Quantum Phase Transitions and Conformality in Nuclear Structure

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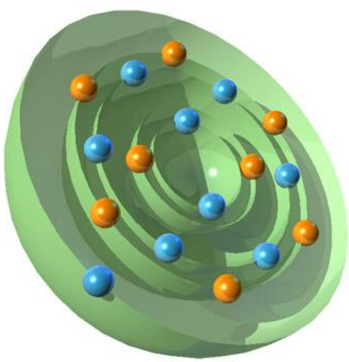
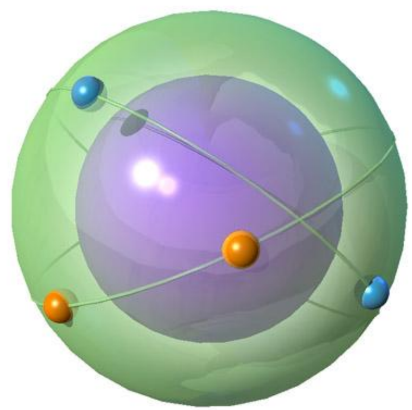
Thanks to Piet Van Isacker,
Dennis Bonatsos and Francesco Iachello .

Thessaloniki, April 12, 2014

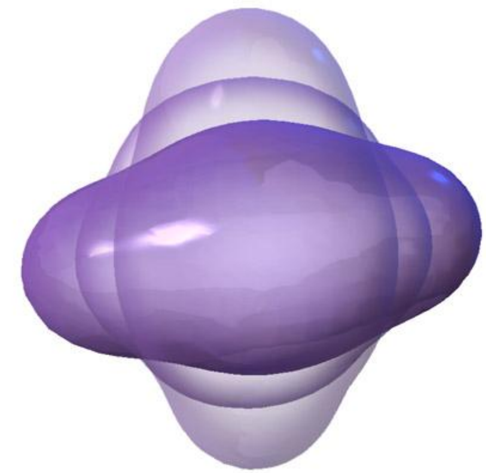
Nuclear collective motions: Simplicity out of complexity



valence nucleons



Cooper pairs:
N s,d boson system

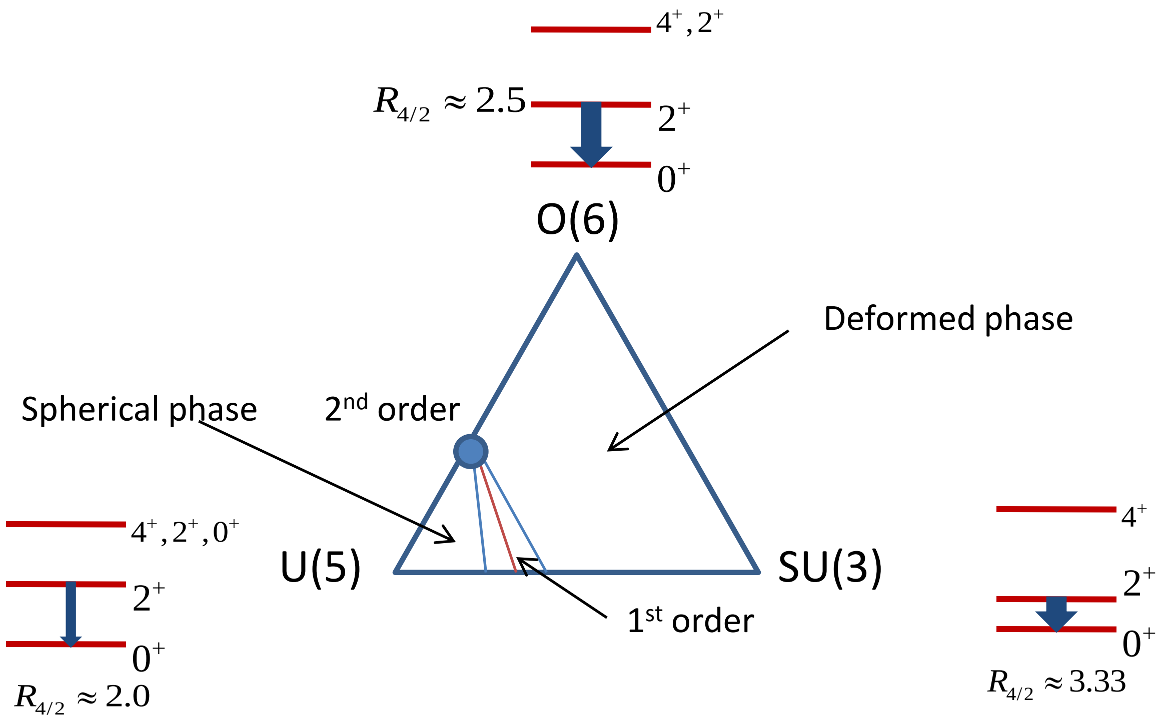


Collective motion:
nuclear shapes

F. Iachello, A. Arima, *The Interacting Boson Model,* Cambridge University Press, 1987.

The Interacting Boson Model

Phase structure of the U(6)



$N \to \infty$

Geometrical limit: Coherent states of U(6) translate the Dynamical symmetries into phases of nuclear structure, expressed as energy surfaces in Bohr coordinates. But the correspondent hamiltonians differ from Bohr's. *A.E.L Dieperink, O. Scholten, F. Iachello, Phys. Rev. Lett. 44, 1747, (1980).*

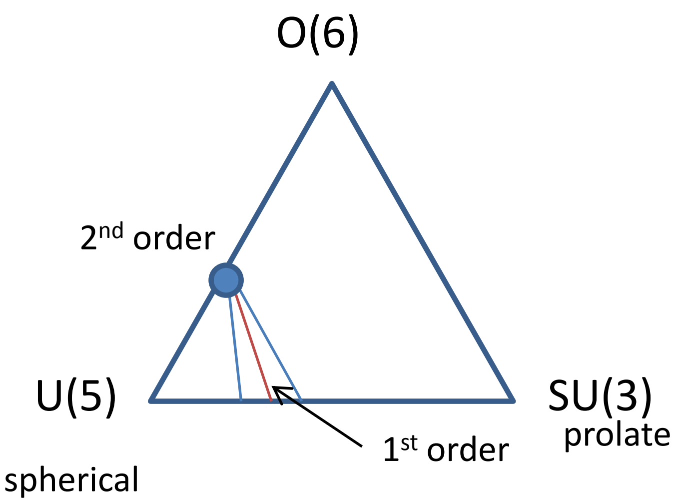
Structural evolution in atomic nuclei is studied through Shape Transitions which in the context of the Interacting Boson Model are the Quantum Phase Transitions (Initially termed as ground state Phase Transitions)

$$H(\xi_1,\xi_2)=\varepsilon\left(H_1+\xi_1 H_2+\xi_2 H_3+\ldots\right)$$

QPT: The control parameter is a coupling constant in the Quantal Hamiltonian. The order parameter is the mean value of a suitable chosen operator. In the IBM:

$$\hat{H}(\eta,\chi)=a\left[\eta\hat{n}_d-\frac{(1-\eta)}{N}\hat{Q}_\chi.\hat{Q}_\chi\right]$$

Two control parameters H(η,χ).
Three phases are characterized by the symmetries



QPTs and Conformality in Nuclear Structure.

- J.W Gibbs : Phase transitions occur only in infinite systems:
In the IBM this is satisfied by the Large N limit.

- Ehrenfest classification: The discontinuities in the energy sufaces of the IBM in the Large N limit.

- Landau phase Transitions: Also, in the Classical limit of the IBM the Energy Surface accepts the Landau analysis as it contains the β^4 term.

- **Scale Invariance? Today we know that the most fundamental property of a second order phase transition is the local scale (conformal) invariance.**

A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, Nucl.Phys.B241, 333,1984.

***This is the study of scale invariance in certain shapes or, in symmetry language, is the study of geometries with conformalities*.**

Critical Point Symmetries

Dynamical symmetries at the critical point: E(5) and X(5) symmetries

F. Iachello, Phys. Rev. Lett. 85, 3580 (2000)

F. Iachello, Phys. Rev. Lett. 87, 052502 (2001)

Shape variables

$$q^1=\beta,\quad q^2=\gamma,\quad q^3=\Phi,\quad q^4=\Theta,\quad q^5=\Psi$$

$$ds^2=g_{ij}dq^i dq^j$$

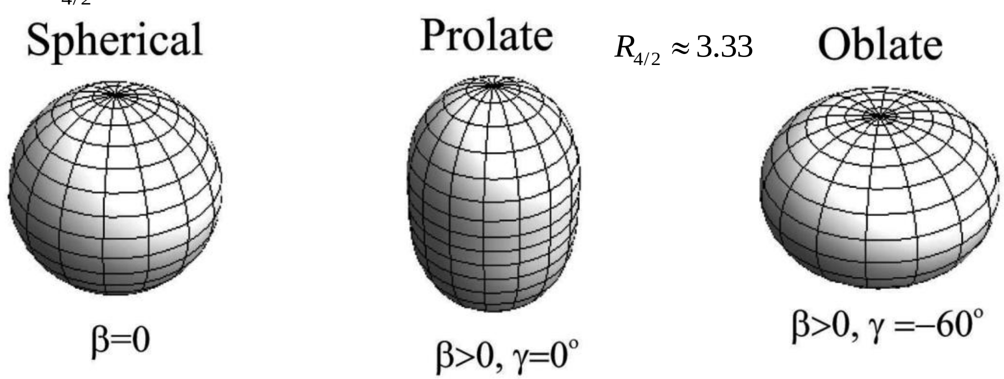
$$H_B=\frac{-\hbar^2}{2B}\nabla^2+\frac{1}{2}C\beta^2$$

Bohr Hamiltonian, *A. Bohr, Dan. Mat.Fys. Medd. 26, (1952),14.*

$R_{4/2}\approx 2.0$

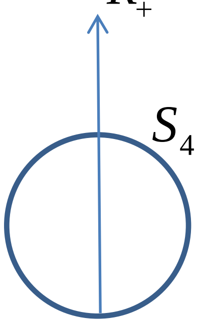
$R_{4/2}\approx 3.33$

(β,Ω_4)



R_+

S_4



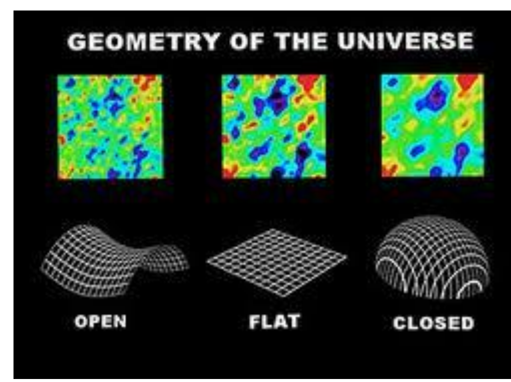
$$R^5=R_+\times S_4$$

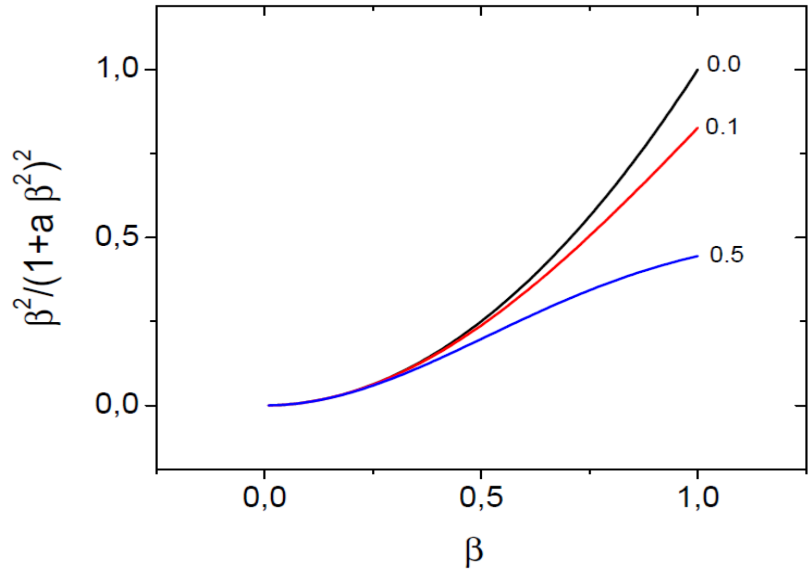
$$ds^2=d\beta^2+\beta^2 d\Omega_4^2$$

P.E. Georgoudis, Phys. Lett. B, 731,122, 1014

D.J.Rowe, T.A. Welsh, M.A.Caprio, Phys.Rev.C 79,054304 ,(2009)

Formal correspondence with cosmological models





*D. Bonatsos, P. E. Georgoudis, D. Lenis, N. Minkov, and C. Quesne, Phys. Rev. C **83**, 044321 (2011).*

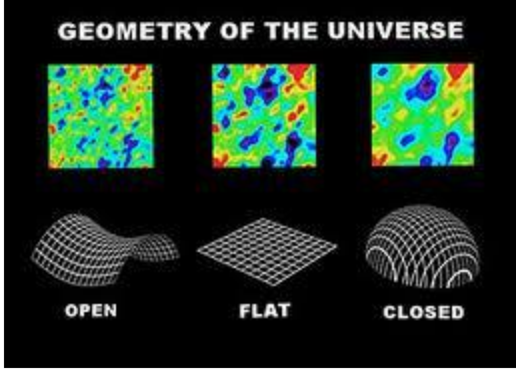
$$B(\beta) = \frac{B(0)}{\left(1 + a\beta^2\right)^2}$$

$$g_{ij} \rightarrow \tilde{g}_{ij} = \frac{g_{ij}}{f^2(\beta)}$$

P.E. Georgoudis, Ph.D. Thesis, to be defended at the National Technical University of Athens, Athens, Greece.

Piet Van Isacker and Kris Heyde, Exactly solvable models of nuclei, Scholarpedia 9(2): 31279

A conformal transformation can "bring the infinity in". This is how the correspondence with the infinite number of bosons of the IBM is achieved in *P.E. Georgoudis, Phys. Lett. B, 732, 122, 2014.*



$$g_{ij} \to \tilde{g}_{ij} = \frac{1}{\left(1+a\beta^2\right)^2} g_{ij}$$

$$ds^2 = d\beta^2 + \beta^2 d\Omega_4^2$$

$$dl^2 = \frac{1}{\left(1+a\beta^2\right)^2}\left(d\beta^2 + \beta^2 d\Omega_4\right)$$

$R^5 \cong R_+ \times S_4$ *D.J.Rowe, T.A. Welsh, M.A.Caprio, Phys.Rev.C 79,054304 ,(2009) .*

Formal Correspondence
with the cosmological
Robertson-Walker
line element, which maps
a hypersphere onto a tangent
plane.

P.E. Georgoudis, Phys. Lett. B, 731, 122, 2014

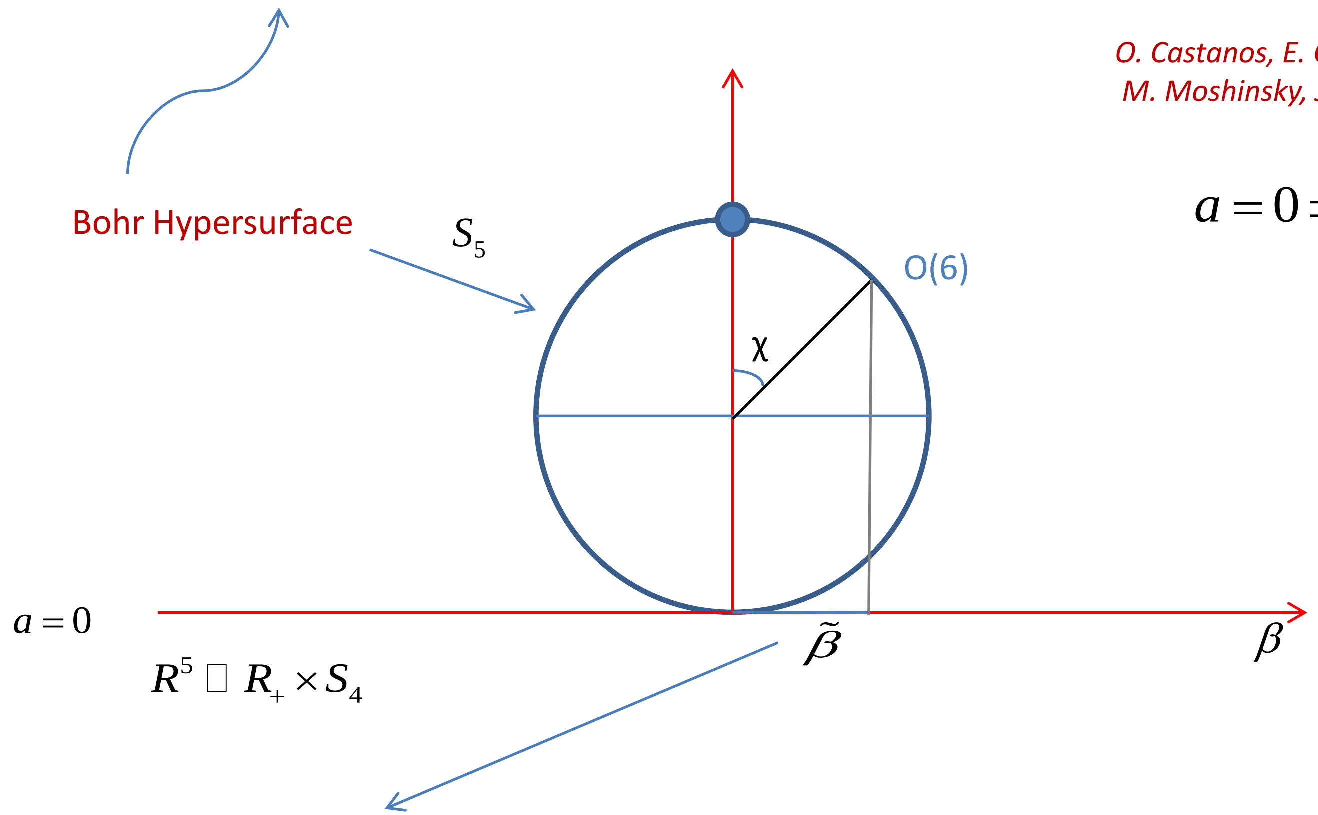
Re-formulation of the Bohr Hamiltonian

Thanks to Piet Van Isacker!

$$dl^2 = \frac{1}{4a}\left(d\chi^2 + \sin^2\chi \; d\Omega_4^2\right) \rightarrow H = \frac{-\hbar^2}{2B}\left(\frac{4a}{\sin^4\chi}\frac{\partial}{\partial\chi}\sin^4\chi\frac{\partial}{\partial\chi} - \frac{4a}{\sin^2\chi}\Lambda^2\right)$$

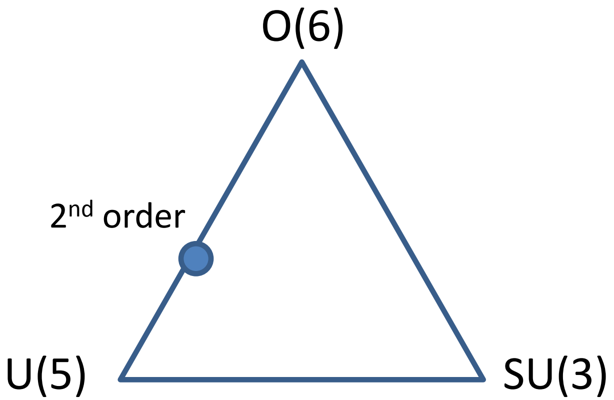
O. Castanos, E. Chacon, A. Frank, M. Moshinsky, J.Math.Phys. 20,1,1979

$$a = 0 \Rightarrow N \rightarrow \infty$$



$$dl^2 = \frac{1}{\left(1+a\beta^2\right)^2}\left(d\beta^2 + \beta^2 d\Omega_4\right) \rightarrow H = \frac{-\hbar^2}{2B}\left(\frac{\left(1+a\beta^2\right)^5}{\beta^4}\frac{\partial}{\partial\beta}\frac{\beta^4}{\left(1+a\beta^2\right)^3}\frac{\partial}{\partial\beta} - \frac{\left(1+a\beta^2\right)^2}{\beta^2}\Lambda^2\right)$$

$$H = \frac{-\hbar^2}{2B}\left(\frac{4a}{\sin^4\chi}\frac{\partial}{\partial\chi}\sin^4\chi\frac{\partial}{\partial\chi} - \frac{4a}{\sin^2\chi}\Lambda^2\right)$$



P.E. Georgoudis, Phys. Lett. B, 731, 122, 2014

$$H = \frac{-\hbar^2}{2B}\left(\frac{\left(1+a\beta^2\right)^5}{\beta^4}\frac{\partial}{\partial\beta}\frac{\beta^4}{\left(1+a\beta^2\right)^3}\frac{\partial}{\partial\beta} - \frac{\left(1+a\beta^2\right)^2}{\beta^2}\Lambda^2\right)$$

$$a = 0$$

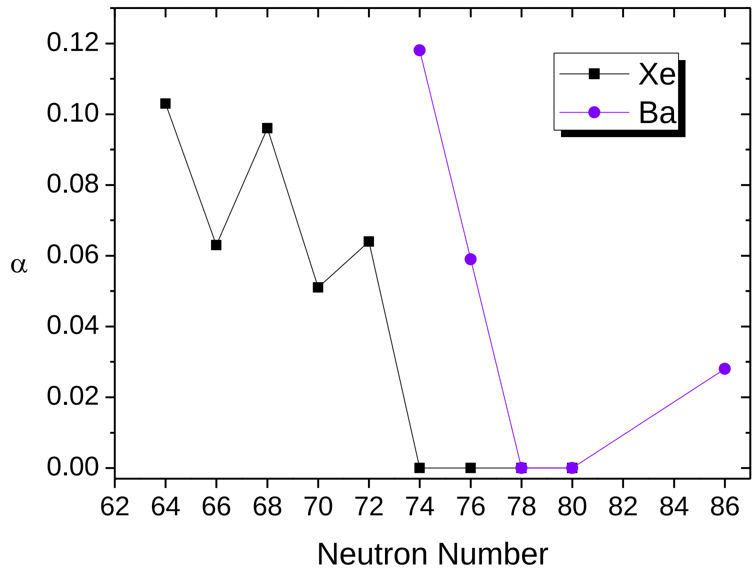
$$H = \frac{-\hbar^2}{2B}\left(\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} - \frac{1}{\beta^2}\Lambda^2\right)$$

M.A. Caprio, F. Iachello, Nucl. Phys. A 781, 26, 2007

E(5) is revealed in the limit of the Large N of the IBM
In the U(5)-O(6) transitional region. This contributes
to the theoretical connection of the E(5) with the 2nd
Order critical point of the IBM, because of the Gibbs criterion.

New E(5) Manifestations?

P.E. Georgoudis, Phys. Lett. B, 731, 122, 2014



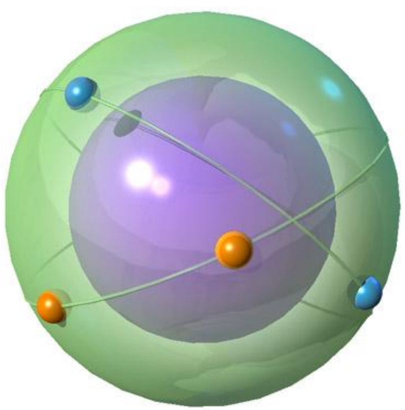
$$a = 0: \quad {}^{128}Xe, {}^{130}Xe, {}^{132}Xe, {}^{134}Xe \qquad {}^{134}Ba, {}^{136}Ba$$

Data taken from Phys. Rev. C. *D. Bonatsos, P.E. Georgoudis, D. Lenis, N. Minkov and C. Quesne, Phys. Rev. C* ***83****, 044321, 2011*

Why: QPTs and CPS

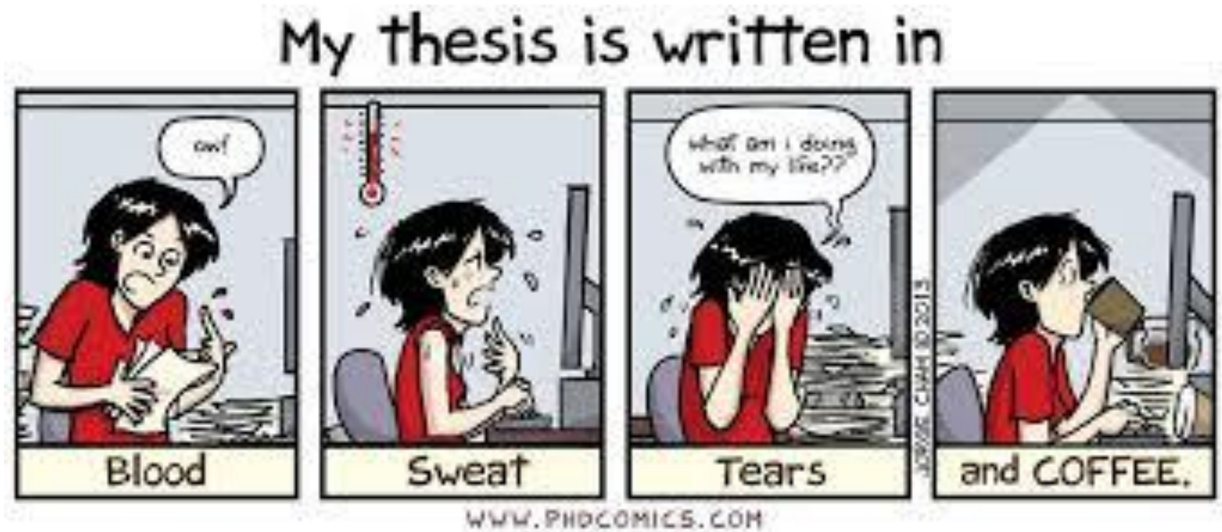
$$H(\eta,\chi)=\varepsilon_0\left[\eta n_d+\frac{1-\eta}{N}Q^\chi\cdot Q^\chi\right]$$

Transitions in the symmetries of the ground states are manifested as QPT in the IBM through the reference to infinity. Only then the energy surfaces exhibit non-analyticity. This is a reference to the continuum, quantitatively expressed with the number of bosons N in the IBM. The conformal factor in the Bohr Hamiltonian inroduces the correspondence with the number of bosons .

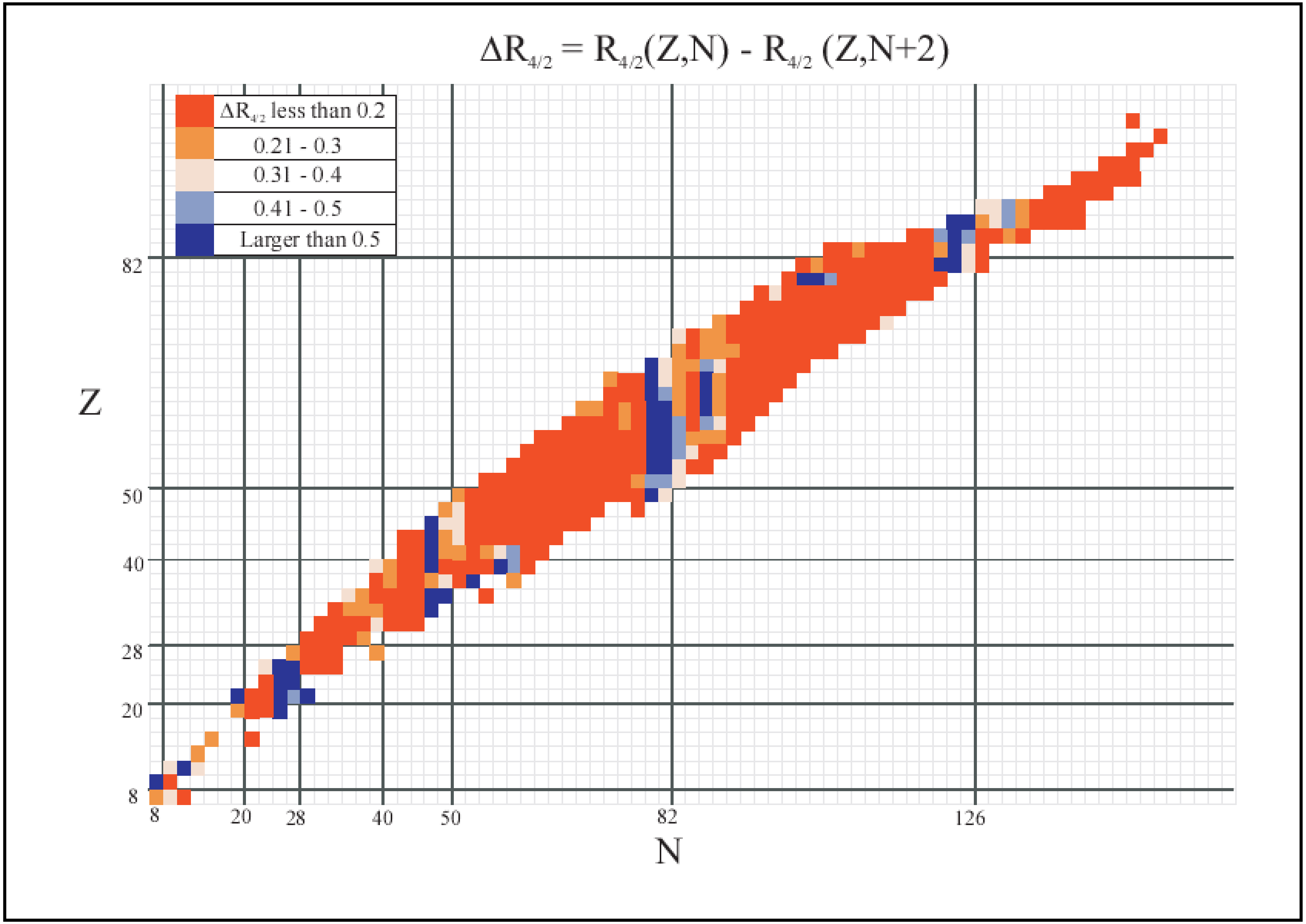


$$N\to\infty$$

$$a=0$$

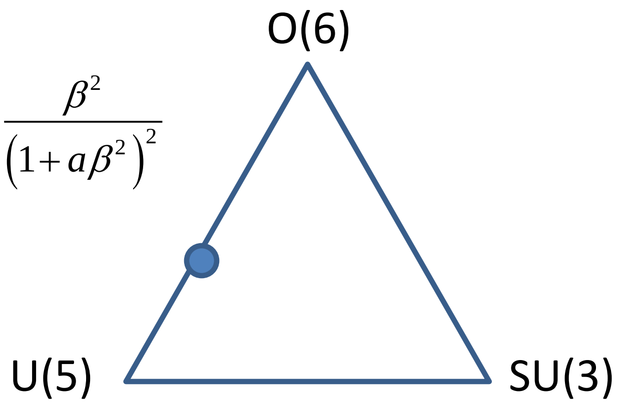

My thesis is written in
ow!
Blood
Sweat
what am i doing with my life??
Tears
and COFFEE.
WWW.PHDCOMICS.COM

Pavel Cejnar, Jan Jolie and Richard F. Casten, Rev. Mod. Phys.,82,3,2155,2010.



$$\Im_k = 4B\left(\frac{\beta^2}{\left(1+a\beta^2\right)^2}\right)\sin^2\left(\gamma - k\frac{2\pi}{3}\right)$$

$$\Im \propto \frac{\beta^2}{\left(1+a\beta^2\right)^2}$$



$$\Im \propto \beta^2$$

O(6) and the IBM

O. Castanos, E. Chacon, A. Frank, M. Moshinsky, J.Math.Phys. 20,1,1979

$$\hat{N} = \frac{1}{2}\left(\frac{1}{b^5}\frac{\partial}{\partial b} b^5 \frac{\partial}{\partial b} + \frac{1}{b^2}L^2 + b^2\right) - 3$$

$$b^2 = \frac{1}{4a}$$ Is a constraint on the number operator leaving only the Casimir of the O(6).

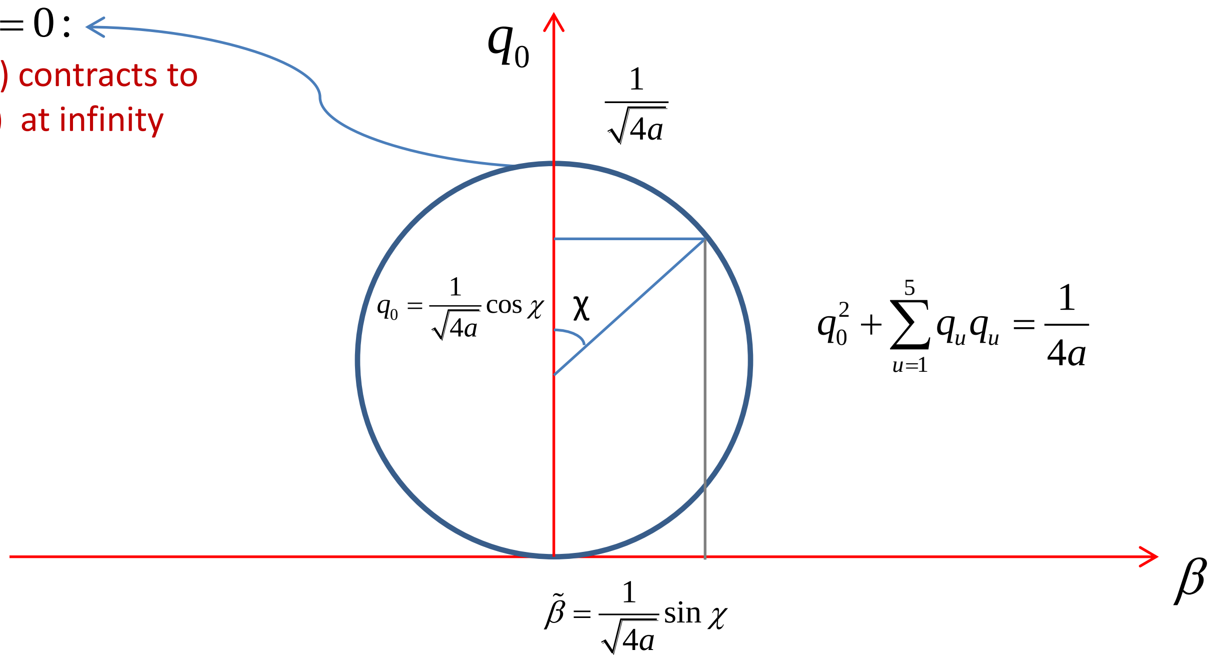
Correspondence between the number of bosons and the radius of the S_5

$$a = 0 \Rightarrow N \to \infty$$

P.E. Georgoudis, Phys. Lett. B731, 122, 2014

$a = 0:$

O(6) contracts to E(5) at infinity



$$q_0^2 + \sum_{u=1}^{5} q_u q_u = \frac{1}{4a}$$

Five Sphere

$$\left(\beta,\Omega_4\right)\rightarrow\left(\chi(\beta),\Omega_4\right):\quad S_5$$

P.E. Georgoudis, Phys. Lett. B, 731, 122, 2014

$$dl^2 = b^2\left(d\chi^2+\sin^2\chi\ \ d\Omega_4^2\right)$$

$$b^2=\frac{1}{4a},\ \sin\chi=\tilde{\beta}\sqrt{4a},\ \tilde{\beta}=\frac{\beta}{1+a\beta^2}$$

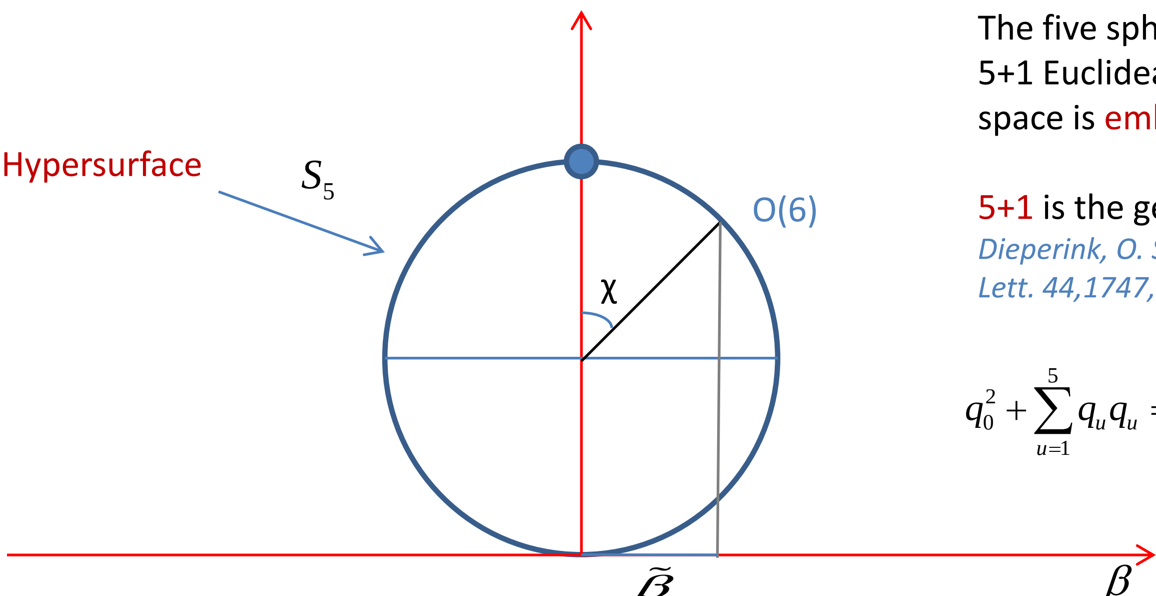
$$dl^2=\frac{1}{\left(1+a\beta^2\right)^2}\left(d\beta^2+\beta^2 d\Omega_4\right)$$

The five sphere lives in a 5+1 Euclidean space. Bohr space is embedded in 5+1 dimensions.

5+1 is the geometry of the IBM , *A.E.L. Dieperink, O. Scholten, F. Iachello, Phys.Rev. Lett. 44,1747, 1980.*

$$q_0^2+\sum_{u=1}^{5}q_u q_u=\frac{1}{4a}$$

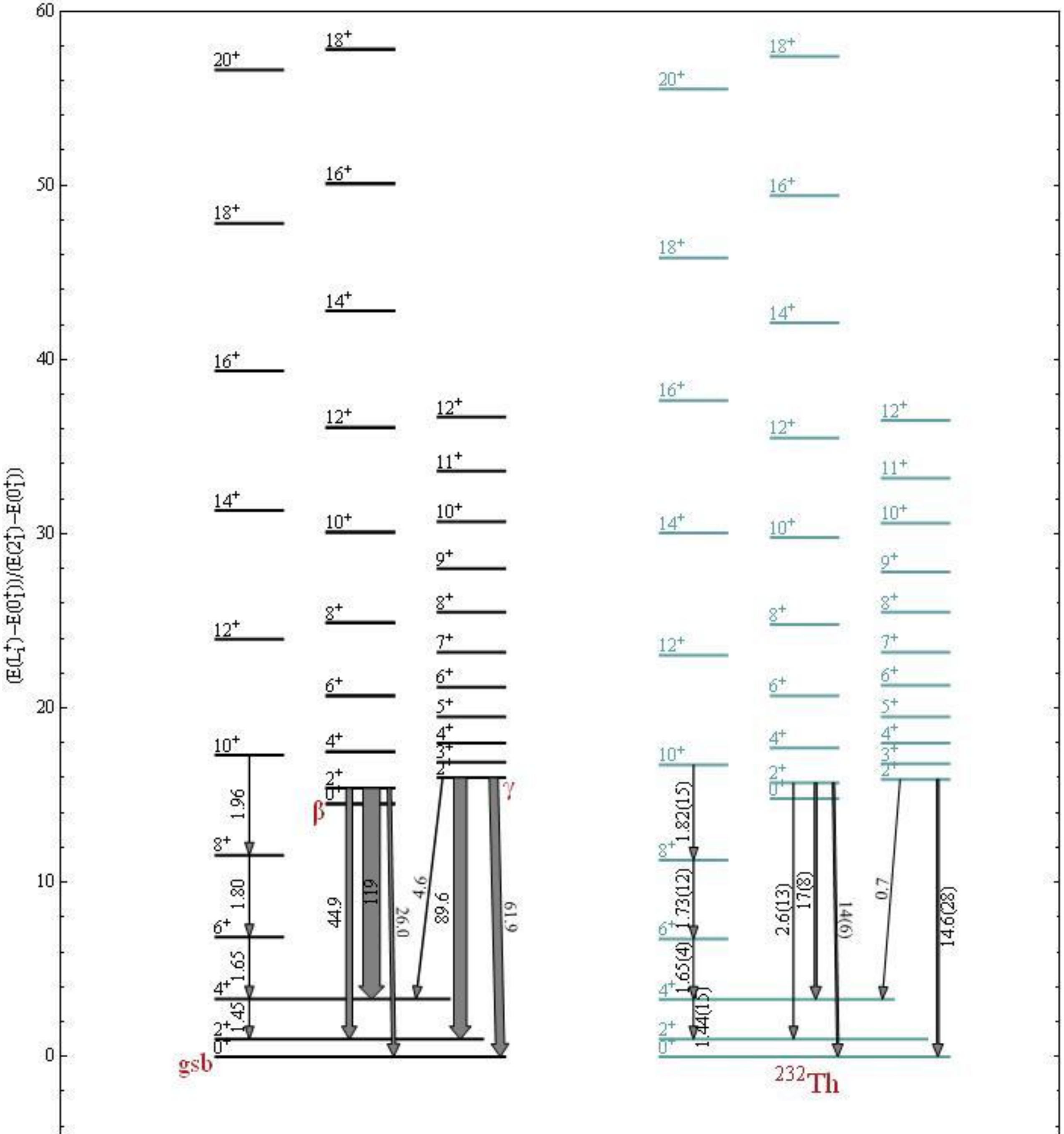
Bohr Hypersurface



Bohr Space

$a=0$

$R^5 \cong R_+\times S_4$



(E(L+i)−E(0+1))/(E(2+1)−E(0+1))
gsb
β
γ
232Th
1.96
1.80
1.65
1.45
44.9
119
26.0
4.6
89.6
61.9
1.82(15)
1.73(12)
1.65(4)
1.44(15)
2.6(13)
17(8)
14(6)
0.7
14.6(28)

Bohr Model

$$R(\vartheta,\varphi,t) = R_0\left[1+\sum_{\mu}\alpha^*_{2\mu}(t)\Upsilon^{(2)}_{\mu}(\vartheta,\varphi)\right]$$

- Quadrupole Vibrations of the nuclear shape (~MeV)
- Monopole term fixed: Nuclear Radius.
- Dipole term ignored: Would represent center of mass motion

$$\alpha_{2\mu} : \alpha_{22,}\ \alpha_{21},\ \alpha_{20},\ \alpha_{2-1,}\ \alpha_{2-2}$$

$$\alpha^{2\mu} \equiv \alpha^*_{2\mu} = (-1)^{\mu}\alpha_{2-\mu}$$

$$\alpha_{2\nu} = \sum_{-2}^{2}\alpha_{2\mu}D^2_{\mu\nu}(\Omega)$$

$$D^2_{\mu\nu}(\Omega) = \langle 2\mu|R(\Omega)|2\nu\rangle$$ → SO(3)

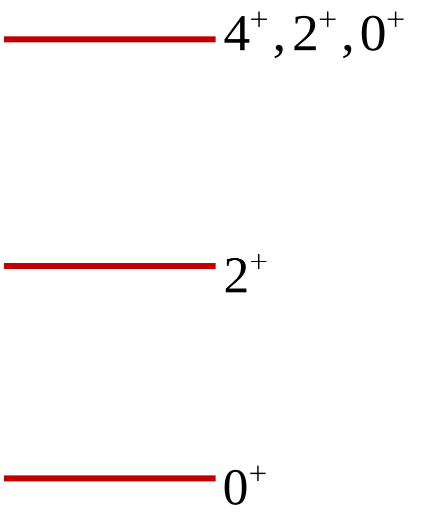
U(5)

$$[\alpha_{2\mu}, \pi_{2\nu}] = i\hbar\delta_{\mu\nu} \longrightarrow \left\{ d^{\dagger}_{2\mu} = \frac{1}{\sqrt{2}}\left(\lambda\alpha_{2\mu} - \frac{i}{\lambda\hbar}\pi_{\mu}\right), \lambda = \sqrt{\frac{B\omega}{\hbar}} \right.$$

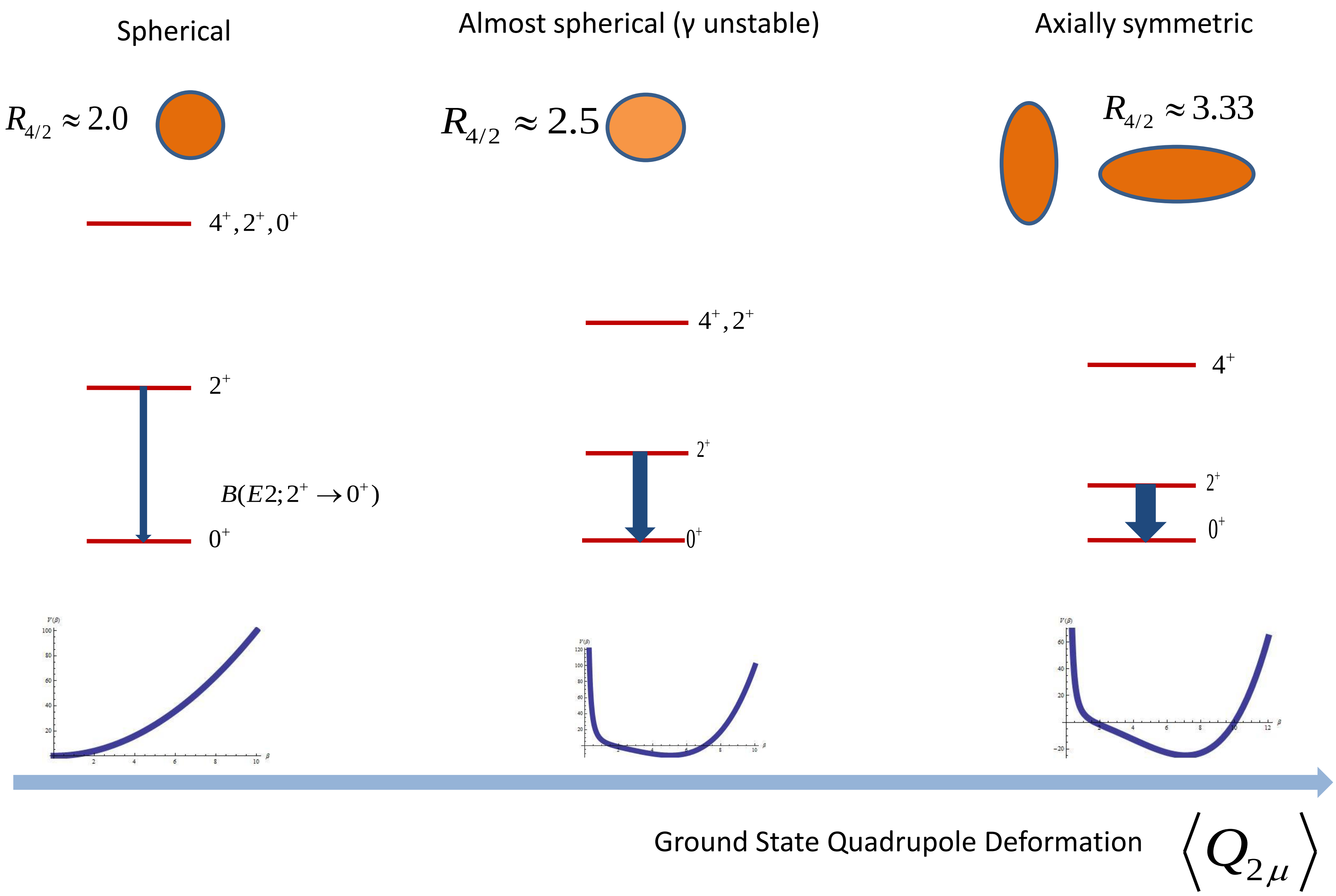
$$\left[d^{\dagger}_{2\mu} \times \tilde{d}_{2\nu}\right]^{0}, \left[d^{\dagger}_{2\mu} \times \tilde{d}_{2\nu}\right]^{2}, \left[d^{\dagger}_{2\mu} \times \tilde{d}_{2\nu}\right]^{4}$$

$$T = \frac{1}{2B}\sum_{\mu}\left|\pi_{\mu}\right|^{2} = \frac{-\hbar^{2}}{2B}\sum_{\mu}\frac{\partial^{2}}{\partial\alpha_{\mu}\partial\alpha_{-\mu}}$$

$$V = \frac{1}{2}C\sum_{\mu}\left|\alpha_{\mu}\right|^{2} = \frac{1}{2}C\sum_{\mu}(-1)^{\mu}\alpha_{\mu}\alpha_{-\mu}$$



Bohr Model: Nuclear Shapes



Intrinsic Frame