

# Symmetry energy effects on isovector properties of neutron rich nuclei with a density functional approach

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# Abstract

- ✧ Model: Variational approach to study the effect of the symmetry energy on isovector properties of various medium and heavy nuclei.
- ✧ Case study:  $^{208}\text{Pb}$ ,  $^{124}\text{Sn}$ ,  $^{90}\text{Zr}$ ,  $^{48}\text{Ca}$ .
- ✧ Calculations:  $R_{\text{skin}}$ ,  $a_A$ , etc.
- ✧ Crucial Point: The Coulomb interaction managed in a self-consistent way.
- ✧ Conclusion: Strong dependence of the SE on the various isovector properties for medium and heavy nuclei.

# Introduction

## ✧ Nuclei Properties

### ✧ Asymmetry Coefficient

$$E_B = \alpha_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - \mathbf{a_A} \frac{(N - Z)^2}{A} - \delta(A, Z)$$

$$\mathbf{a_A} \sim S(\rho)$$

### ✧ Neutron Skin Thickness

$$\mathbf{R_{skin}} = R_n - R_p$$

$$\mathbf{R_{skin}} \sim a(\rho)$$

# Model & Approach

## ✧ Total Energy Functional

$$F_0 \left( \frac{d\rho(r)}{dr} \right)^2 \quad \frac{e^2}{2} \int \frac{\rho(r')(1-a(r))\rho(r)}{|r-r'|} d^3r'$$

$$E[\rho, \alpha] = \int_v \left[ \epsilon_{ANM}(\rho, a) + F_0 |\nabla \rho(r)|^2 + \frac{1}{4} \rho(r)(1 - \alpha(r)) V_c(r) \right] d^3r$$

$$\rho(r) T_0 \left( a \left( \frac{\rho(r)}{\rho_0} \right)^{2/3} - b \left( \frac{\rho(r)}{\rho_0} \right) + c \left( \frac{\rho(r)}{\rho_0} \right)^{5/3} \right) + \alpha^2(r) \rho(r) S(\rho(r))$$

$$J \longleftarrow S(\rho_0) \left( \frac{\rho(r)}{\rho_0} \right)^\gamma \longrightarrow \frac{L}{3J}$$

# Model & Approach

✧ Basic Idea: Minimization of Total Energy

$$E[\rho, \alpha] = \int_v \left[ \epsilon_{ANM}(\rho, a) + F_0 |\nabla \rho(r)|^2 + \frac{1}{4} \rho(r)(1 - \alpha(r)) V_c(r) \right] d^3r$$

→ Total density  $\rho(r)$

→ Isospin asymmetry function  $a(r)$

$$\int \rho(r) d^3r = A$$

$$\int r^2 a(r) \rho(r) dr = N - Z$$

# Model & Approach

## ✧ Variational Method – Lagrangian Function

$$\mathcal{L} = 4\pi r^2 \left( \epsilon_{ANM}(\rho(r), a(r)) + F_0 \left( \frac{dp}{dr} \right)^2 + \frac{1}{4} \rho(1 - \alpha) V_c(r) \right) - \lambda_1 4\pi r^2 \rho - \lambda_2 4\pi r^2 \alpha \rho$$

→ Total density  $\rho(r)$

→ Isospin asymmetry function  $a(r)$

$$\rho(r) = \frac{n_0}{1 + e^{\frac{r-d}{w}}}$$

$$\frac{\partial \mathcal{L}}{\partial a} - \frac{d}{dr} \left( \frac{\partial \mathcal{L}}{\partial a'} \right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \rho} - \frac{d}{dr} \left( \frac{\partial \mathcal{L}}{\partial \rho'} \right) = 0$$

# Model & Approach

## ✧ LDE – Solutions

$$\frac{\partial \mathcal{L}}{\partial a} - \frac{d}{dr} \left( \frac{\partial \mathcal{L}}{\partial \alpha'} \right) = 0$$

$$\rho'' + \frac{2\rho'}{r} - \frac{1}{2F_0} \left[ \frac{\partial \epsilon_{SNM}(\rho)}{\partial \rho} + a^2 \left( S(\rho) + \rho \frac{\partial S(\rho)}{\partial \rho} \right) + \frac{1}{4} (1 - \alpha) V_c(r) - \lambda_1 - \lambda_2 \alpha \right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \rho} - \frac{d}{dr} \left( \frac{\partial \mathcal{L}}{\partial \rho'} \right) = 0$$

$$a(r) = \frac{1}{8S(\rho)} (V_c(r) + 4\lambda_2) \quad 0 \leq a(r) \leq 1$$

# Model & Approach

## ✧ Coulomb Interactions

### ✧ Coulomb Potential

$$V_c(r) = \frac{e^2}{2} \int \frac{\rho(r')(1 - a(r))\rho(r)}{|r - r'|} d^3r'$$

### ✧ Discontinuity Behavior on Integral

$$a(r) = 1$$

$$r \leq r_c$$

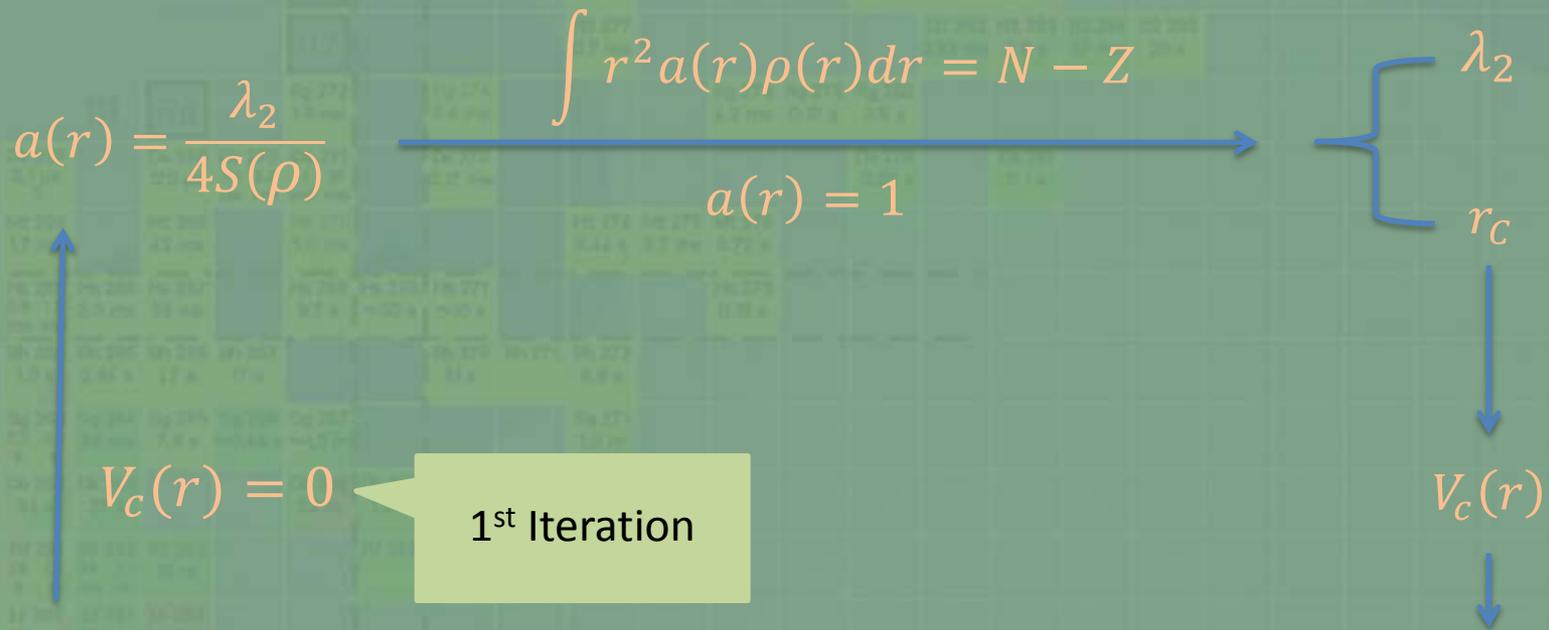
$$V_c(r) = 2\pi e^2 \left[ \int_0^r \frac{1}{r} (1 - a(r')) \rho(r') r'^2 d^2r' + \int_r^{r_c} (1 - a(r')) \rho(r') r' dr' \right]$$

$$r \geq r_c$$

$$V_c(r) = 2\pi e^2 \left[ \int_0^{r_c} \frac{1}{r} (1 - a(r')) \rho(r') r'^2 d^2r' \right]$$

# Model & Approach

## ✧ Self-Consistency



$$a(r) = \frac{1}{8S(\rho)} (V_c(r) + 4\lambda_2)$$

$$0 \leq a(r) \leq 1$$

# Model & Approach

## ✧ Calculations

Inputs (Constants)

$^{48}\text{Ca}$

$^{90}\text{Zr}$

$^{208}\text{Pb}$

$^{124}\text{Sn}$

Inputs (Variables)

$L$

$J$

Outputs

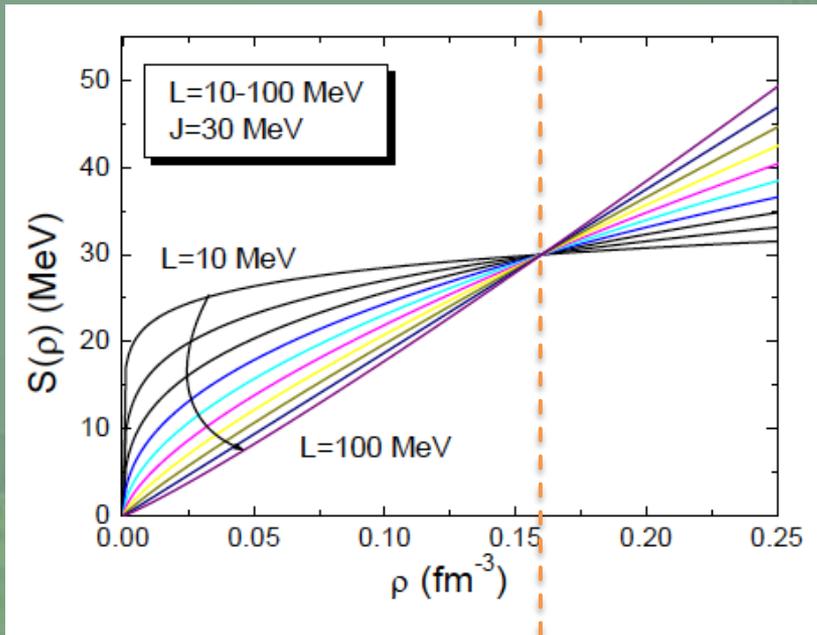
$E_{total}$

$\Delta R$

$a_A$

- ◆ All the calculated properties are studied as a function of the slope of the SE ( $L$ ) and the value of the SE at the nuclear saturation density ( $J$ ).

# Results and Discussion



finite nuclei | neutron stars

$$S(\rho_0) \left( \frac{\rho(r)}{\rho_0} \right)^\gamma$$

- ◆ Lower values of  $L \rightarrow$  higher values of the  $S(\rho)$ . (nucleons less bound,  $N \gg Z$ )
- ◆ Higher values of  $L \rightarrow$  lower contribution of the  $S(\rho)$ . (nucleons more bound,  $N \approx Z$ )

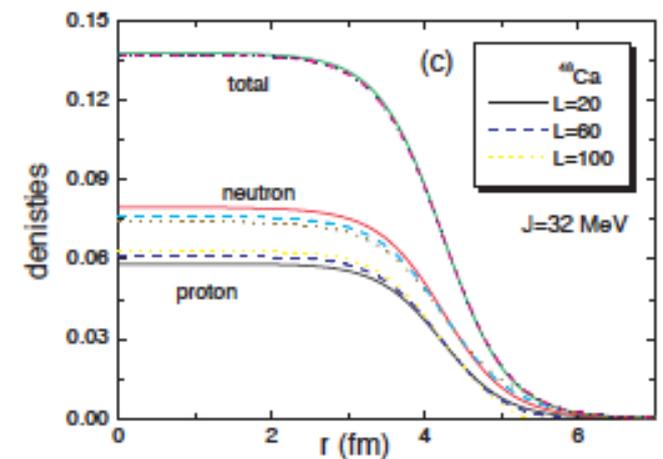
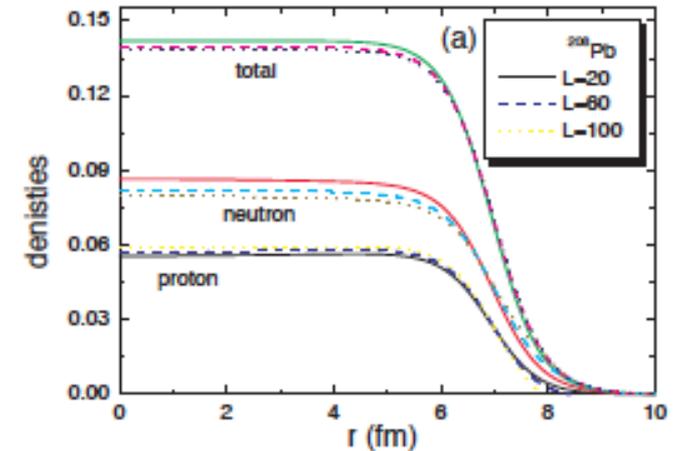
$L$	$E$	$R_n$	$R_p$	$R_{skin}$	$a_A$
10	-1581.36	5.629	5.623	0.006	28.52
20	-1593.44	5.659	5.604	0.055	26.77
30	-1606.35	5.695	5.586	0.109	24.85
40	-1620.28	5.723	5.560	0.163	22.90
50	-1633.62	5.756	5.537	0.219	20.87
60	-1646.40	5.780	5.517	0.263	19.03
70	-1658.36	5.803	5.500	0.303	17.32
80	-1669.35	5.816	5.478	0.338	15.80
90	-1679.37	5.828	5.458	0.370	14.41
100	-1688.46	5.846	5.448	0.398	13.11

- ◆ The isovector properties of nuclei are related with the trend of the symmetry energy in the region  $10 \text{ MeV} < L < 100 \text{ MeV}$

# Results and Discussion

Higher values of  $L$ :

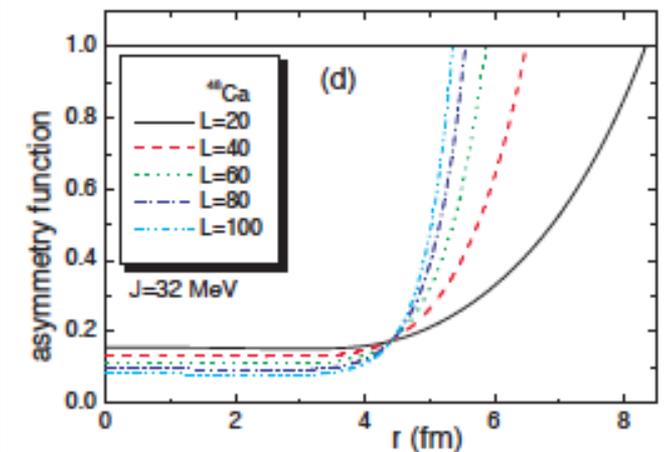
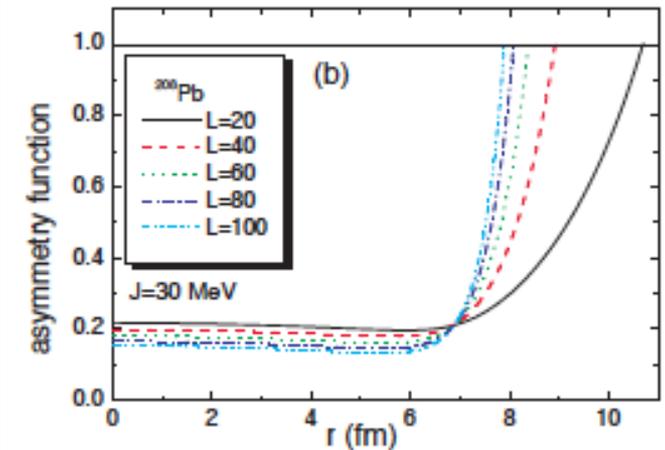
- Shift the neutron distribution to the outer part of the nucleus.
- Concentrates deeper the protons.
- Shift of  $r_c$ , increasing dramatically the neutron skin, forming a kind of neutron halo inside the nucleus.



# Results and Discussion

The asymmetry function  $\alpha(r)$ :

- Acts as a regulator on the proton and neutron distributions in order to minimize the total energy of the nucleus.
- Coulomb potential  $V_c(r)$  acts inversely.
- Long range Coulomb forces – Isovector part of nuclear forces → creation of the neutron skin thickness.



# Results and Discussion

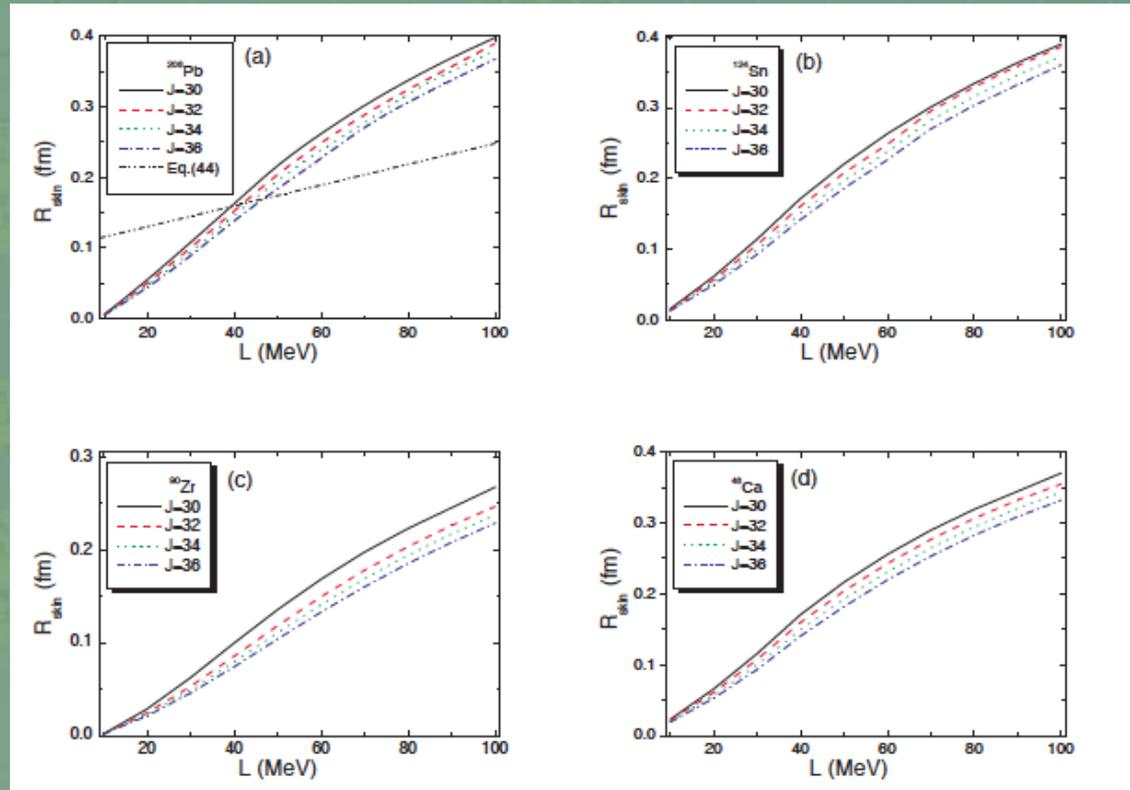
Strong dependence of  $R_{skin}$  on  $L$ :

$$R_{skin}(fm) = 0.101 + 0.00147L (MeV)$$

Centelles et. al.

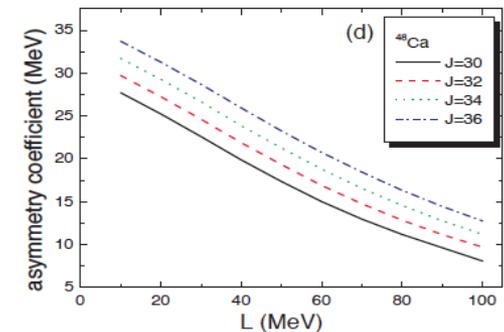
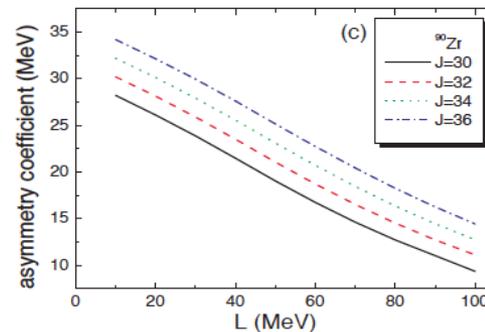
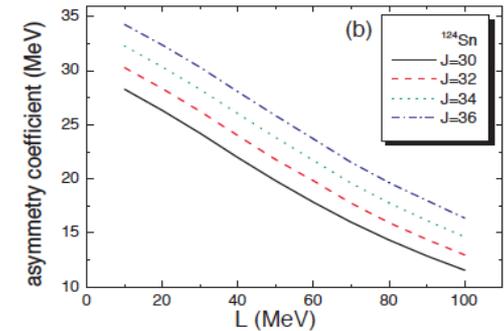
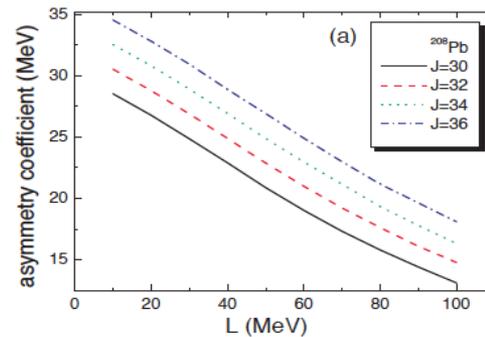
◆ That relation supports a softer dependence of  $R_{skin}$  on  $L$  compared to the present study.

◆ The intersection between our results and the results compatible with Centelles approximation corresponds to values of binding energy are very close to the experimental data for the specific nuclei.



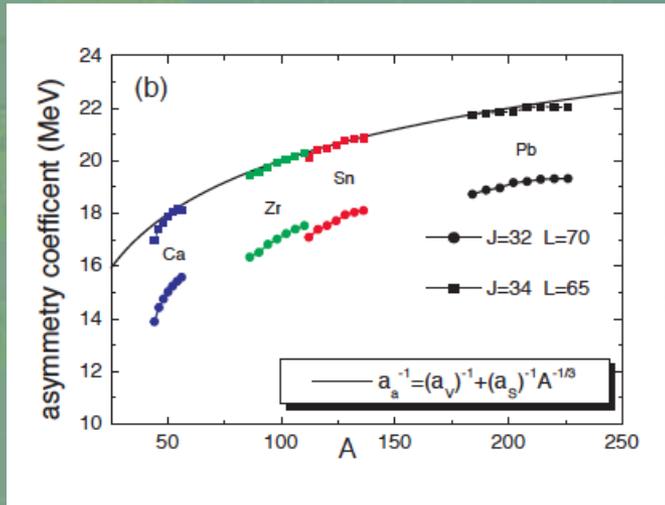
# Results and Discussion

- ◆  $a_A$  is a decreasing function of  $L$ .
- ◆ Higher values of  $L \rightarrow$  Low values of  $\alpha(\rho)$ .
- ◆  $a_A$  exhibits a mass depended A behavior.

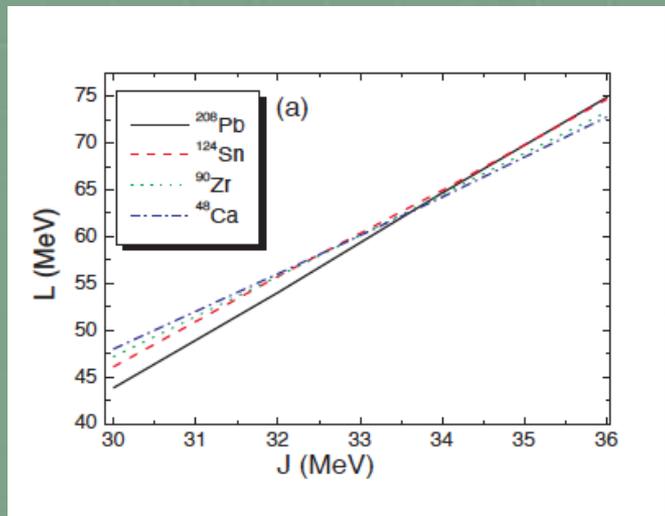


# Results and Discussion

- ◆ The set  $J=34$  MeV and  $L=65$  MeV reproduces very well the empirical values of  $a_A$  for almost all of the medium and heavy isotopes.

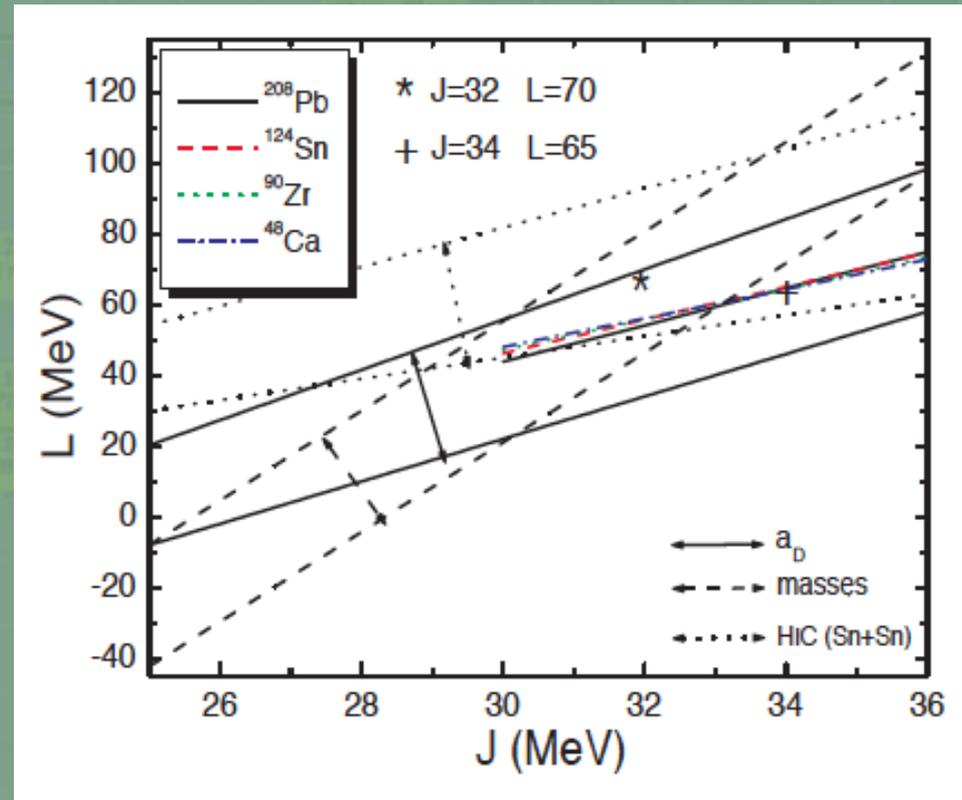


- ◆ Possible universal dependence holds between  $L$  and  $J$  for nuclei at least in the mass region  $A = 40-200$ .
- ◆ The same set of  $L$  and  $J$  reproduce in a very good accuracy the  $a_A$  for medium as well as heavy nuclei.



# Results and Discussion

- ◆ Heavy ion collision
  - ◆ Nuclear structure observables.
- The present results lies inside the relevant area.



# Results and Discussion

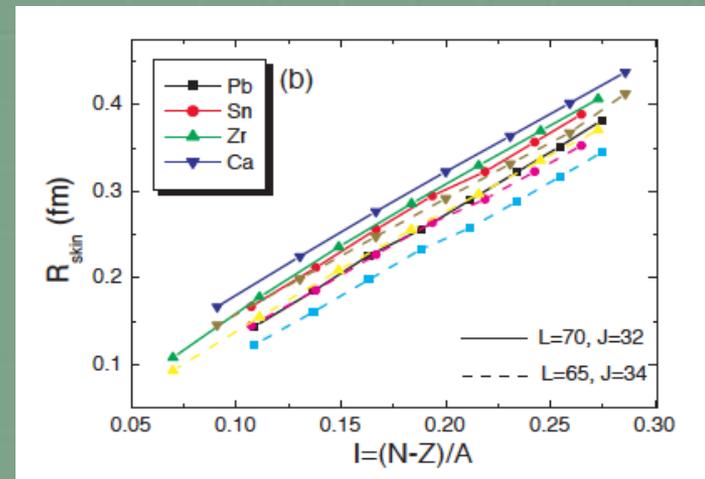
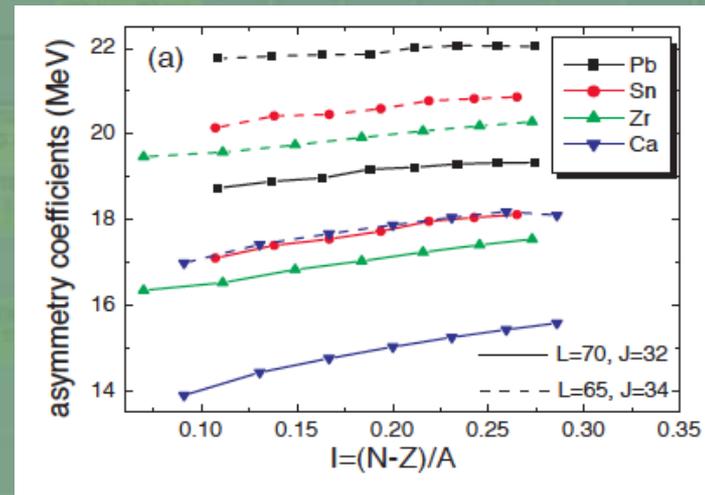
- ◆ Dependence of the coefficient  $a_A$  on the asymmetry parameter  $I$  for all the relevant isotopes.
- ◆ In almost all cases there is a soft dependence of  $a_A$  on  $I$  but a strong dependence on the value of  $J$ .
- ◆ Linear dependence of  $R_{skin}$  on  $I$ .

$$R_{skin} = a + bI$$

$$-0.02 \leq a \leq 0.045$$

$$1.31 \leq b \leq 1.45$$

- ◆ Intervals are dependent on the specific set of values of  $L$  and  $J$ .



# Conclusion

## ✧ About the present work

- Neutron skin thickness is very sensitive on  $L$ .
- Case of  $^{208}\text{Pb}$ : The present approximation supports a stronger sensitivity of the neutron skin thickness on  $L$ .
- Comparison to the empirical formula.
- $J$  imposes strong constraints on  $L$ .

# Conclusion

## ✧ About the present work

- The method can be applied in the totality of medium and heavy neutron rich nuclei.
- Coulomb interaction can easily be separated from the nucleon-nucleon interaction.
- The approach can be easily extended to include more complicated expressions:
  - SE
  - SNM

# Conclusion

## ✧ About future works (open problems)

- ◆ Experimental and theoretical work is necessary.
  - ➔ Finite nuclei.
  - ➔ Neutron star structure.
- ◆ SE contribution for low densities.
- ◆ Correlation between the nuclear EOS of nuclear matter and the DFT of infinite nuclei.

# The End

Thank you for your attention!