Proxy-SU(3) symmetry in heavy nuclei

Dennis Bonatsos
INPP, NCSR Demokritos

The people behind the work

NTUA

I.E. Assimakis

A. Martinou

S. Sarantopoulou

U. Athens

S. Peroulis

K. Blaum (Heidelberg)

R.B. Cakirli (Istanbul)

R.F. Casten (Yale, MSU)

N. Minkov (INRNE Sofia)

Elliott SU(3)

sd shell

J.P. Elliott, Proc. Roy. Soc. Ser. A 245 (1958) 128, 562

J.P. Elliott and M. Harvey, 272 (1963) 557

classification in terms of SU(3)

SU(3)

8 generators

angular momentum L_μ, $\mu=-1,0,1$

quadrupole operator Q_ν, $\nu=-2,-1,0,1,2$

irreducible representations (irreps) (λ,μ)

Young diagrams $[f_1,f_2]$

$\lambda=f_1-f_2$, $\mu=f_2$

$SU(3) \supset SO(3)$ rules

(λ,μ) K L

K=missing quantum number

$K=\min\{\lambda,\mu\}, \min\{\lambda,\mu\}-2, \ldots, 1$ or 0

$L=K, K+1, K+2, ,,,, K+\max\{\lambda,\mu\}$

except $K=0$

$L=\max\{\lambda,\mu\}, \max\{\lambda,\mu\}-2, \ldots, 1$ or 0

example: 24Mg

12 protons, 12-8 valence protons ->(4,2)

12 neutrons, 12-8 valence neutrons->(4,2)

total (8,4)

K=4, 2, 0

ground state band K=0, L=8,6,4,2,0

K=2 band, L=2,3,4,5,6,7,8,9,10

K=4 band, L=4,5,6,7,8,9,10,11,12

Shell model algebras with SU(3) subalgebras

shell	algebra
sd	U(6)
pf	U(10)
sdg	U(15)
pfh	U(21)
sdgi	U(28)
pfhj	U(36)

D.B. and A. Klein, Ann. Phys. 169 (1986) 61

$U(N) \supset SU(3)$

J.P. Draayer, Y. Leschber, S.C. Park, R. Lopez,
Comput. Phys. Commun. 56 (1989) 279

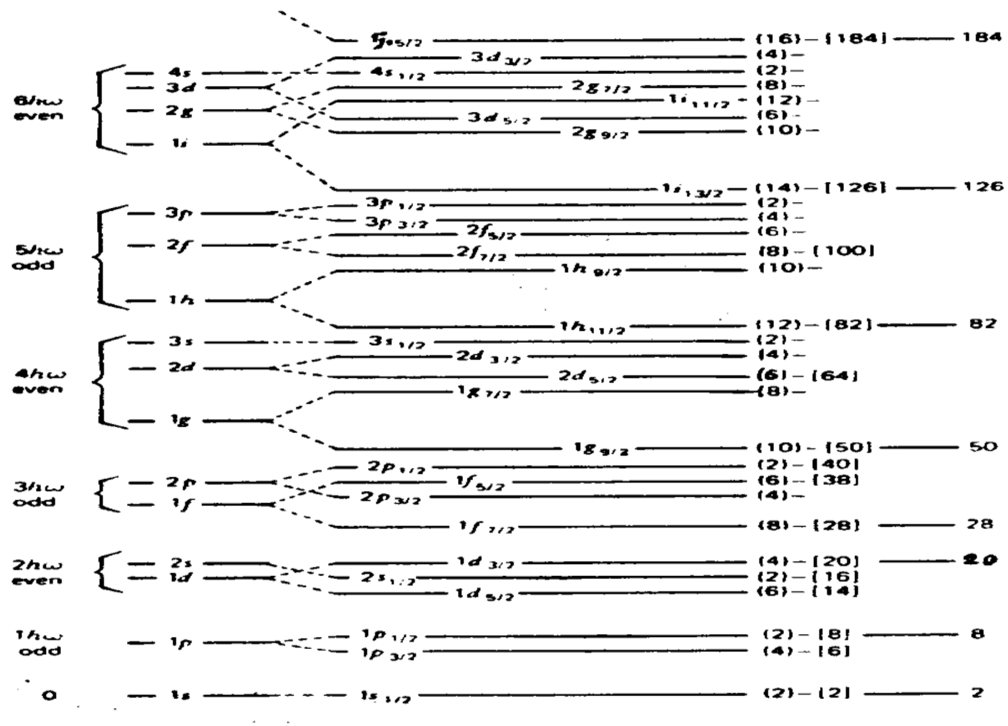
	8-20	28-50	50-82	82-126	126-184	184-258
	sd	pf	sdg	pfh	sdgi	pfhj
	U(6)	U(10)	U(15)	U(21)	U(28)	U(36)
N						
2	(4,0)	(6,0)	(8,0)	(10,0)	(12,0)	(14,0)
4	(4,2)	(8,2)	(12,2)	(16,2)	(20,2)	(24,2)
6	(6,0)	(12,0)	(18,0)	(24,0)	(30,0)	(36,0)
8	(2,4)	(10,4)	(18,4)	(26,4)	(34,4)	(42,4)
10	(0,4)	(10,4)	(20,4)	(30,4)	(40,4)	(50,4)
12	(0,0)	(12,0)	(24,0)	(36,0)	(48,0)	(60,0)

spin-orbit interaction

- Lowers the orbit with highest j
- Mixes the harmonic oscillator shells
- Destroys SU(3) symmetry of h.o. shells

Approximations needed

shell model



6ħω even
5ħω odd
4ħω even
3ħω odd
2ħω even
1ħω odd
0
4s
3d
2g
1i
3p
2f
1h
3s
2d
1g
2p
1f
2s
1d
1p
1s
3d 3/2
4s 1/2
2g 7/2
1i 11/2
3d 5/2
2g 9/2
1i 13/2
3p 1/2
3p 3/2
2f 5/2
2f 7/2
1h 9/2
1h 11/2
3s 1/2
2d 3/2
2d 5/2
1g 7/2
1g 9/2
2p 1/2
1f 5/2
2p 3/2
1f 7/2
1d 3/2
2s 1/2
1d 5/2
1p 1/2
1p 3/2
1s 1/2
(16) – [184]
(4) –
(2) –
(8) –
(12) –
(6) –
(10) –
(14) – [126]
(2) –
(4) –
(6) –
(8) – [100]
(10) –
(12) – [82]
(2) –
(4) –
(6) – [64]
(8) –
(10) – [50]
(2) – [40]
(6) – [38]
(4) –
(8) – [28]
(4) – [20]
(2) – [16]
(6) – [14]
(2) – [8]
(4) – [6]
(2) – [2]
184
126
82
50
28
20
8
2

Interacting Boson Model

N=number of bosons (valence fermion pairs)

(2N,0) K=0	ground state band
(2N-4,2) K=0,2	β1, γ1 bands
(2N-8,4) K=0,2,4	β2, γ2, first K=4 bands

F. Iachello and A. Arima, The Interacting Boson Model (Cambridge UP, 1987)

Example:168Er in IBM

68 protons, 82-68=14 valence protons

100 neutrons, 100-82=18 valence neutrons

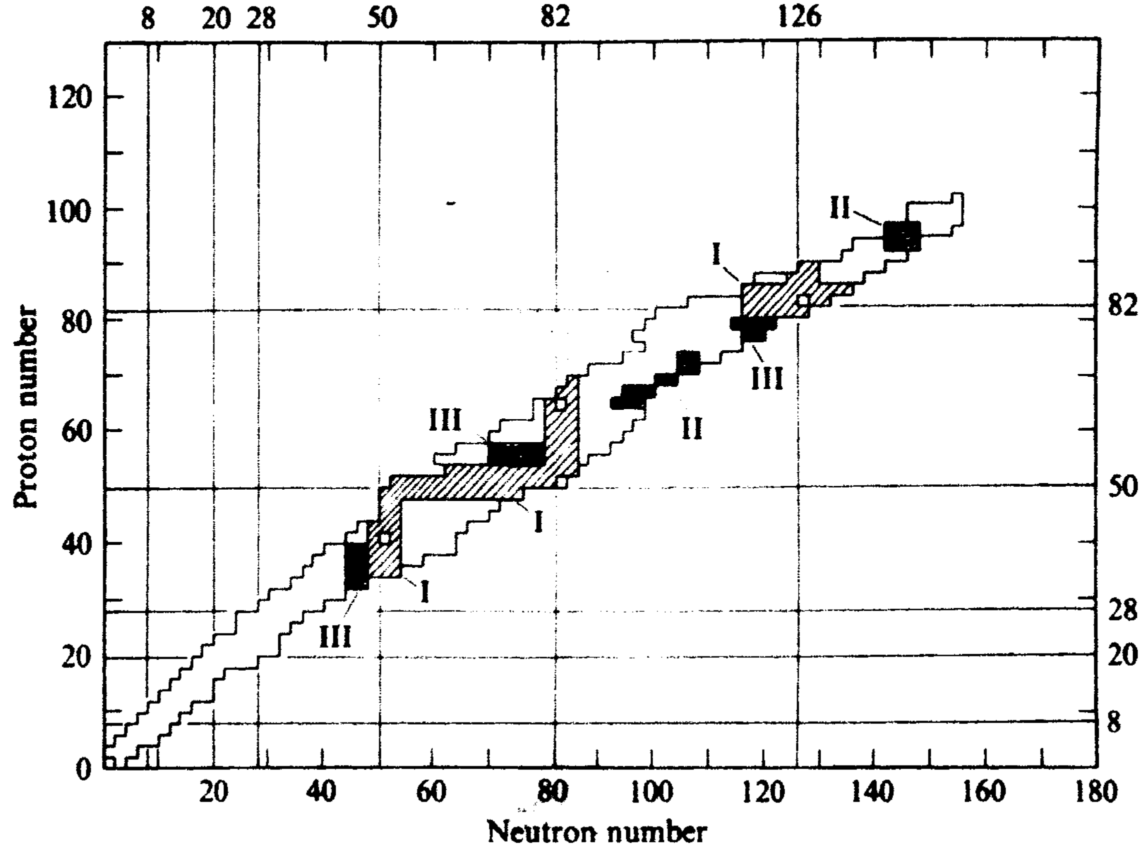
(14+18)/2=16 bosons

(32,0) ground state band

(28,2) β1, γ1

(24,4) β2, γ2, K=4

Interacting Boson Model



Pseudo-SU(3)

- Map the levels of normal parity
- Leave levels of intruder parity unchanged

Proxy-SU(3)

- Map the levels of intruder parity
- Leave levels of normal parity unchanged

Nilsson model

H-7

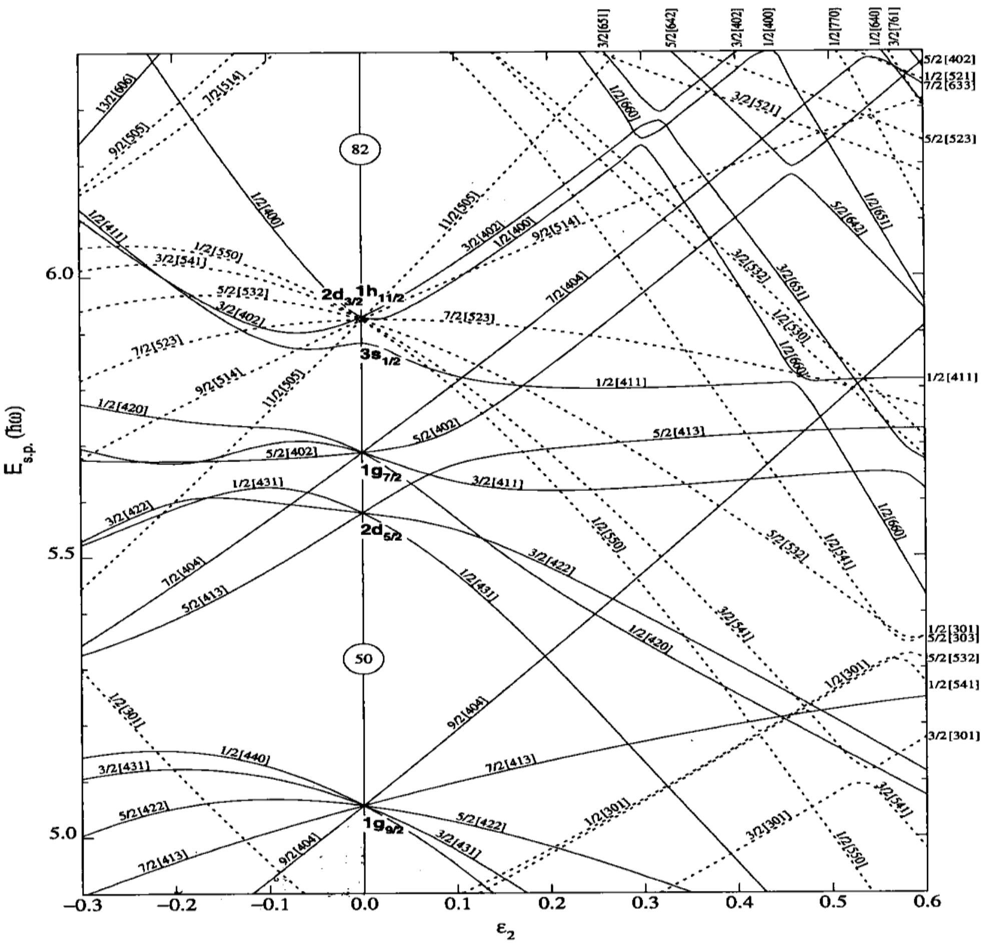


Figure 5. Nilsson diagram for neutrons, $50 \le N \le 82$ ($\varepsilon_4 = \varepsilon_2^2/6$).

14164

Nilsson model

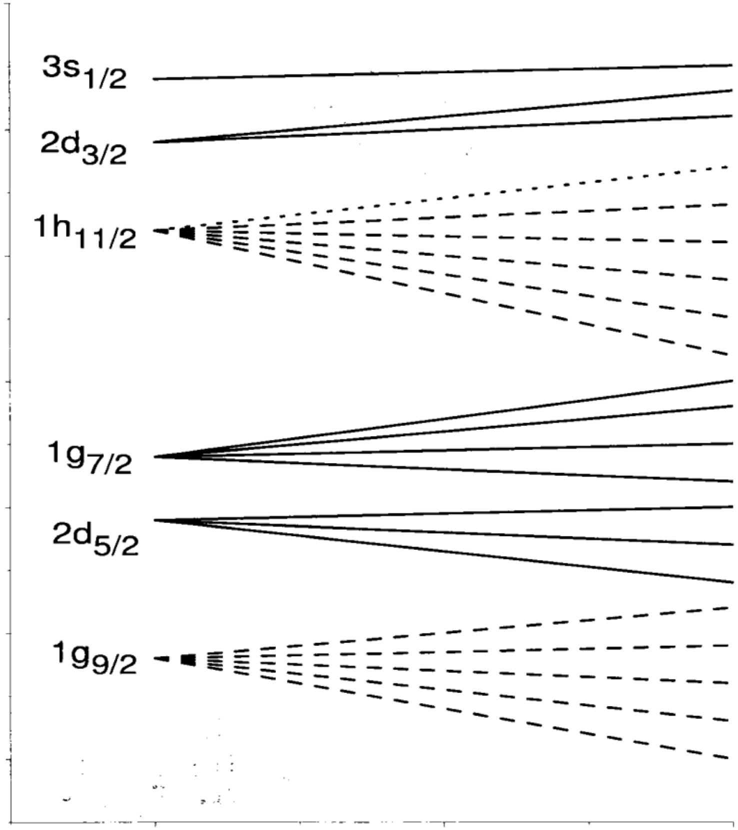
Notation of levels K[N Nz Λ]

K z-projection of total angular momentum
N oscillator quanta
Nz oscillator quanta in z-direction
Λ z-projection of orbital ang. momentum

K=Λ+Σ   Σ=projection of spin

S.G. Nilsson and I. Ragnarsson, Shapes and Shells in Nuclear Structure (Cambridge UP, 1995)

50-82 shell



Pseudo-SU(3)

- Reduces N by one unit
- Changes Λ by one unit
- Inverts spin Σ

Proxy-SU(3)

- Uses 0[110] pairs
- Leaves Λ, K, Σ unchanged
- Reduces N, Nz by one unit

Pseudo-SU(3)

R.D. Ratna Raju, J. P. Draayer, and K. T. Hecht, Nucl. Phys. A 202 (1973) 433

J.P. Draayer, K.J. Weeks, an K.T. Hecht, Nucl. Phys. A 381 (1982) 1

Pseudo-SU(3)

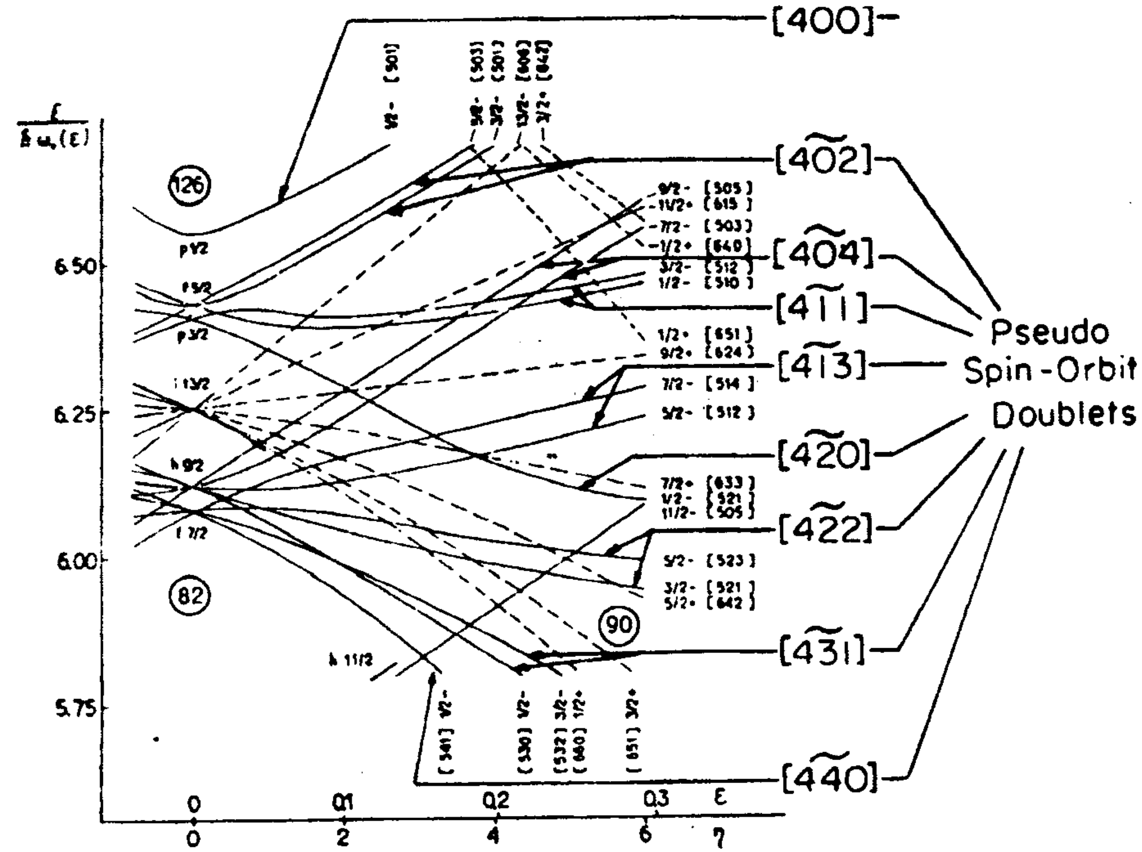


Fig. 2. The Nilsson diagram, 82 ≦ *N* ≦ 126, labelled with pseudo oscillator quantum numbers

Proxy-SU(3)

Uses 0[110] pairs

First used for proton-neutron interaction

R.B. Cakirli, K. Blaum, and R.F. Casten,

Phys. Rev. C 82 (2010) 061304(R)

Same angular momentum content

Large overlaps

D. B., S. Karampagia, R.B. Cakirli, R.F. Casten, K. Blaum, L. Amon Susam, Phys. Rev. C 88 (2013) 054309

proxy-SU(3) vs. pseudo-SU(3)

50-82	present	pseudo
1/2[400]	1/2[400]	**1/2[301]**
1/2[411]	1/2[411]	**1/2[310]**
3/2[402]	3/2[402]	**3/2[301]**
1/2[420]	1/2[420]	**1/2[321]**
3/2[411]	3/2[411]	**3/2[312]**
5/2[402]	5/2[402]	**5/2[303]**
1/2[431]	1/2[431]	**1/2[330]**
3/2[422]	3/2[422]	**3/2[321]**
5/2[413]	5/2[413]	**5/2[312]**
7/2[404]	7/2[404]	**7/2[303]**
1/2[550]	**1/2[440]**	1/2[550]
3/2[541]	**3/2[431]**	3/2[541]
5/2[532]	**5/2[422]**	5/2[532]
7/2[523]	**7/2[413]**	7/2[523]
9/2[514]	**9/2[404]**	9/2[514]
11/2[505]		11/2[505]

proxy-SU(3) vs. pseudo-SU(3)

TABLE VII: Levels appearing in the various major shells in the framework of the Nilsson model [15, 16] (labelled by the nucleon number ranges for each shell), in the present proxy SU(3) scheme (labelled by "present"), and in the pseudo-SU(3) scheme [8, 9] (labelled by "pseudo"). The orbitals that have been replaced by alternate substitutions in the last two cases are indicated by boldface. See subsection IV.C for further discussion.

28-50	present	pseudo	82-126	present	pseudo	126-184	present	pseudo
1/2[301]	1/2[301]	**1/2[220]**	1/2[501]	1/2[501]	**1/2[400]**	1/2[611]	1/2[611]	**1/2[510]**
1/2[321]	1/2[321]	**1/2[220]**	1/2[521]	1/2[521]	**1/2[420]**	1/2[600]	1/2[600]	**1/2[501]**
3/2[312]	3/2[312]	**3/2[211]**	3/2[512]	3/2[512]	**3/2[411]**	3/2[602]	3/2[602]	**3/2[501]**
1/2[310]	1/2[310]	**1/2[211]**	1/2[510]	1/2[510]	**1/2[411]**	1/2[631]	1/2[631]	**1/2[530]**
3/2[301]	3/2[301]	**3/2[202]**	3/2[501]	3/2[501]	**3/2[402]**	3/2[622]	3/2[622]	**3/2[521]**
5/2[303]	5/2[303]	**5/2[202]**	5/2[503]	5/2[503]	**5/2[402]**	5/2[613]	5/2[613]	**5/2[512]**
1/2[440]	**1/2[330]**	1/2[440]	1/2[541]	1/2[541]	**1/2[440]**	1/2[620]	1/2[620]	**1/2[521]**
3/2[431]	**3/2[321]**	3/2[431]	3/2[532]	3/2[532]	**3/2[431]**	3/2[611]	3/2[611]	**3/2[512]**
5/2[422]	**5/2[312]**	5/2[422]	5/2[523]	5/2[523]	**5/2[422]**	5/2[602]	5/2[602]	**5/2[503]**
7/2[413]	**7/2[303]**	7/2[413]	7/2[514]	7/2[514]	**7/2[413]**	7/2[604]	7/2[604]	**7/2[503]**
9/2[404]		9/2[404]	1/2[530]	1/2[530]	**1/2[431]**	1/2[651]	1/2[651]	**1/2[550]**
			3/2[521]	3/2[521]	**3/2[422]**	3/2[642]	3/2[642]	**3/2[541]**
			5/2[512]	5/2[512]	**5/2[413]**	5/2[633]	5/2[633]	**5/2[532]**
50-82	**present**	**pseudo**	7/2[503]	7/2[503]	**7/2[404]**	7/2[624]	7/2[624]	**7/2[523]**
1/2[400]	1/2[400]	**1/2[301]**	9/2[505]	9/2[505]	**9/2[404]**	9/2[615]	9/2[615]	**9/2[514]**
1/2[411]	1/2[411]	**1/2[310]**	1/2[660]	**1/2[550]**	1/2[660]	1/2[640]	1/2[640]	**1/2[541]**
3/2[402]	3/2[402]	**3/2[301]**	3/2[651]	**3/2[541]**	3/2[651]	3/2[631]	3/2[631]	**3/2[532]**
1/2[420]	1/2[420]	**1/2[321]**	5/2[642]	**5/2[532]**	5/2[642]	5/2[622]	5/2[622]	**5/2[523]**
3/2[411]	3/2[411]	**3/2[312]**	7/2[633]	**7/2[523]**	7/2[523]	7/2[613]	7/2[613]	**7/2[514]**
5/2[402]	5/2[402]	**5/2[303]**	9/2[624]	**9/2[514]**	9/2[514]	9/2[604]	9/2[604]	**9/2[505]**
1/2[431]	1/2[431]	**1/2[330]**	11/2[615]	**11/2[505]**	11/2[615]	11/2[606]	11/2[606]	**11/2[505]**
3/2[422]	3/2[422]	**3/2[321]**	13/2[606]		13/2[606]	1/2[770]	**1/2[660]**	1/2[770]
5/2[413]	5/2[413]	**5/2[312]**				3/2[761]	**3/2[651]**	3/2[761]
7/2[404]	7/2[404]	**7/2[303]**				5/2[752]	**5/2[642]**	5/2[752]
1/2[550]	**1/2[440]**	1/2[550]				7/2[743]	**7/2[633]**	7/2[743]
3/2[541]	**3/2[431]**	3/2[541]				9/2[734]	**9/2[624]**	9/2[734]
5/2[532]	**5/2[422]**	5/2[532]				11/2[725]	**11/2[615]**	11/2[725]
7/2[523]	**7/2[413]**	7/2[523]				13/2[716]	**13/2[606]**	13/2[716]
9/2[514]	**9/2[404]**	9/2[514]				15/2[707]		15/2[707]
11/2[505]		11/2[505]						

approximation schemes

Shell model	pseudo-SU(3)		proxy-SU(3)	
28-50	sd	U(6)	pf	U(10)
50-82	pf	U(10)	sdg	U(15)
82-126	sdg	U(15)	pfh	U(21)
126-184	pfh	U(21)	sdgi	U(28)
184-258	sdgi	U(28)	pfhj	U(36)

Example: 168 Er in pseudo-SU(3)

68-50=18 valence protons

From Nilsson diagrams:

10 with normal parity -> [22222] of U(10)

8 with intruder parity 1h11/2

100-82=18 valence protons

From Nilsson diagrams:

10 with normal parity -> [22222] of U(15)

8 with intruder parity 1i13/2

Example: 168Er in pseudo-SU(3)

10 protons in U(10) -> (10,4)

10 neutrons in U(15) -> (20,4)

total (30,8)

SU(3) + 8 protons in 1h11/2

+ 8 neutrons in 1i13/2

Example: 168Er in proxy-SU(3)

18 protons in U(15) -> (18,6)

18 neutrons in U(21) -> (36,6)

total (54,12)

next irrep: (50,14)

(54,12) contains:

gsb, γ1, first K=4, first K=6,…

(50,14) contains:

β1, γ2, 2^{nd} K=4, 2^{nd} K=6, …

Why does proxy-SU(3) work?

Ioannis E. Assimakis

Compare

usual Nilsson calculation

proxy-SU(3) calculation

Few and small extra matrix elements

What does proxy-SU(3) predict for deformations and spectra?

Andriana Martinou

Parameter-free predictions for β and γ

Need 3-body and/or 4-body SO(3) scalars in the integrity basis of SU(3)

Get parameter-free predictions of ratios of energy levels and B(E2)s

What does proxy-SU(3) predict for the prolate-oblate transition ?

Smaragda Sarantopoulou

Location of prolate-oblate shape/phase transition

Particle-hole symmetry breaking

Outlook

- Superheavy elements (SHE) V. Prassa
- Scissors mode Th. Mertzimekis
- p-n pairs Ch. Moustakidis
- Nuclear astrophysics S. Karampagia

  Nuclear level density (NLD)

  γ-ray strength function (γSF)

Outlook

- Two shells

  sd+pf = spdf U(16)

  pf+sdg = spdfg U(25)

  sdg+pfh=spdfgh U(36)

  pfh+sdgi=spdfghi U(49)

- Proxy-O(6)

  sd shell U(6) $\supset$ SU(3) J.P. Elliott

  pf shell U(10) $\supset$ SU(3)

  D. Kusnezov, J. Phys. A 22 (1989) 4271, 5673

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