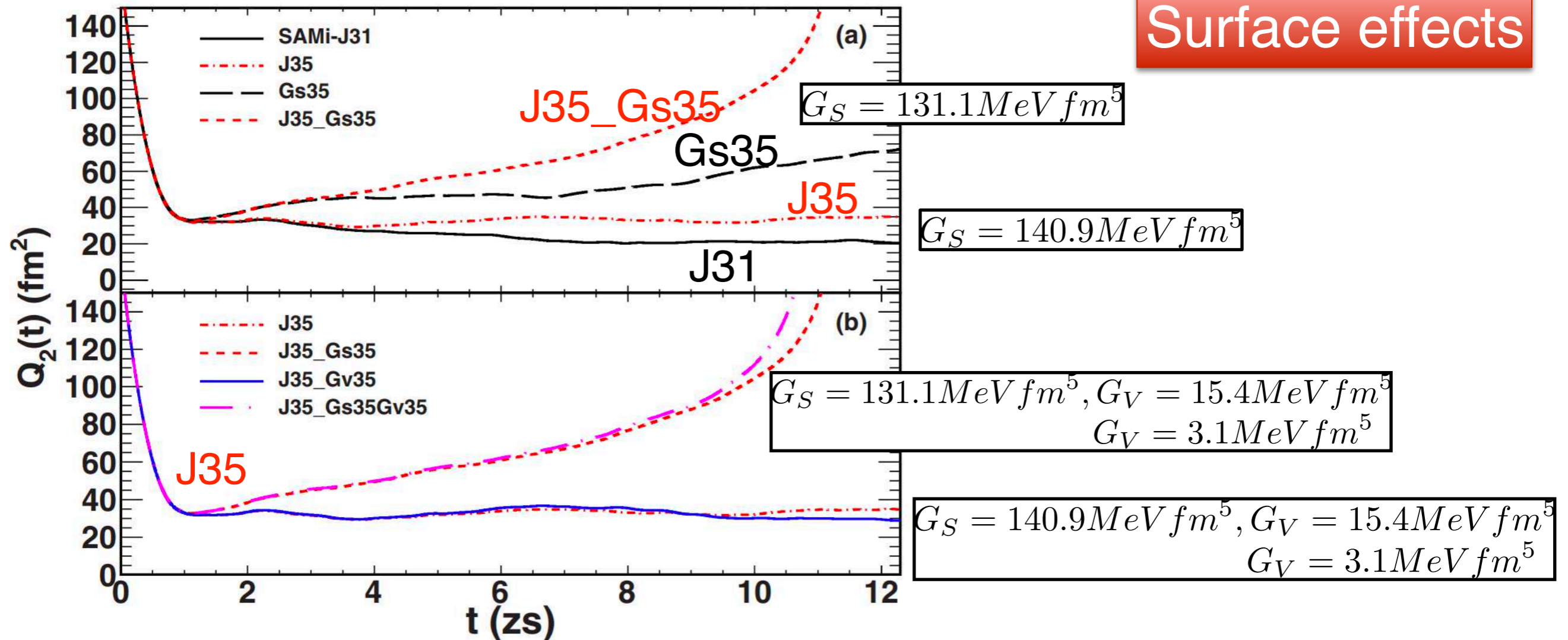
- SAMi-J35 shows different result from J35  
(Larger effects are due to the surface term)

# TDHF simulations

Side collisions,  $b=0$  fm

$^{238}\text{U} + ^{40}\text{Ca}$  at  $E_{cm} = 203\text{MeV}$

Surface effects



- Isoscalar surface term  $\rightarrow$  large effect  
( $G_S$  reduced, favor the formation of more elongated configuration)
- Isovector surface term  $\rightarrow$  tiny effect

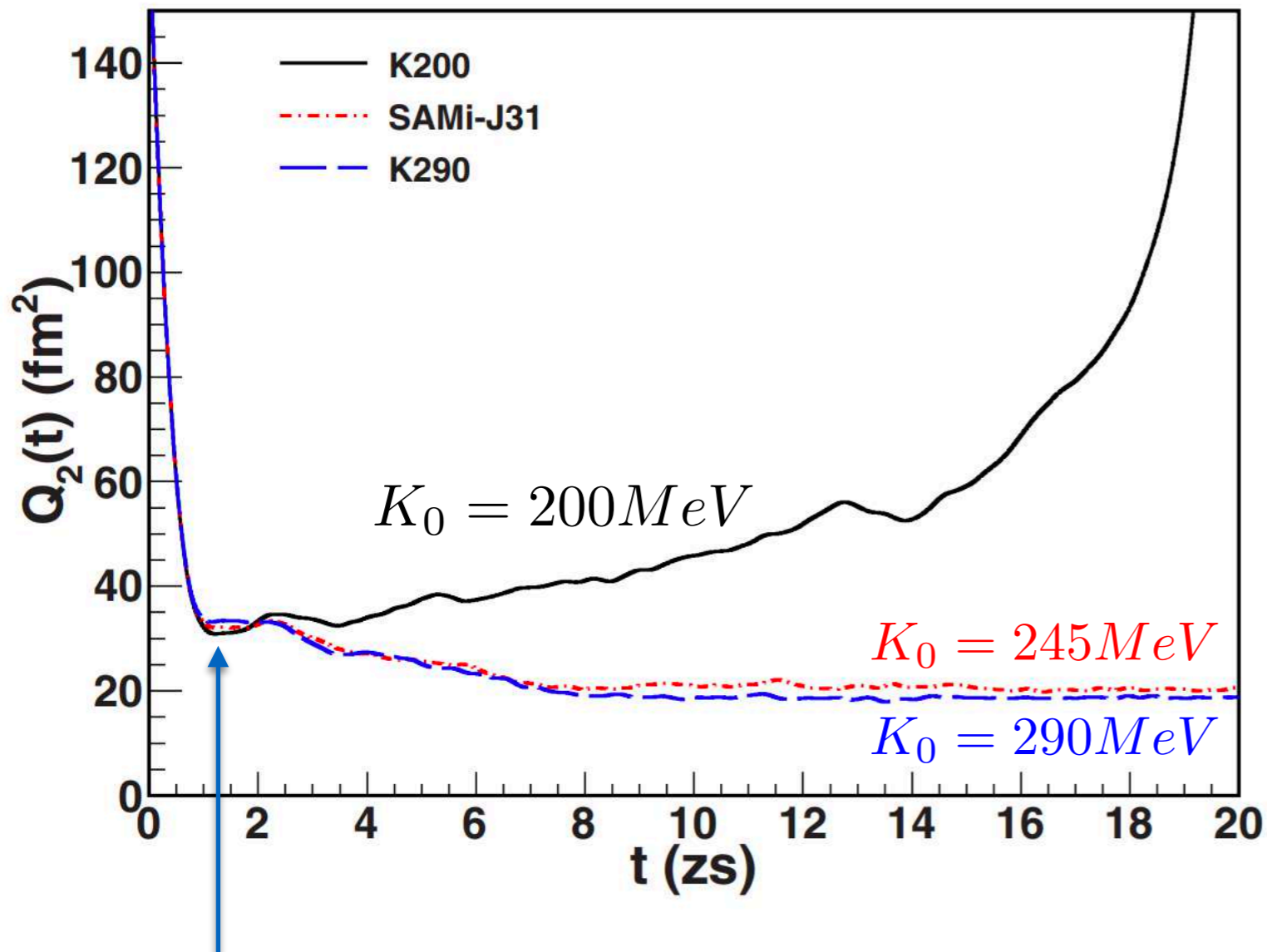
# TDHF simulations

Side collisions,  $b=0$  fm



## Incompressibility effects

- Smaller  $K_0$ , easier to compress and expand
- Smaller  $K_0$ , larger amplitude density oscillations, helps the system to fission



At the compression stage, the smaller  $K_0$  corresponds to the smaller quadrupole moment

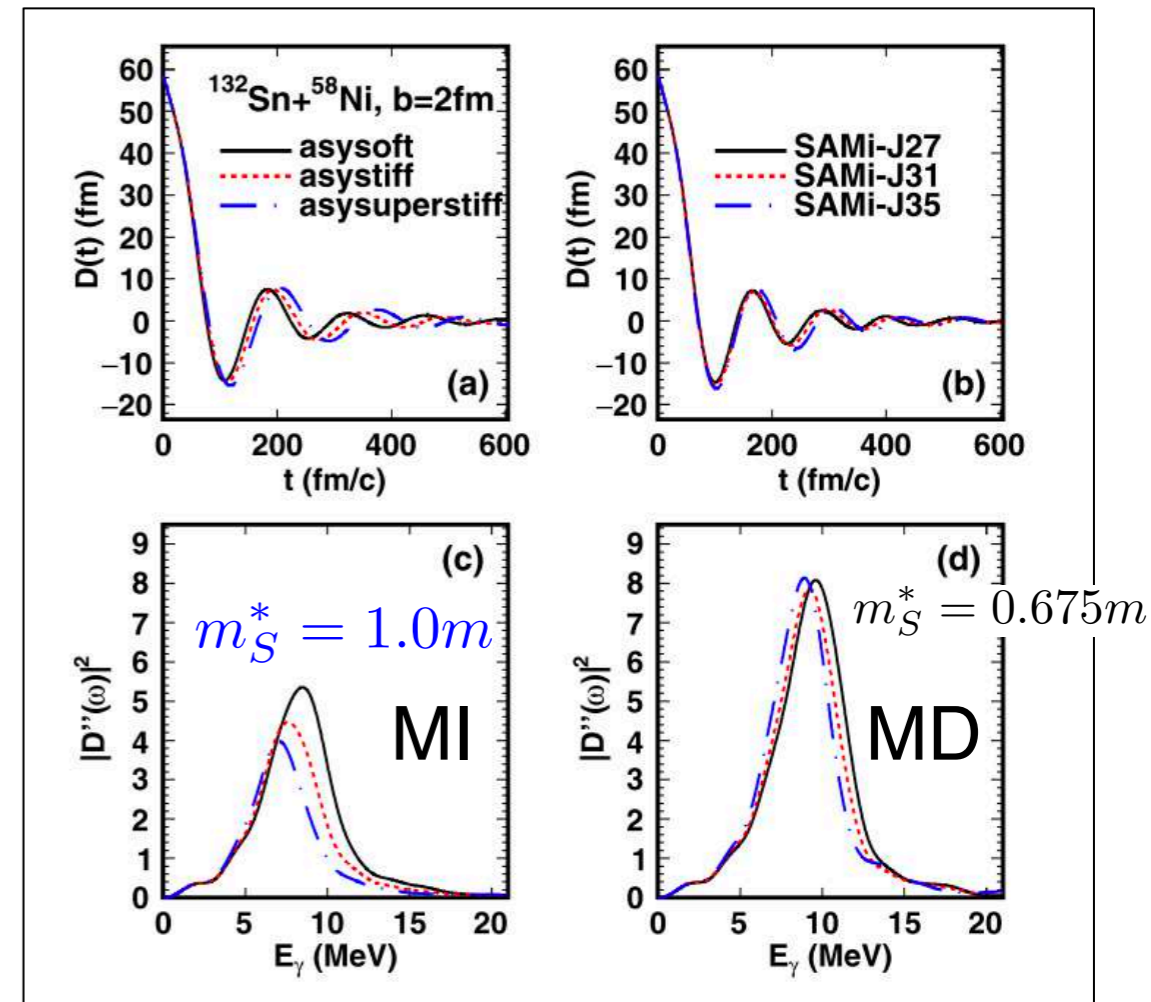
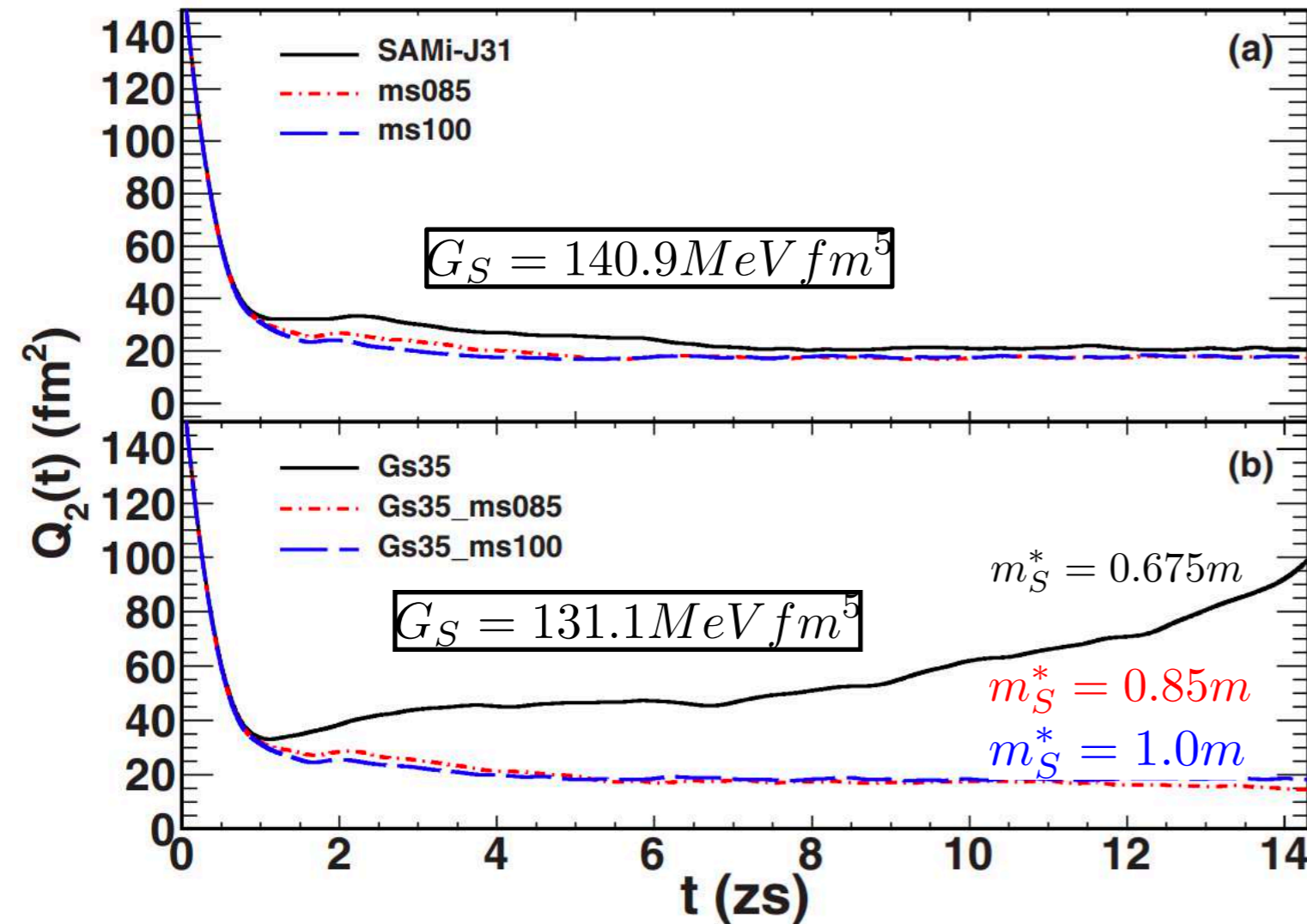


# TDHF simulations

Side collisions,  $b=0$  fm

$^{238}\text{U} + ^{40}\text{Ca}$  at  $E_{cm} = 203\text{MeV}$

Isoscalar effective mass



- With increased effective mass, jump from quasi-fission to fusion

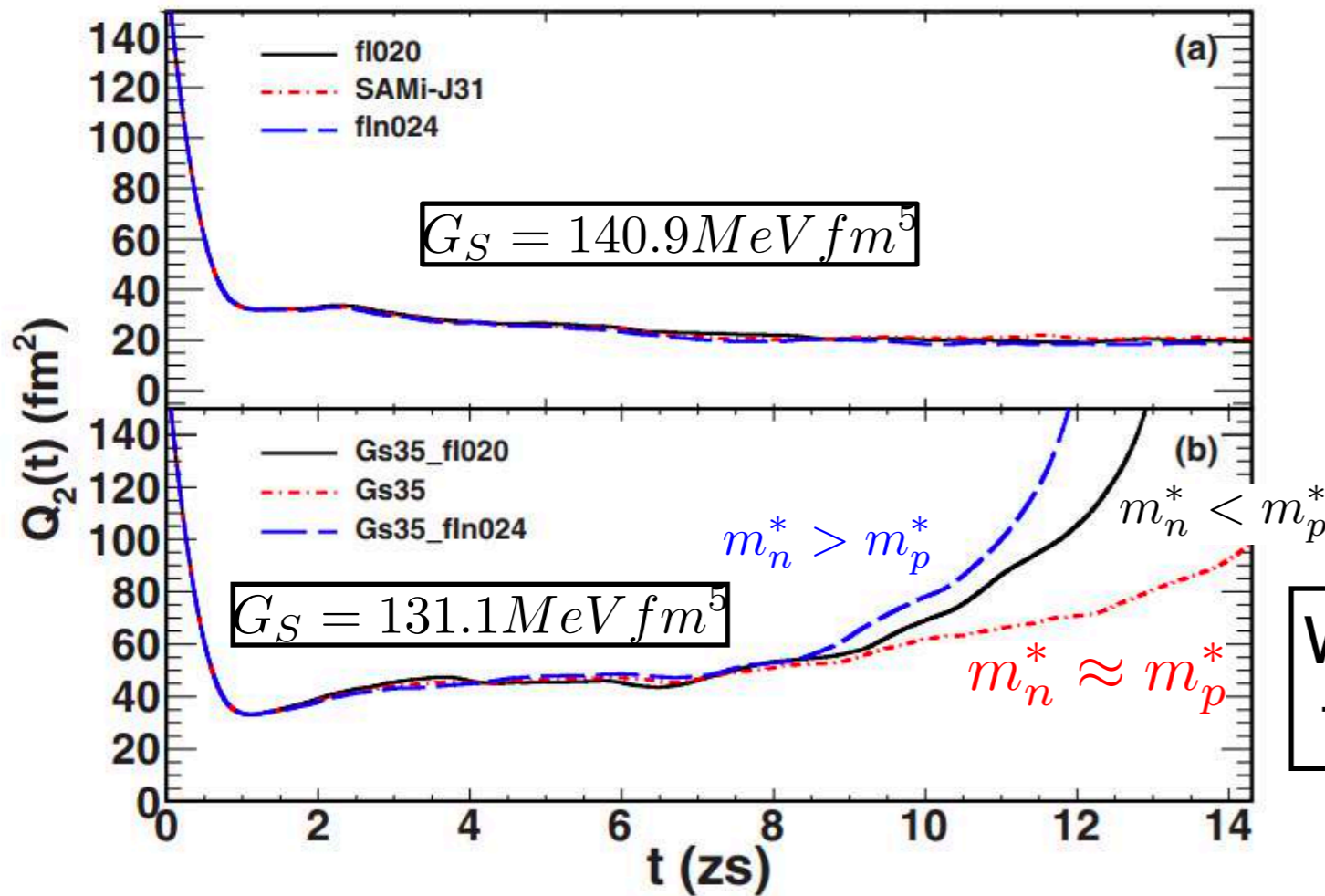
# TDHF simulations

Side collisions,  $b=0$  fm



Isovector effective mass

$$f_I = \frac{m}{m_s^*} - \frac{m}{m_v^*}$$



With increased effective mass splitting faster quasi-fission

- For  $fl > 0$ , leads to larger neutron repulsion, in addition to symmetry energy
- For  $fl < 0$ , tends to counterbalance symmetry energy effects but enhance the Coulomb repulsion

# Conclusions

- Dissipative reactions at low energies open the opportunity to learn about fundamental properties of the nuclear effective interaction of interest also in the astrophysical context
- **Competition between fusion and quasi-fission**  
In  $^{40}\text{Ca} + ^{238}\text{U}$  reactions at energies close to the Coulomb barrier an important sensitivity is observed to nuclear EoS properties:  
surface - incompressibility – effective mass – symmetry energy



**Thank you  
for  
your attention**