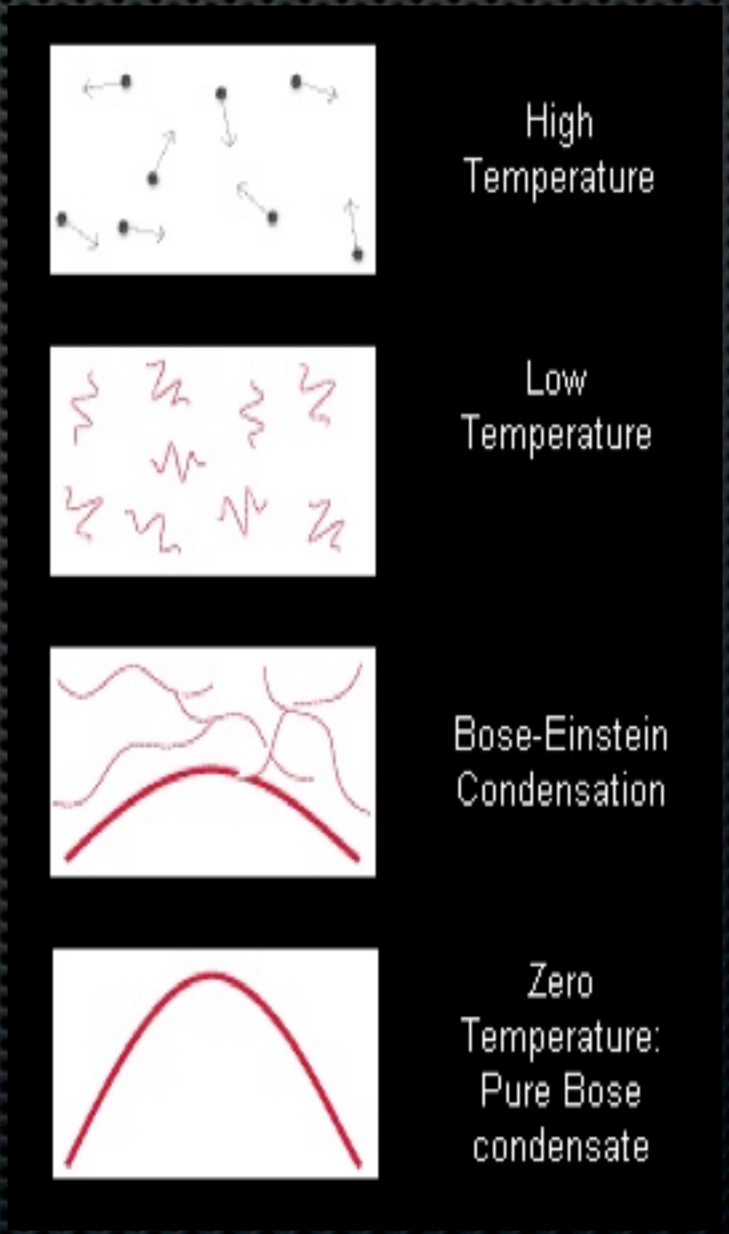
Parafermionic behavior in Bose Einstein condensates.

Martinou Andriana

Supervisor: Bonatsos Dennis

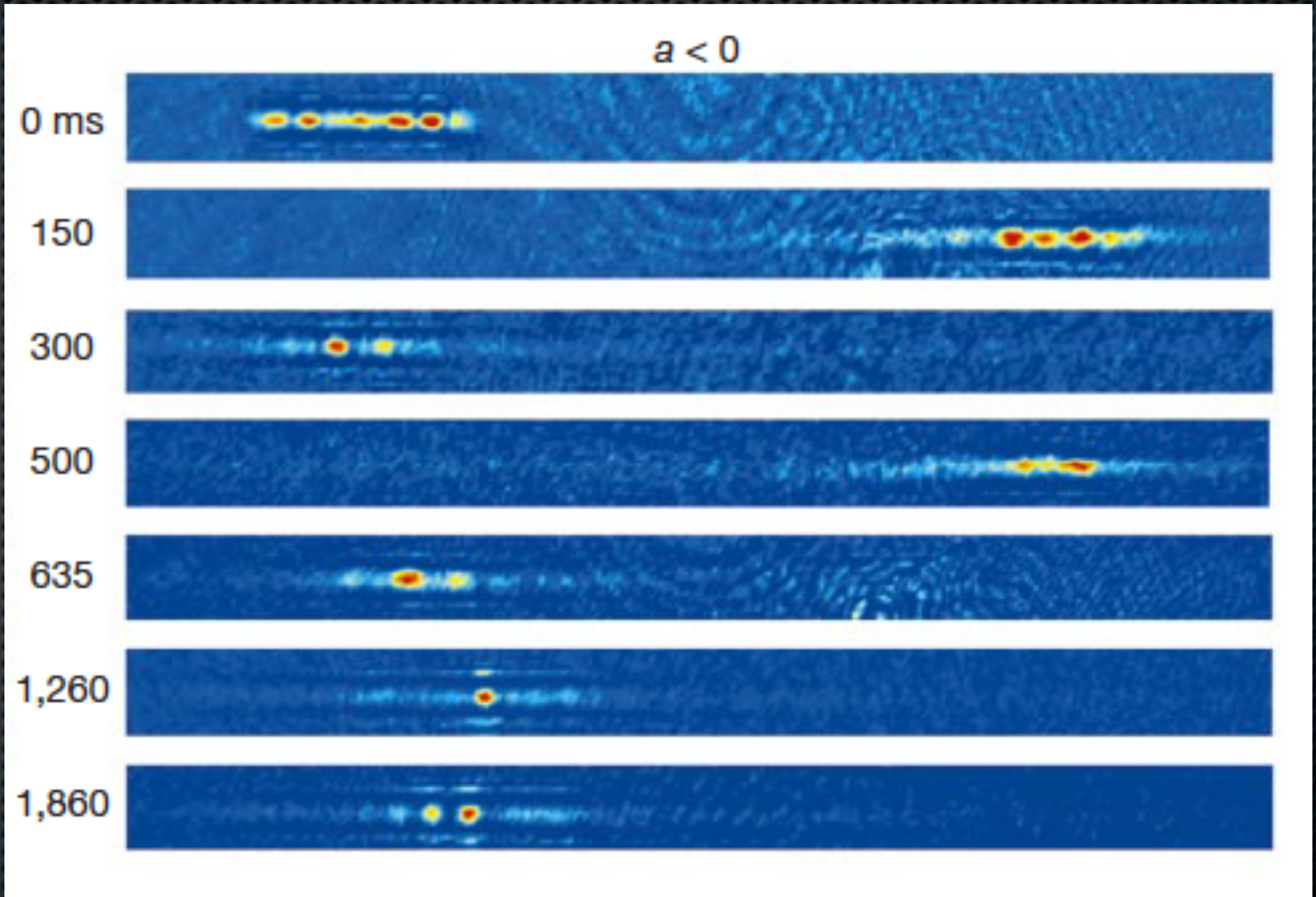
Bose-Einstein Condensates

- The atoms are being cooled down at almost **zero** temperature and trapped in an **magneto-optical trap**. Due to Heisenberg's Principle, their wave-functions expand in space. The resulting particle consists of N atoms and is called **BEC**.



The experiment in Rice University

- Four BECs were created. Each of them consisted of 5000 Lithium atoms that were tuned to **attract** each other. The specific atoms are considered to be bosons. But a gap between the BECs indicates a fermionic behavior of them.



Parafermion

- A Parafermion is every non elementary particle. Thus, it is nor a boson neither a fermion.
- The "Order of the Parafermion" is a quantity proportional to the number of the atoms that can occupy the same quantum state.

The Order of the Parafermion

- If $\rho = 1$ then the particles are fermions.
- If $\rho \to \infty$ then the particles are bosons.
- If $\rho \approx 1$ then the particles have a fermionic behavior.
- If $\rho \gg 1$ the particles have a bosonic behavior.

Parafermionic Oscillators

- We define *deformed creation and annihilation operators*.

$$a^\dagger a = \Phi(N) = [N], \qquad aa^\dagger = \Phi(N+1) = [N+1],$$

$$[a, a^\dagger] = [N+1] - [N], \qquad \{a, a^\dagger\} = [N+1] + [N].$$

- The function Φ(N) is called *deformation function*.

$$\Phi(\mathrm{N}) = \mathrm{N}(\rho + 1 - \mathrm{N})(\lambda + \mu \cdot \mathrm{N} + \nu \cdot \mathrm{N}^2 + \rho \cdot \mathrm{N}^3 + \sigma \cdot \mathrm{N}^4 + \ldots)$$

The energy of the Parafermion

- The appropriate Hamiltonian for the BEC is the Bose-like Hamiltonian. The energy is a function of the number of atoms in the BEC.

$$H = \frac{\hbar\omega}{2}(aa^\dagger + a^\dagger a)$$

$$E(N) = \frac{1}{2}(\Phi(N) + \Phi(N+1))$$

The Gross-Pitaevskii equation

- The **Gross-Pitaevskii** equation is a **non linear Schrödinger equation**.

$$i\hbar\frac{\partial\Phi(\mathbf{r},t)}{\partial t} = (-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r}) + g|\Phi(\mathbf{r},t)|^2)\Phi(\mathbf{r},t).$$

- V_{ext} is the potential of the magneto-optical trap.

$$V_{ext} = \frac{1}{2}m\omega_r^2 r^2 + \frac{1}{2}m\omega_z^2 z^2.$$

- V_{int} is the the mean field potential of the mutual interaction of the atoms.

$$V_{int} = g\,|\,\Phi(\vec{r},t)\,|^2$$

- $\Phi(r)$ is the ground state.

$$\Phi(\mathbf{r}) = P(r)U(z)$$

The energy of the BEC

- The energy of a BEC as given from the Gross-Pitaevskii equation is a non linear function of the number of atoms N.

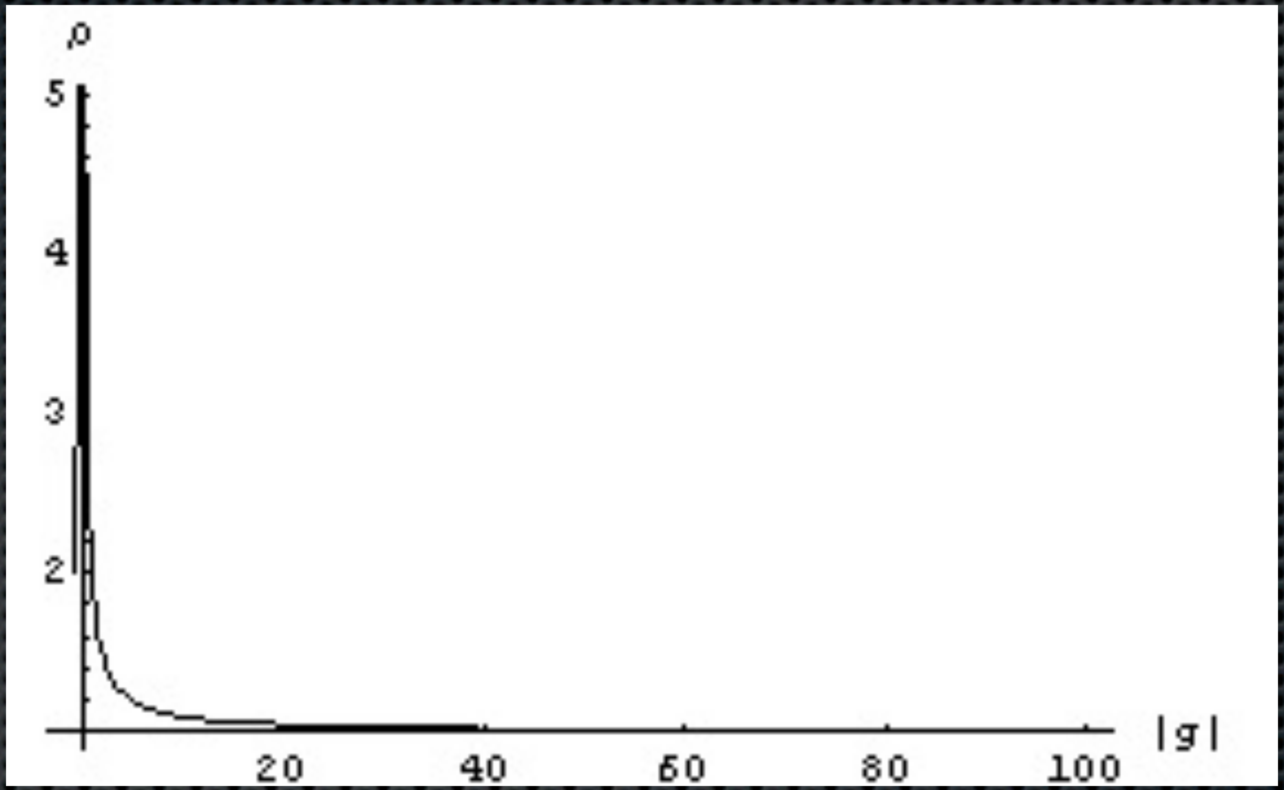
$$E = \frac{\hbar^2}{2m}(k^2 + \frac{\eta^2}{3})N + \frac{g\eta}{6}N^2.$$

$$E = uN + vN^2,$$

The energy of gas of bosons in zero temperature should be a linear function of N, since all of them would be in the ground state:

$$E = uN$$

The order of the Parafermion of the BEC



$$\frac{bosonic - behavior}{fermionic - behavior} = |\frac{u}{v}| = |\frac{3\hbar^2}{-g\eta m}(k^2 + \frac{\eta^2}{3})| = \rho$$

- Download link: http://dspace.lib.ntua.gr/handle/123456789/6199
- Mail: amartinou@inp.demokritos.gr