Nuclear Equation of State Effects on the r-mode Instability of Neutron Stars

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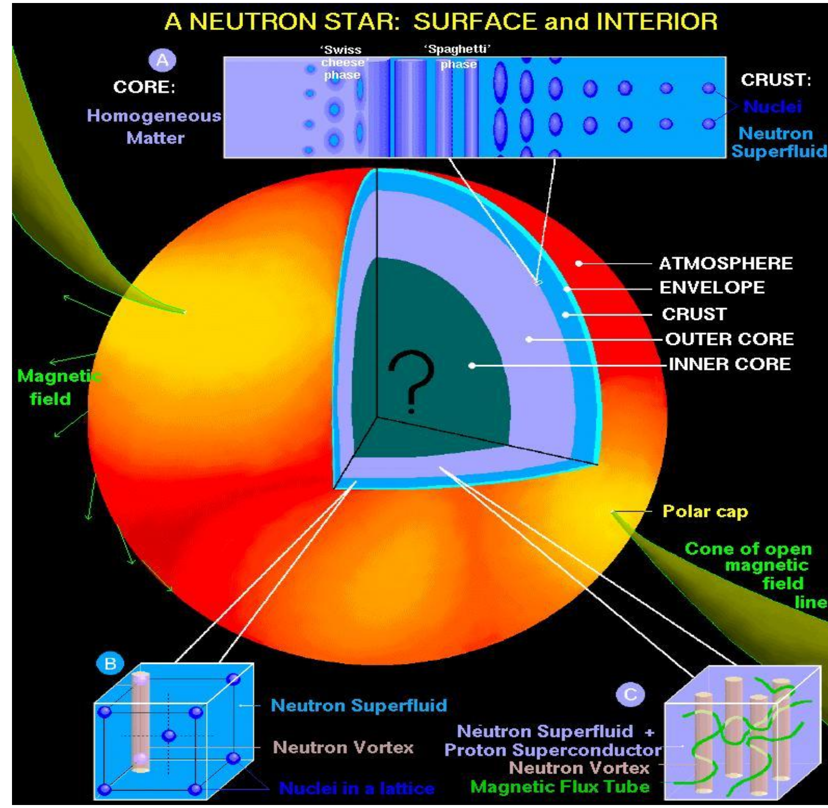
Outline

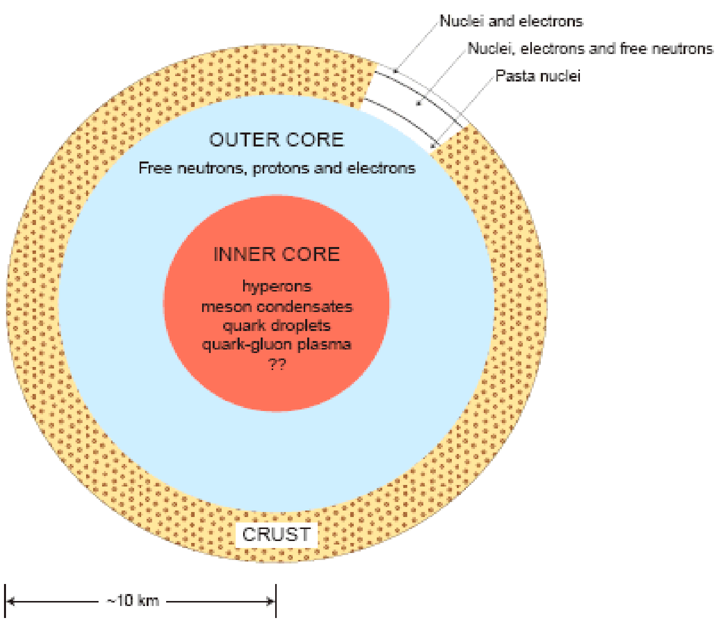
A. Introduction on the r-mode instability of neutron stars

B. Equation of state of neutron stars matter

C. Relative formulae

D. Presentation of the results





Properties of Neutron Stars

•Radius: R ~ 10 km
Mass: M ~ 1.4 – 2.5 Msun
Mean density: r ~ 4x10^14 g/cm3
Period: ~ a few ms – a few s
Magnetic field: B ~ 108 – 1015 G

r-mode instability

- **Neutron stars suffer a number of instabilities with a common feature: they can be directly associated with unstable modes of oscillation lead to the production of gravitational radiation**
- **The first prediction for the r-mode made by Chandrasekhar (1970), Friedeman and Schutz (1978) (CFS Instability)**
- **The r-modes are always retrograde in the rotating frame and prograde in the inertial frame (satisfy the CFS criterion at all rates of rotation ----> the r-modes are generically unstable in rotating perfect fluid (interior neutron stars))**
- **The CFS instability allows some oscillation modes of a fluid body to be driven rather than damped by radiation reaction (gravitational waves), essentially due to a disagreement between two frames of reference.**
- **The role of nuclear equation of state and density dependence of the nuclear symmetry energy is very important.**

Schematic mechanism for spin down of rotating neutron star

Rotating newborn neutron star:

→ random density or velocity oscillation in nuclear fluid

→ r-mode instability (if r-mode unstable growth)

→ emission of gravitational waves (curry away angular momentum)

→rotating neutron star spin down over time

The time dependence of an r-mode instability is given by

$$e^{i\omega t - t/\tau} \tag{39}$$

where ω is the frequency of the mode and τ is the overall time scale of the mode which describes both its exponential growth, driven by the CFS mechanism and its decay due to viscus damping and can be written as

$$\frac{1}{\tau(\Omega, T)} = \frac{1}{\tau_{GR}(\Omega)} + \frac{1}{\tau_v(\Omega, T)} \tag{40}$$

The r-mode will be unstable only when $1/\tau$ is negative that is when τ_{GR} is shorter than τ_v.

The critical angular velocity Ω_c defined by the equation

$$\frac{1}{\tau(\Omega_c, T)} = \frac{1}{\tau_{GR}(\Omega_c)} + \frac{1}{\tau_v(\Omega_c, T)} = 0 \tag{41}$$

In total

$$\begin{array}{ll} \Omega > \Omega_c & \mathrm{unstable\ r-mode} \\ \Omega < \Omega_c & \mathrm{stable\ r-mode} \end{array}$$

Gravitational radiation instability in the r-modes

$$\frac{1}{\tau} = \frac{1}{\tau_{GR}} + \frac{1}{\tau_v}$$

$$\tau_v = \frac{1}{2\Omega}\frac{2^{m+3/2}(m+1)!}{m(2m+1)!!\mathcal{I}_m}\sqrt{\frac{2\Omega R_c^2\rho_c}{\eta_c}}\int_0^{R_c}\frac{\rho}{\rho_c}\left(\frac{r}{R_c}\right)^{2m+2}\frac{dr}{R_c}$$

$$\frac{1}{\tau_{GR}} = -\frac{32\pi G\Omega^{2m+2}}{c^{2m+3}}\frac{(m-1)^{2m}}{[(2m+1)!!]^2}\left(\frac{m+2}{m+1}\right)^{2m+2}\int_0^{R_c}\rho r^{2m+2}dr$$

The r mode corresponds to the value m=2

Viscous dissipation mechanisms in the r-modes

- **In neutron star colder than about 10^9 K the shear viscosity dominate by electron-electron scattering**

$$\eta_{ee} = 6.0 \times 10^6 \rho^2 T^{-2}, \qquad (\mathrm{g\ cm^{-1}}\ s^{-1})$$

- **For temperature above 10^9 K, neutron-neutron scattering provides the dominant dissipation mechanics**

$$\eta_{nn} = 347 \rho^{9/4} T^{-2}, \qquad (\mathrm{g\ cm^{-1}\ s^{-1}})$$

- **The bulk viscosity arises as the mode oscillation drives the fluid away from β-equilibrium (strong at very high temperature)**

$$\zeta = 6 \times 10^{25} \rho_{15}^2 T_9^6 \left(\frac{\omega_r}{1\,\mathrm{Hz}}\right)^{-2} \mathrm{g\ cm^{-1}\ s^{-1}}.$$

Times scales- Critical angular velocity- Critical temperature

The fiducial viscous time scale defined as

$$\tau_v = \tilde{\tau}_v \left(\frac{\Omega_0}{\Omega}\right)^{1/2} \left(\frac{T}{10^8\ K}\right)$$

The fiducial gravitational radiation time scale defined as

$$\tau_{GR} = \tilde{\tau}_{GR} \left(\frac{\Omega_0}{\Omega}\right)^{2m+2}$$

$$\Omega_0 = \sqrt{\pi G \overline{\rho}}$$

$\overline{\rho}$ is the mean value of the density.

The critical angular velocity, above which the r-mode is unstable is defined by the condition

$$\tau_{GR} = \tau_v$$

$$\frac{\Omega_c}{\Omega_0} = \left(\frac{\tilde{\tau}_{GR}}{\tilde{\tau}_v}\right)^{2/11} \left(\frac{10^8\ K}{T}\right)^{2/11}$$

The critical angular velocity marks the boundary between stability and instability

The angular velocity of a neutron star can never exceed some maximum value $\Omega_{max}=\Omega_{Kepler}=2/3\Omega_0$. So there is a critical temperature below which the gravitational radiation instability is completed suppressed by viscosity

$$\frac{T_c}{10^8\ K} = \left(\frac{\Omega_0}{\Omega_{max}}\right)^{11/2} \left(\frac{\tilde{\tau}_{GR}}{\tilde{\tau}_v}\right) \approx \left(\frac{3}{2}\right)^{11/2} \left(\frac{\tilde{\tau}_{GR}}{\tilde{\tau}_v}\right)$$

$$\frac{\Omega_c}{\Omega_0} = \frac{\Omega_{max}}{\Omega_0} \left(\frac{T_c}{T}\right)^{2/11} \approx \frac{2}{3} \left(\frac{T_c}{T}\right)^{2/11}$$

The nuclear model

The energy per baryon of neutron star matter

$$\begin{aligned} E_b(n,I) &= \frac{3}{10}E_F^0 u^{2/3}\left[(1+I)^{5/3}+(1-I)^{5/3}\right]+\frac{1}{3}A\left[\frac{3}{2}-(\frac{1}{2}+x_0)I^2\right]u \\ &+ \frac{\frac{2}{3}B\left[\frac{3}{2}-(\frac{1}{2}+x_3)I^2\right]u^{\sigma}}{1+\frac{2}{3}B'\left[\frac{3}{2}-(\frac{1}{2}+x_3)I^2\right]u^{\sigma-1}} \\ &+ \frac{3}{2}\sum_{i=1,2}\left[C_i+\frac{C_i-8Z_i}{5}I\right]\left(\frac{\Lambda_i}{k_F^0}\right)^3\left(\frac{((1+I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}-\tan^{-1}\frac{((1+I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}\right) \\ &+ \frac{3}{2}\sum_{i=1,2}\left[C_i-\frac{C_i-8Z_i}{5}I\right]\left(\frac{\Lambda_i}{k_F^0}\right)^3\left(\frac{((1-I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}-\tan^{-1}\frac{((1-I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}\right) \end{aligned}$$

Nuclear symmetry energy

$$E_b(n,I)=E_b(n,I=0)+E_{sym,2}(n)I^2+E_{sym,4}(n)I^4+\cdots+E_{sym,2k}(n)I^{2k}+\ldots,$$

$$E_{sym,2k}(n)=\frac{1}{(2k)!}\frac{\partial^{2k}E_b(n,I)}{\partial I^{2k}}\bigg|_{I=0}.$$

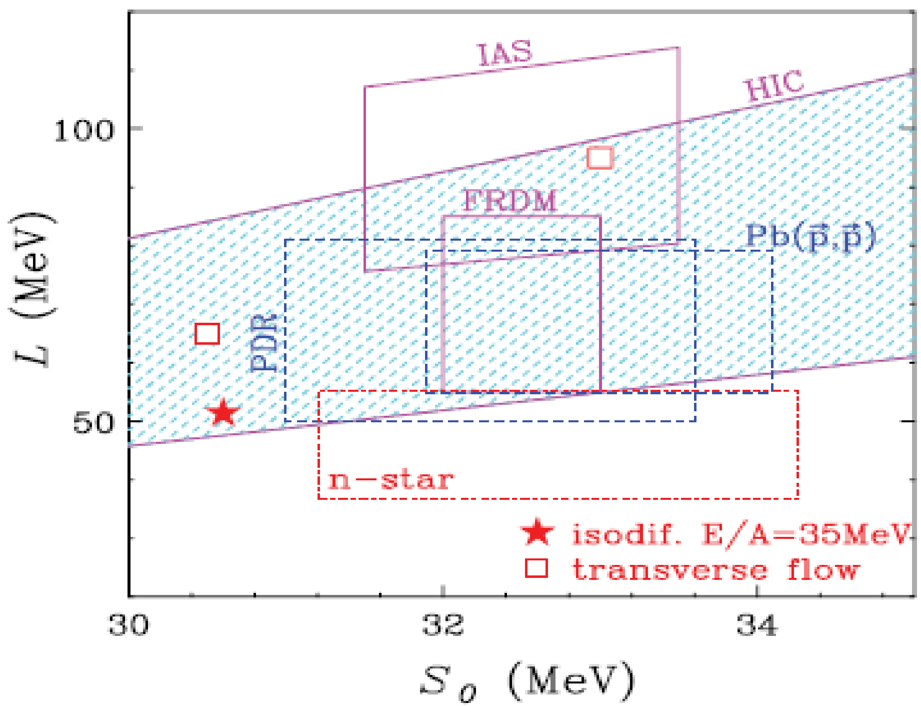
$$E_{sym}(n)=E_{sym,2}(n)=\frac{1}{2!}\frac{\partial^2 E(n,I)}{\partial I^2}\bigg|_{I=0}$$

$$E_{sym}(n_0)=30\ \mathrm{MeV}$$

$$L=3n_0\frac{\partial E_{sym}(n)}{\partial n}\bigg|_{n=n_0}$$

$$50\ \mathrm{MeV}\leq L\leq 110\ \mathrm{MeV}$$

Experimental constraints on the symmetry energy

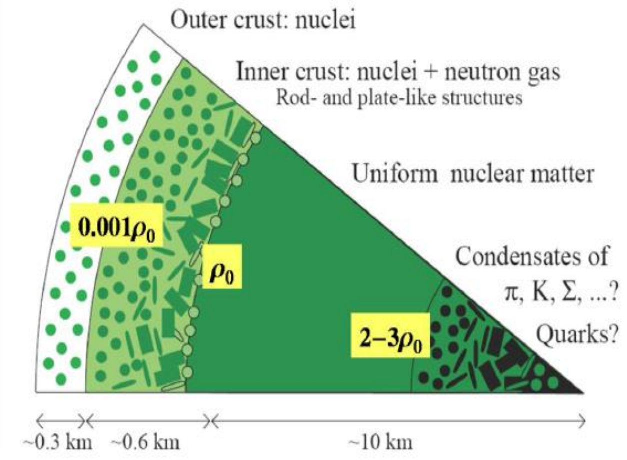


M. B. TSANG *et al.*

PHYSICAL REVIEW C **86**, 015803 (2012)

FIG. 2. (Color online) Constraints on the slope L and magnitude S_0 of the symmetry energy at saturation density from different experiments. The experimental methods are labeled next to the boxes with the estimated uncertainties. The symbols are results without the analysis of the errors. See text for details.

Core-crust interface



The determination of n_t is very complicated problem

The main theoretical approaches are:

- **The dynamical method**
- **The thermodynamical method**
- **The random phase approximation (RPA)**

The location of the inner edge of the crust is a Phase Transition Problem

Inner Crust <=========>Outer Core

Solid Phase <=========>Liquid Phase

Thermodynamical method

We consider uniform matter consists mainly by neutrons, few percent of protons and electrons under beta equilibrium

The first low of thermodynamics reads:

$$\mathrm{d}u = -P\mathrm{d}v - \hat{\mu}\mathrm{d}q,$$

The stability of the uniform phase requires that the energy density **u(v,q) is a convex function** with the following two constraints:

$$-\left(\frac{\partial P}{\partial v}\right)_q - \left(\frac{\partial P}{\partial q}\right)_v \left(\frac{\partial q}{\partial v}\right)_{\hat{\mu}} > 0,$$

$$-\left(\frac{\partial \hat{\mu}}{\partial q}\right)_v > 0.$$

$$u(v,q) = E_b(v,q) + E_e(v,q).$$

For a given equation of state, the quantity C(n) is plotted as a function of the baryon density n and the equation C(n)=0 defines the transition density n_t

$$C(n) = 2n\frac{\partial E_b(n,x)}{\partial n} + n^2\frac{\partial^2 E_b(n,x)}{\partial n^2} - \left(\frac{\partial^2 E_b(n,x)}{\partial n \partial x} n\right)^2 \left(\frac{\partial^2 E_b(n,x)}{\partial x^2}\right)^{-1} > 0,$$

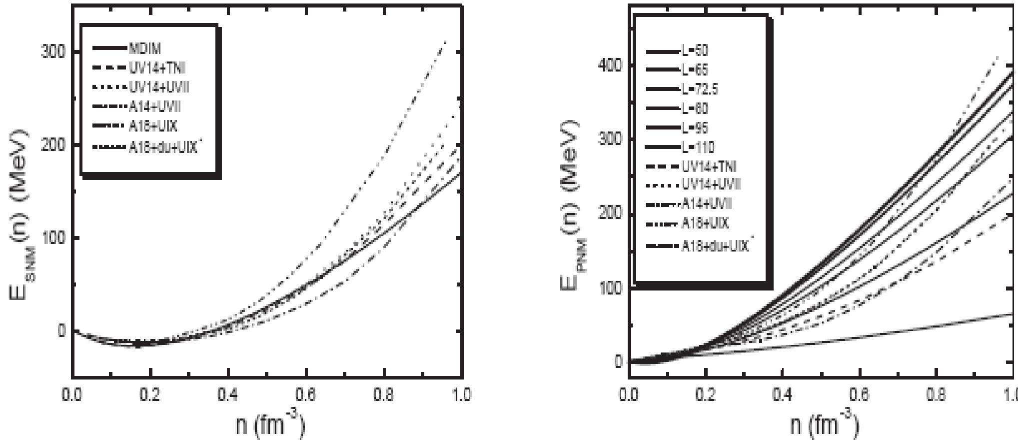


Figure 1: a) The energy per baryon of symmetric nuclear matter and b) the energy per baryon of pure neutron matter.

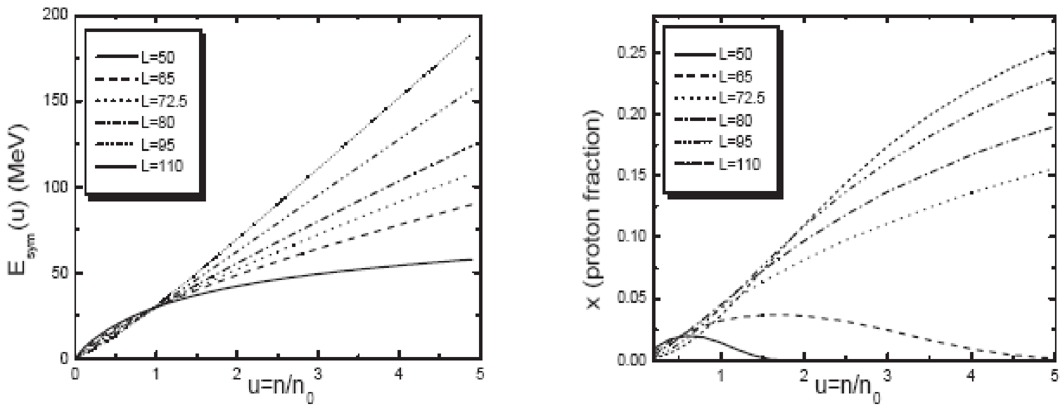


Figure 2: a) The nuclear symmetry energy and b) the proton fraction as a function of the baryon density for varies values of the slope L.

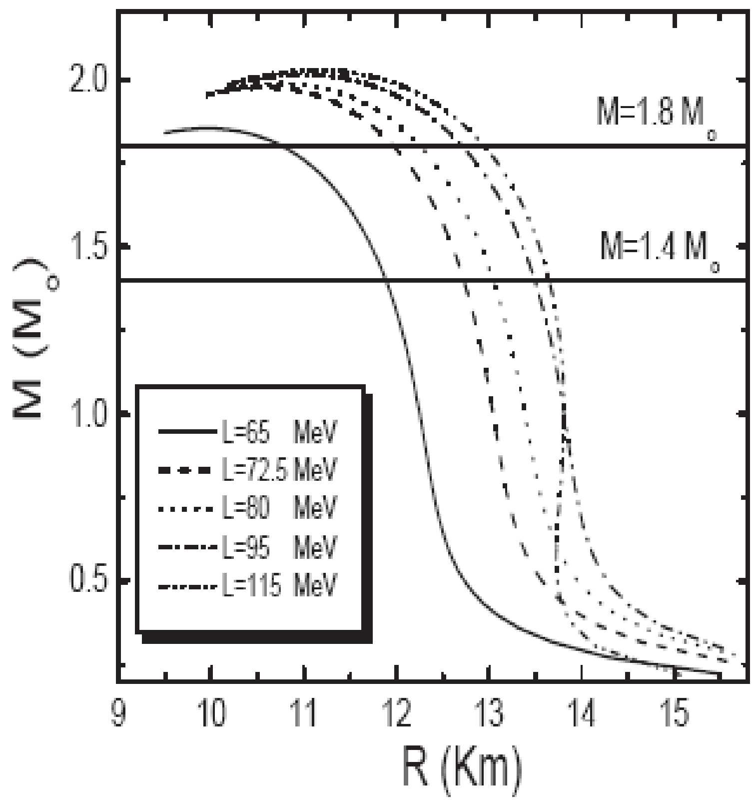


Figure 4: The mass-radius relations for the selected EOSs

The fiducial gravitational radiation time scale correlated with the slope parameter as

$$\tilde{\tau}_{GR} = 2.70736 \left(\frac{R}{10\mathrm{Km}}\right)^{9} \frac{1}{0.7 + 0.084L},$$

The fiducial viscus time scales correlated with the slope parameter as

$$\tilde{\tau}_{ee} = 13.3053 \left(\frac{R}{10\mathrm{Km}}\right)^{3/4} \left(\frac{10\mathrm{Km}}{R_c}\right)^{6} \frac{1.24 + 0.084L}{(2.112 - 0.0121L)^{3/2}}, \quad (M = 1.4M_{\odot})$$

$$\tilde{\tau}_{nn} = 31.09 \left(\frac{R}{10\mathrm{Km}}\right)^{3/4} \left(\frac{10\mathrm{Km}}{R_c}\right)^{6} \frac{1.24 + 0.084L}{(2.112 - 0.0121L)^{13/8}}, \quad (M = 1.4M_{\odot})$$

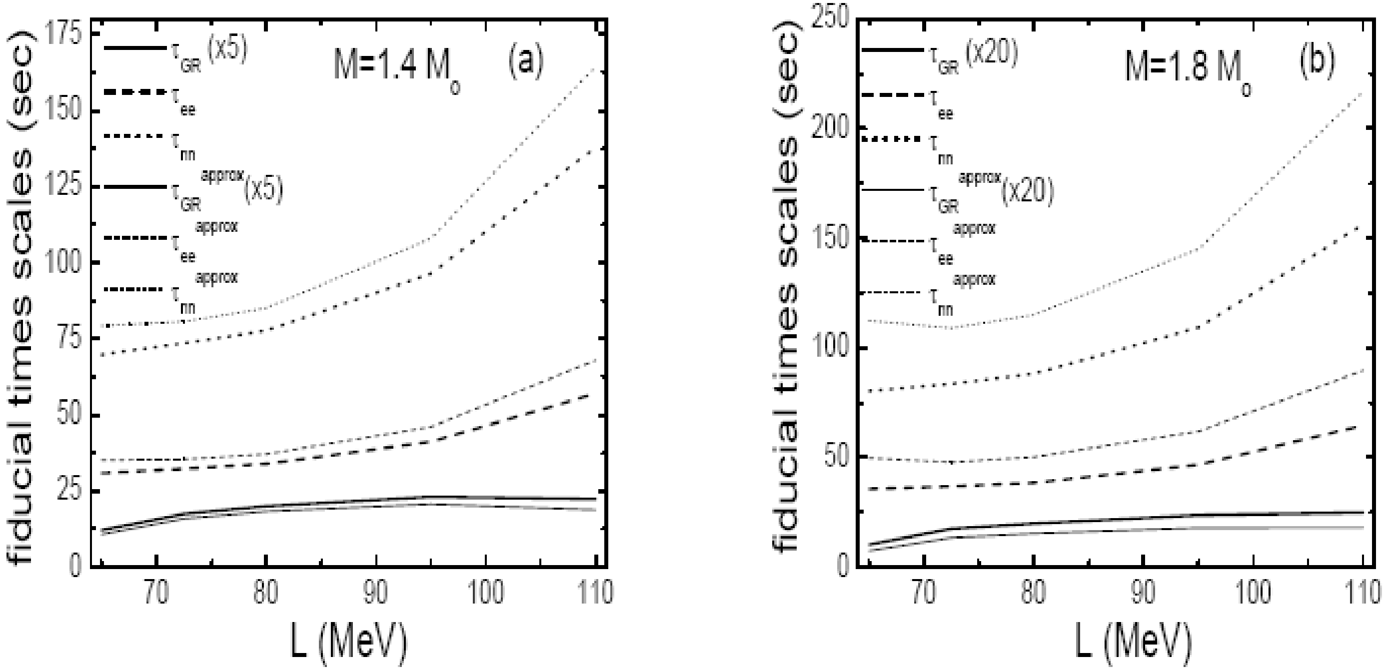


Figure 6: The fiducial time scales $\tilde{\tau}_{GR}$, $\tilde{\tau}_{ee}$, $\tilde{\tau}_{nn}$ as well as $\tilde{\tau}_{GR}^{approx}$, $\tilde{\tau}_{ee}^{approx}$, $\tilde{\tau}_{nn}^{approx}$ as a function of the slope parameter L for a neutron star with mass $M = 1.4M_{\odot}$ (a) and $M = 1.8M_{\odot}$ (b). In case $M = 1.4M_{\odot}$ the gravitational times scales are multiplied by a factor 5 and in case $M = 1.8M_{\odot}$ are multiplied by a factor 20.

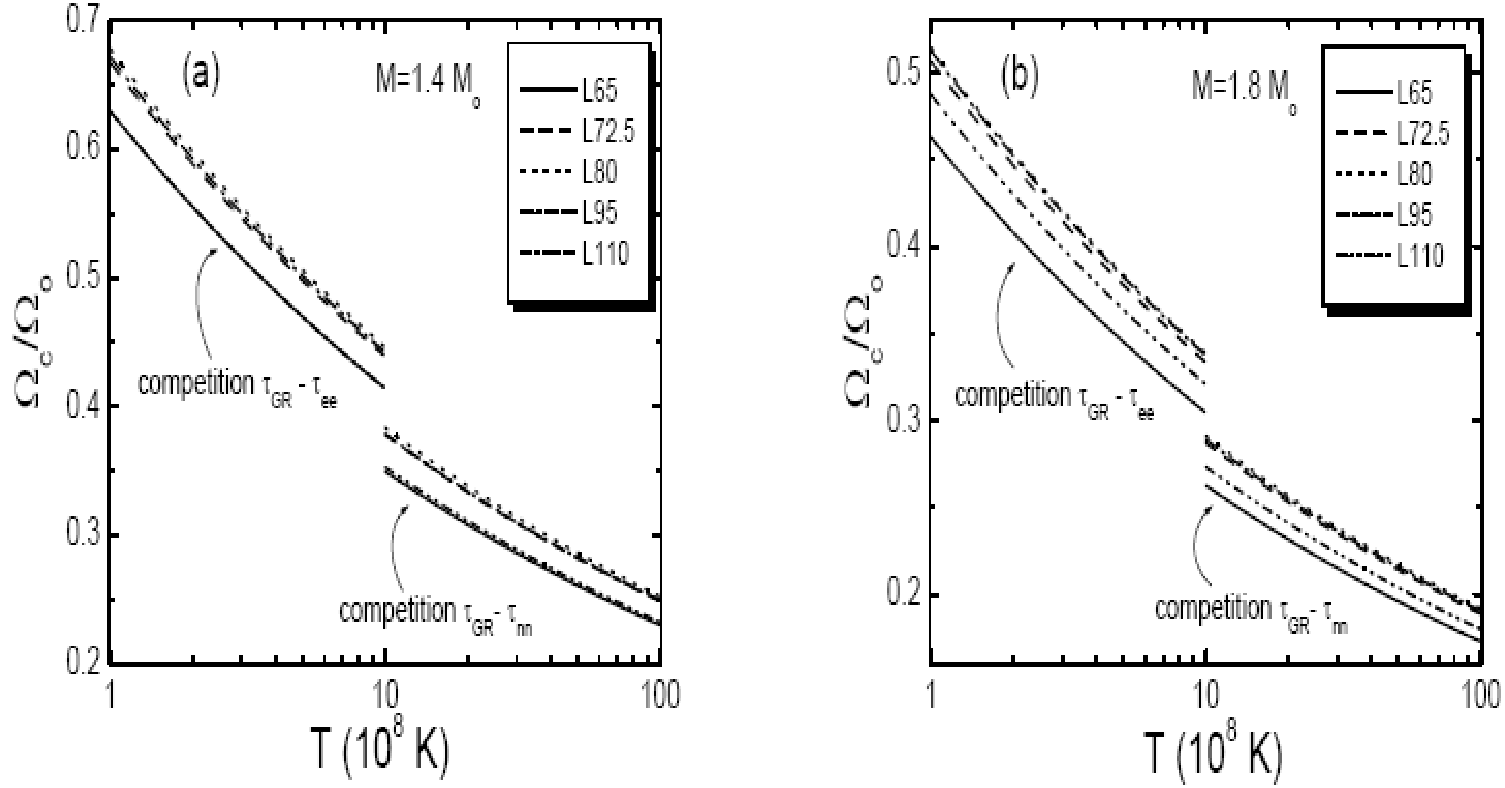


Figure 7: Temperature dependence of the critical angular velocity ratio Ω_c/Ω_0 for a neutron star with mass $M = 1.4M_\odot$ (a) and $M = 1.8M_\odot$ (b) constructed for the selected EOSs.

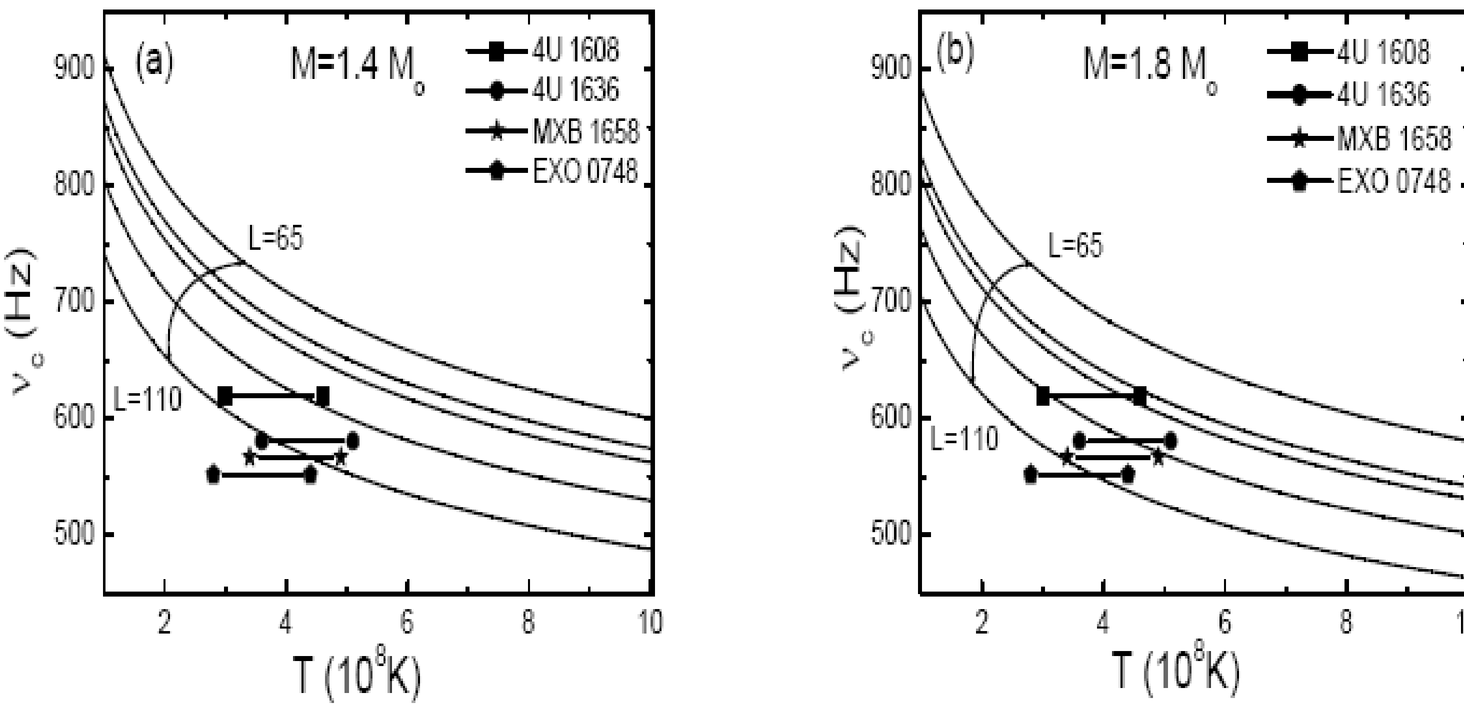


Figure 9: The critical frequency temperature dependence for a neutron star with mass $M = 1.4 M_{\odot}$ (a) and $M = 1.8 M_{\odot}$ (b) constructed for the selected EOSs. In addition, the location of the observed short-recurrence-time LMXBs [42, 43]

Conclusions-Outlook

- We tried to clarify the nuclear equation of state effect on the possible r-mode instabilities
- The slope parameter L has been connected with the main quantities characterize the r-mode instability (time scales, critical angular velocity and critical temperature)
- The nuclear equation of state affects the r-mode in two ways:
  a) affect the location of the crust and
  b) the structure of the density of the neutron star core
- The parameter L has been varied in the interval 50-110 MeV
- Accordingly the fiducial time scales τ_{GR}, τ_{ee}, τ_{nn}, increases by a factor 1.8, 1.85 and 2 (for M=1.4 Mo) and 2.4, 1.8 and 1.95 (for M=1.8 Mo)
- Accordingly $\Omega c/\Omega o$ increases 7-8 % (for M=1.4 Mo) and 10% (for M=1.8 Mo)
- Tc increases 35% (for M=1.4 Mo) and 45% (for M=1.8 Mo)

•We found that four well known cases of low-mass X-ray binaries (LMXBs) lie inside instability window contrary to observations. One can presume that:

a) Either the LMXBs masses are even lower than M=1.4 Mo

b) Or the softer equation of state is more preferred.

•<u>It is very interesting problem if it is possible to constrain the nuclear physics input from the related observation data (very complex problem).</u>

•The present model can be extended to include also the temperature effect on equation of state and properties of r-mode instabilities. This is important for a new-born neutron star (very hot and rapidly rotating) where bulk viscosity is the dominant dissipation mechanism.

Open problems

- The role of magnetic field in the evolution of the r-modes (suppress the instability ? Limits its growth ? Merely change the values of the frequency and growth times ?)
- The role of the crust: in the present work we consider perfectly rigid crust (the viscosity is maximal between core and crust). Considering elastic crust the oscillation of the core could partially penetrate into the crust
- Do superfluid effects suppress the r-mode instability completely in neutron stars ?