JUSTIFICATION OF PROXY SU(3) THROUGH A NILSSON CALCULATION

- SU(3) is the symmetry group of the harmonic oscillator –present in the sd shell of a nucleus

- The symmetry of the harmonic oscillator is destroyed at higher major shells, mostly due to spin –orbit interaction

- The proxy SU(3) is a new symmetry scheme valid for deformed nuclei ,based on SU(3), which is established by replacing the “intruder” parity orbitals with their 0[110] partners

- The new shells that will emerge will have SU(3) symmetry, but how will be known that the approximation is valid?

- A mathematical justification through a Nilsson calculation is necessary

The Nilsson Hamiltonian for large deformations

$$H = H_{osc} + u_{ls}\hbar(l \cdot s) + u_{ll}\hbar\omega_0(l^2 - \langle l^2 \rangle_{\mathrm{N}})$$

-The eigenvalues of the terms H_{osc} and $\langle l^2 \rangle_{\mathrm{N}}$ can be calculated directly because they are functions of N and n_z

-The eigenvalues of the terms $l \cdot s$ and l^2 must be calculated in the new basis

-The u_{ls} and u_{ll} are known parameters

region	v_{ls}	v_{ll}	κ	μ
$N, Z < 50$	-0.16	0	0.08	0
$50 < Z < 82$	-0.127	-0.0382	0.0635	0.602
$82 < N < 126$	-0.127	-0.0268	0.0635	0.422
$82 < Z < 126$	-0.115	-0.0375	0.0575	0.652
$126 < N$	-0.127	-0.0206	0.0635	0.324

A. Bohr and B. R. Mottelson, Nuclear Structure, Vol. II: Nuclear Deformations (Benjamin, New York, 1975).

MATRIX ELEMENTS OF THE SPIN ORBIT TERM $l \cdot s$

- Diagonal matrix elements $\langle n_z r s \Sigma | l \cdot s | n_z r s \Sigma \rangle$=(r-s)Σ=ΛΣ

- Non diagonal matrix elements $\langle n_z - 1, r, s, \Sigma - 1 | l \cdot s | n_z r s \Sigma \rangle$=$-\frac{1}{\sqrt{2}}\sqrt{n_z(r+1)}$

$\langle n_z + 1, r, s - 1, \Sigma - 1 | l \cdot s | n_z r s \Sigma \rangle$=$-\frac{1}{\sqrt{2}}\sqrt{(n_z+1)s}$

$\langle n_z r s \Sigma | l \cdot s | n_z r s \Sigma \rangle$=(r-s)Σ=ΛΣ

$\langle n_z r s \Sigma | l \cdot s | n_z r s \Sigma \rangle$=(r-s)Σ=ΛΣ

Shapes and Shells in Nuclear Structure, by Ingemar Ragnarsson , Sven Gvsta Nilsson, Cambridge, UK: Cambridge University Press, 2005

MATRIX ELEMENTS OF THE SQUARE OF THE ANGULAR MOMENTUM l^2

- Diagonal matrix elements

$$\langle n_z r s \Sigma | l^2 | n_z r s \Sigma \rangle = 2n_z + (r+s+1) + (r+s) + (r-s)^2$$

- Non diagonal matrix elements

$$\langle n_z + 2, r-1, s, \Sigma | l^2 | n_z r s \Sigma \rangle = -2\sqrt{(n_z+2)(n_z+1)rs}$$

$$\langle n_z r s \Sigma | l^2 | n_z r s \Sigma \rangle = -2\sqrt{(n_z-1)n_z\,(r+1)(s+1)}$$

Shapes and Shells in Nuclear Structure, by Ingemar Ragnarsson , Sven Gvsta Nilsson, Cambridge, UK: Cambridge University Press, 2005

Matrix of $l \cdot s$ for the 50-82 shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[550]	$\frac{3}{2}$[541]	$\frac{5}{2}$[532]	$\frac{7}{2}$[523]	$\frac{9}{2}$[514]	$\frac{11}{2}$[505]
1/2[400]	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		−0.5	0	−1.414	0	0	0	0	0	0	0	0	0	0	0	0
3/2[402]			−1	0	−1.225	0	0	0	0	0	0	0	0	0	0	0
1/2[420]				0	0	0	1.225	0	0	0	0	0	0	0	0	0
3/2[411]					0.5	0	0	1	0	0	0	0	0	0	0	0
5/2[402]						1	0	0	0.707	0	0	0	0	0	0	0
1/2[431]							−0.5	0	0	0	0	0	0	0	0	0
3/2[422]								−1	0	0	0	0	0	0	0	0
5/2[413]									−1.5	0	0	0	0	0	0	0
7/2[404]										−2	0	0	0	0	0	0
1/2[550]											0	0	0	0	0	0
3/2[541]												0.5	0	0	0	0
5/2[532]													1	0	0	0
7/2[523]														1.5	0	0
9/2[514]															2	0
11/2[505]																2.5

Matrix of $l \cdot s$ for the sdg shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[440]	$\frac{3}{2}$[431]	$\frac{5}{2}$[422]	$\frac{7}{2}$[413]	$\frac{9}{2}$[404]
1/2[400]	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		−0.5	0	−1.414	0	0	0	0	0	0	0	0	0	0	0
3/2[402]			−1	0	−1.225	0	0	0	0	0	0	0	0	0	0
1/2[420]				0	0	0	1.225	0	0	0	0	0	0	0	0
3/2[411]					0.5	0	0	1	0	0	0	0	0	0	0
5/2[402]						1	0	0	0.707	0	0	0	0	0	0
1/2[431]							−0.5	0	0	0	**−1.414**	0	0	0	0
3/2[422]								−1	0	0	0	**−1.732**	0	0	0
5/2[413]									−1.5	0	0	0	**−1.732**	0	0
7/2[404]										−2	0	0	0	**−1.414**	0
1/2[440]											0	0	0	0	0
3/2[431]												0.5	0	0	0
5/2[422]													1	0	0
7/2[413]														1.5	0
9/2[404]															2

Matrix of l^2 for the 50-82 shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[550]	$\frac{3}{2}$[541]	$\frac{5}{2}$[532]	$\frac{7}{2}$[523]	$\frac{9}{2}$[514]	$\frac{11}{2}$[505]
1/2[400]	4	0	0	−5.657	0	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		12	0	0	0	0	−6.928	0	0	0	0	0	0	0	0	0
3/2[402]			8	0	0	0	0	−4.899	0	0	0	0	0	0	0	0
1/2[420]				14	0	0	0	0	0	0	0	0	0	0	0	0
3/2[411]					12	0	0	0	0	0	0	0	0	0	0	0
5/2[402]						8	0	0	0	0	0	0	0	0	0	0
1/2[431]							14	0	0	0	0	0	0	0	0	0
3/2[422]								18	0	0	0	0	0	0	0	0
5/2[413]									20	0	0	0	0	0	0	0
7/2[404]										20	0	0	0	0	0	0
1/2[550]											10	0	0	0	0	0
3/2[541]												18	0	0	0	0
5/2[532]													24	0	0	0
7/2[523]														28	0	0
9/2[514]															30	0
11/2[505]																30

Matrix of l^2 of sdg shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[440]	$\frac{3}{2}$[431]	$\frac{5}{2}$[422]	$\frac{7}{2}$[413]	$\frac{9}{2}$[404]
1/2[400]	4	0	0	−5.657	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		12	0	0	0	0	−6.928	0	0	0	0	0	0	0	0
3/2[402]			8	0	0	0	0	−4.899	0	0	0	0	0	0	0
1/2[420]				14	0	0	0	0	0	0	**−6.928**	0	0	0	0
3/2[411]					12	0	0	0	0	0	0	**−6.928**	0	0	0
5/2[402]						8	0	0	0	0	0	0	**−4.899**	0	0
1/2[431]							14	0	0	0	0	0	0	0	0
3/2[422]								18	0	0	0	0	0	0	0
5/2[413]									20	0	0	0	0	0	0
7/2[404]										20	0	0	0	0	0
1/2[440]											**8**	0	0	0	0
3/2[431]												**14**	0	0	0
5/2[422]													**18**	0	0
7/2[413]														**20**	0
9/2[404]															**20**

Matrix of the Hamiltonian for the 50-82 proton shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[550]	$\frac{3}{2}$[541]	$\frac{5}{2}$[532]	$\frac{7}{2}$[523]	$\frac{9}{2}$[514]	$\frac{11}{2}$[505]
1/2[400]	6.28	−0.13	0	0.22	0	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		5.74	0	0.18	0	0	0.27	0	0	0	0	0	0	0	0	0
3/2[402]			6.26	0	0.16	0	0	0.19	0	0	0	0	0	0	0	0
1/2[420]				5.30	0	0	−0.16	0	0	0	0	0	0	0	0	0
3/2[411]					5.61	0	0	−0.13	0	0	0	0	0	0	0	0
5/2[402]						6.00	0	0	−0.09	0	0	0	0	0	0	0
1/2[431]							5.06	0	0	0	0	0	0	0	0	0
3/2[422]								5.27	0	0	0	0	0	0	0	0
5/2[413]									5.56	0	0	0	0	0	0	0
7/2[404]										5.93	0	0	0	0	0	0
1/2[550]											5.88	0	0	0	0	0
3/2[541]												5.81	0	0	0	0
5/2[532]													5.82	0	0	0
7/2[523]														5.90	0	0
9/2[514]															6.06	0
11/2[505]																6.30

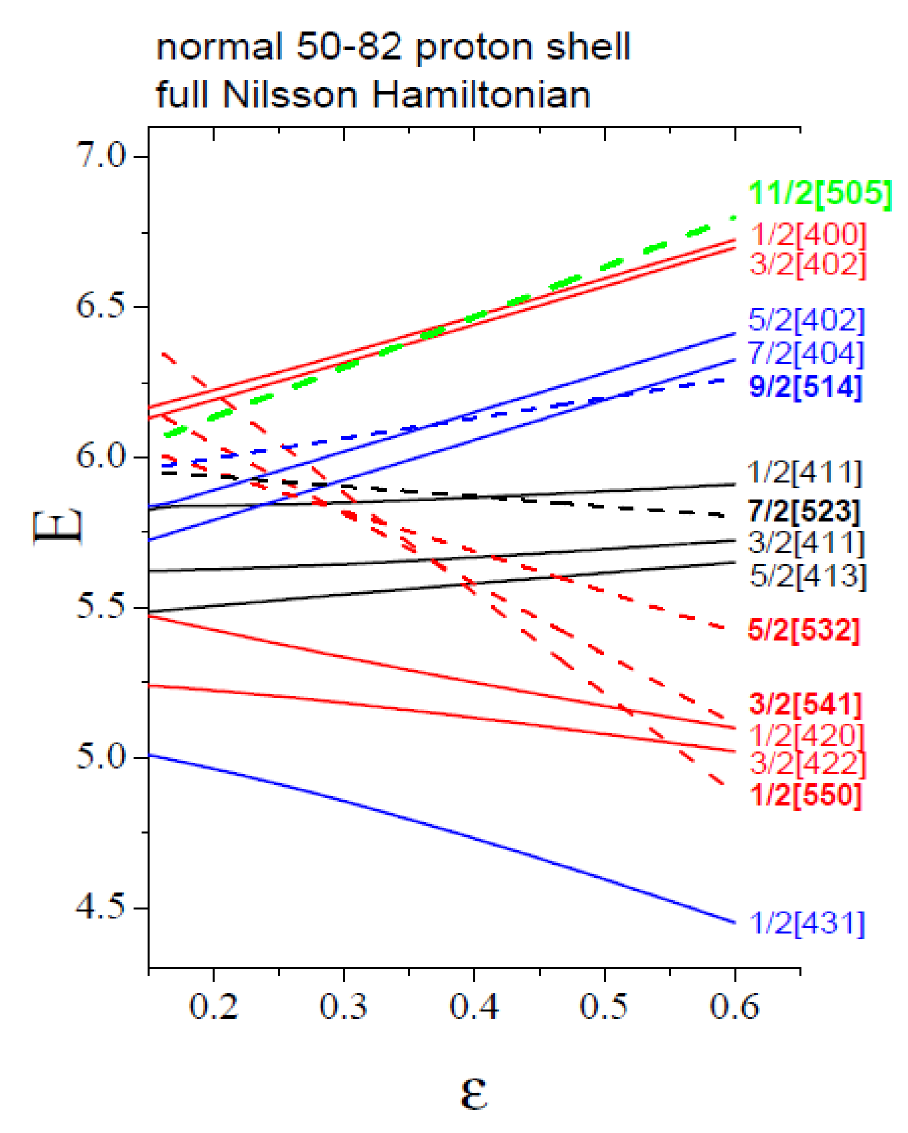
Matrix of the Hamiltonian for the sdg proton shell

	$\frac{1}{2}$[400]	$\frac{1}{2}$[411]	$\frac{3}{2}$[402]	$\frac{1}{2}$[420]	$\frac{3}{2}$[411]	$\frac{5}{2}$[402]	$\frac{1}{2}$[431]	$\frac{3}{2}$[422]	$\frac{5}{2}$[413]	$\frac{7}{2}$[404]	$\frac{1}{2}$[440]	$\frac{3}{2}$[431]	$\frac{5}{2}$[422]	$\frac{7}{2}$[413]	$\frac{9}{2}$[404]
1/2[400]	6.28	−0.13	0	0.22	0	0	0	0	0	0	0	0	0	0	0
1/2[411]		5.74	0	0.18	0	0	0.27	0	0	0	0	0	0	0	0
3/2[402]			6.26	0	0.16	0	0	0.19	0	0	0	0	0	0	0
1/2[420]				5.30	0	0	−0.16	0	0	0	**0.27**	0	0	0	0
3/2[411]					5.61	0	0	−0.13	0	0	0	**0.27**	0	0	0
5/2[402]						6.00	0	0	−0.09	0	0	0	**0.19**	0	0
1/2[431]							5.06	0	0	0	**0.18**	0	0	0	0
3/2[422]								5.27	0	0	0	**0.22**	0	0	0
5/2[413]									5.56	0	0	0	**0.22**	0	0
7/2[404]										5.93	0	0	0	**0.18**	0
1/2[440]											**5.73**	0	0	0	0
3/2[431]												**5.74**	0	0	0
5/2[422]													**5.82**	0	0
7/2[413]														**5.98**	0
9/2[404]															**6.22**

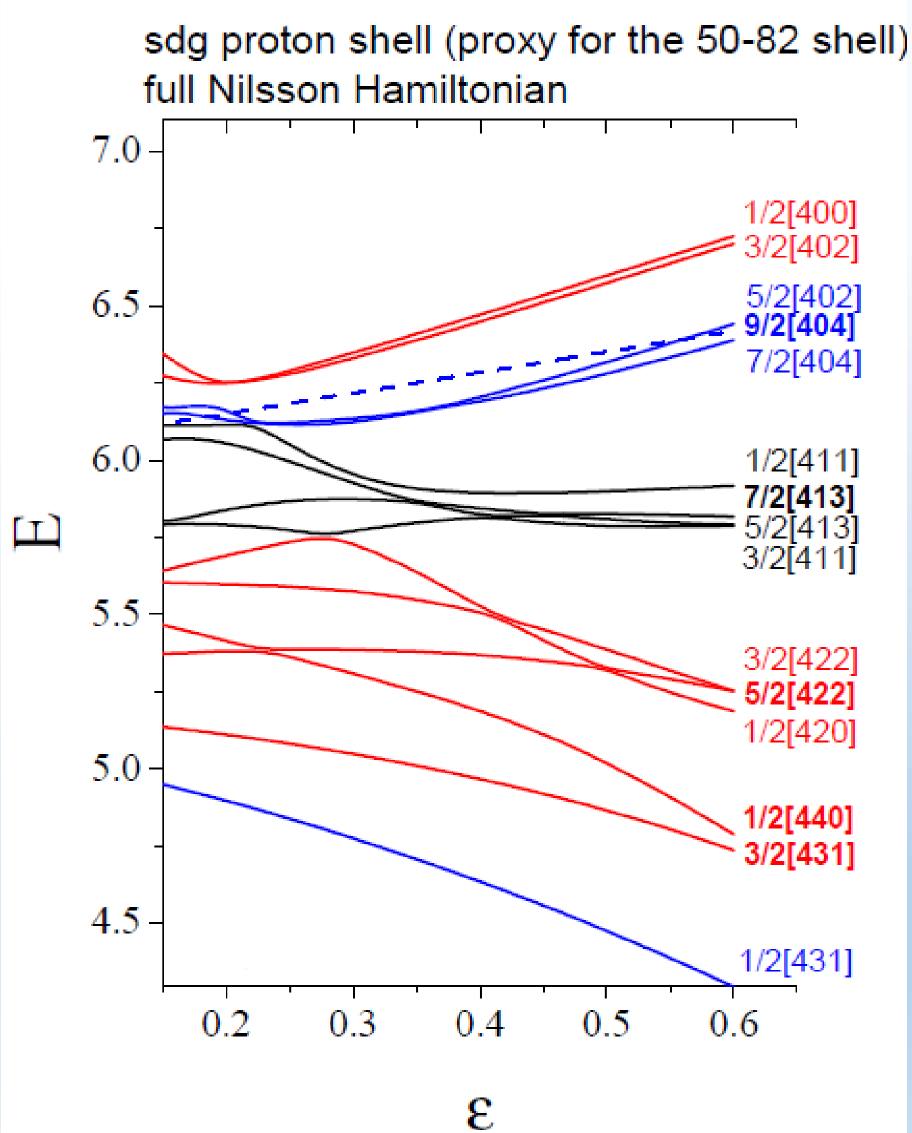
- The matrices are symmetric so only the diagonal and the upper part is shown.

- A very small percentage is affected by the approximations in the proxy SU(3) scheme!!

- The SU(3) symmetry is conserved!!



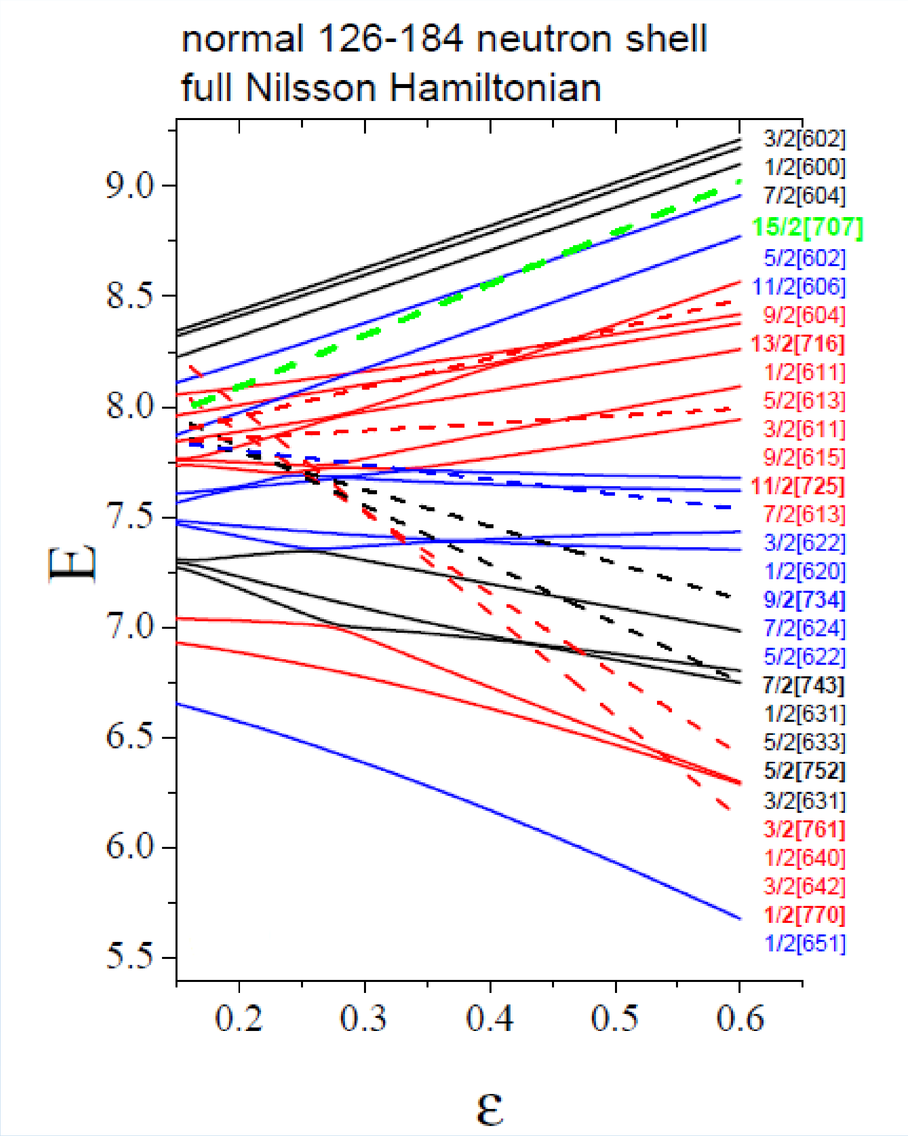
normal 50-82 proton shell
full Nilsson Hamiltonian
7.0
6.5
6.0
5.5
5.0
4.5
E
0.2
0.3
0.4
0.5
0.6
ε
11/2[505]
1/2[400]
3/2[402]
5/2[402]
7/2[404]
9/2[514]
1/2[411]
7/2[523]
3/2[411]
5/2[413]
5/2[532]
3/2[541]
1/2[420]
3/2[422]
1/2[550]
1/2[431]

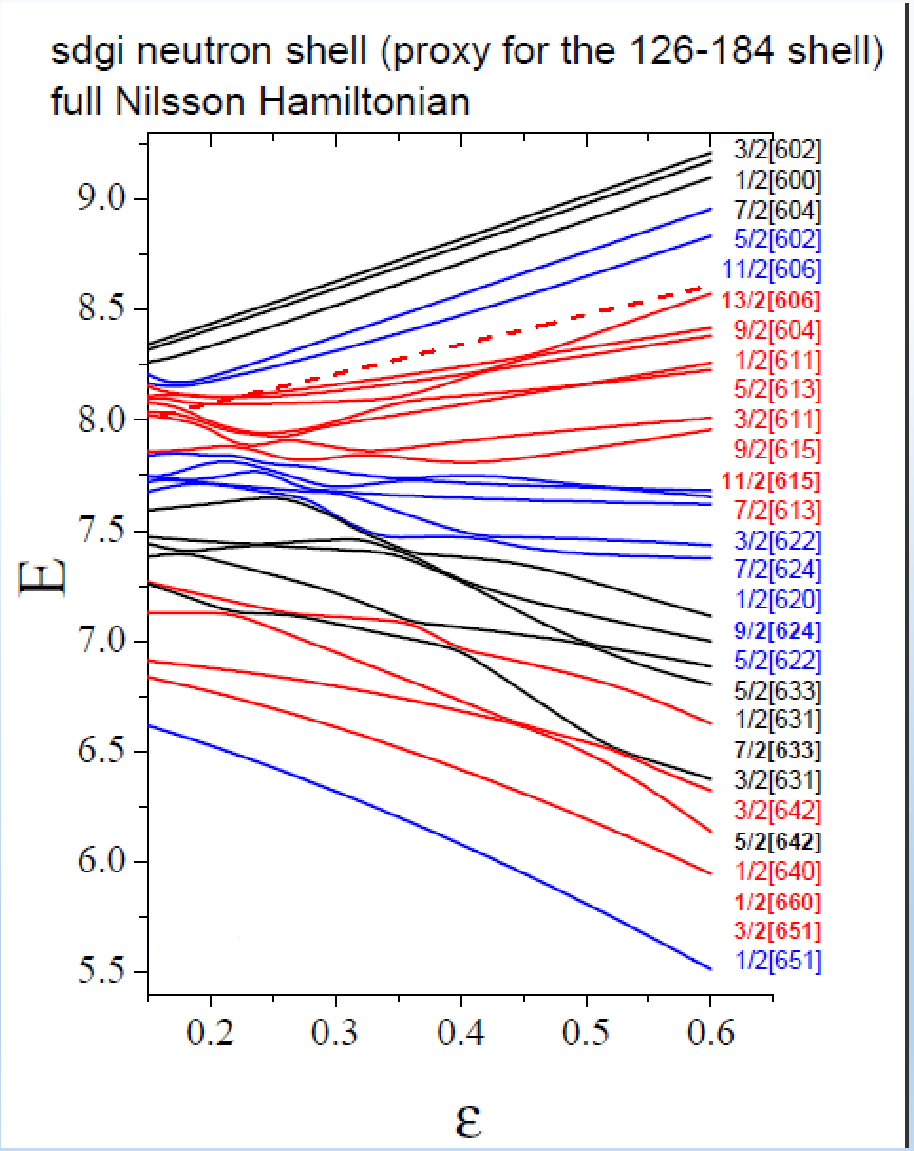


sdg proton shell (proxy for the 50-82 shell)
full Nilsson Hamiltonian
7.0
6.5
6.0
5.5
5.0
4.5
E
0.2
0.3
0.4
0.5
0.6
ε
1/2[400]
3/2[402]
5/2[402]
9/2[404]
7/2[404]
1/2[411]
7/2[413]
5/2[413]
3/2[411]
3/2[422]
5/2[422]
1/2[420]
1/2[440]
3/2[431]
1/2[431]

- As we move to heavier shells the approximation gets better!
- In the sdg shell 8.44% of the matrix elements are affected by the approximations, in the pfh shell the number drops to 5.44% and in the sdgi shell at 3.70%

shell	$\mathbf{l} \cdot \mathbf{s}$	$\mathbf{l}^2$	H	total
pf	6	-	6	100
sdg	8	11	19	225
pfh	10	14	24	441
sdgi	12	17	29	784


normal 126-184 neutron shell
full Nilsson Hamiltonian
E
ε
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
0.2
0.3
0.4
0.5
0.6
3/2[602]
1/2[600]
7/2[604]
15/2[707]
5/2[602]
11/2[606]
9/2[604]
13/2[716]
1/2[611]
5/2[613]
3/2[611]
9/2[615]
11/2[725]
7/2[613]
3/2[622]
1/2[620]
9/2[734]
7/2[624]
5/2[622]
7/2[743]
1/2[631]
5/2[633]
5/2[752]
3/2[631]
3/2[761]
1/2[640]
3/2[642]
1/2[770]
1/2[651]


sdgi neutron shell (proxy for the 126-184 shell)
full Nilsson Hamiltonian
E
ε
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
0.2
0.3
0.4
0.5
0.6
3/2[602]
1/2[600]
7/2[604]
5/2[602]
11/2[606]
13/2[606]
9/2[604]
1/2[611]
5/2[613]
3/2[611]
9/2[615]
11/2[615]
7/2[613]
3/2[622]
7/2[624]
1/2[620]
9/2[624]
5/2[622]
5/2[633]
1/2[631]
7/2[633]
3/2[631]
3/2[642]
5/2[642]
1/2[640]
1/2[660]
3/2[651]
1/2[651]

Matrix of the Hamiltonian for the 126-184 shell

	$\frac{1}{2}$[611]	$\frac{1}{2}$[600]	$\frac{3}{2}$[602]	$\frac{1}{2}$[631]	$\frac{3}{2}$[622]	$\frac{5}{2}$[613]	$\frac{1}{2}$[620]	$\frac{3}{2}$[611]	$\frac{5}{2}$[602]	$\frac{7}{2}$[604]	$\frac{1}{2}$[651]	$\frac{3}{2}$[642]	$\frac{5}{2}$[633]	$\frac{7}{2}$[624]	$\frac{9}{2}$[615]	$\frac{1}{2}$[640]	$\frac{3}{2}$[631]	$\frac{5}{2}$[622]	$\frac{7}{2}$[613]	$\frac{9}{2}$[604]	$\frac{11}{2}$[606]	$\frac{1}{2}$[770]	$\frac{3}{2}$[761]	$\frac{5}{2}$[752]	$\frac{7}{2}$[743]	$\frac{9}{2}$[734]	$\frac{11}{2}$[725]	$\frac{13}{2}$[716]	$\frac{15}{2}$[707]
1/2[611]	8.05	−0.16	0	0.25	0	0	0.22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/2[600]		8.53	0	0	0	0	0.17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3/2[602]			8.58	0	0	0	0	0.18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/2[631]				7.24	0	0	−0.22	0	0	0	0.26	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0	0	0
3/2[622]					7.61	0	0	−0.18	0	0	0	0	0	0	0	0	0.27	0	0	0	0	0	0	0	0	0	0	0	0
5/2[613]						8.01	0	0	−0.13	0	0	0	0.20	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0
1/2[620]							7.56	0	0	0	0	0	0	0	0	0.29	0	0	0	0	0	0	0	0	0	0	0	0	0
3/2[611]								7.92	0	0	0	0	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0	0
5/2[602]									8.32	0	0	0	0	0	0	0	0	0.16	0	0	0	0	0	0	0	0	0	0	0
7/2[604]										8.46	0	0	0	0.13	0	0	0	0	0.20	0	0	0	0	0	0	0	0	0	0
1/2[651]											6.77	0	0	0	0	−0.20	0	0	0	0	0	0	0	0	0	0	0	0	0
3/2[642]												6.97	0	0	0	0	−0.18	0	0	0	0	0	0	0	0	0	0	0	0
5/2[633]													7.21	0	0	0	0	−0.16	0	0	0	0	0	0	0	0	0	0	0
7/2[624]														7.49	0	0	0	0	−0.13	0	0	0	0	0	0	0	0	0	0
9/2[615]															7.81	0	0	0	0	−0.09	0	0	0	0	0	0	0	0	0
1/2[640]																6.92	0	0	0	0	0	0	0	0	0	0	0	0	0
3/2[631]																	7.12	0	0	0	0	0	0	0	0	0	0	0	0
5/2[622]																		7.35	0	0	0	0	0	0	0	0	0	0	0
7/2[613]																			7.63	0	0	0	0	0	0	0	0	0	0
9/2[604]																				7.95	0	0	0	0	0	0	0	0	0
11/2[606]																					8.17	0	0	0	0	0	0	0	0
1/2[770]																						7.53	0	0	0	0	0	0	0
3/2[761]																							7.52	0	0	0	0	0	0
5/2[752]																								7.55	0	0	0	0	0
7/2[743]																									7.62	0	0	0	0
9/2[734]																										7.74	0	0	0
11/2[725]																											7.89	0	0
13/2[716]																												8.089	0
15/2[707]																													8.32

Matrix of the Hamiltonian for the sdgi shell

	1/2[611]	1/2[600]	3/2[602]	1/2[631]	3/2[622]	5/2[613]	1/2[620]	3/2[611]	5/2[602]	7/2[604]	1/2[651]	3/2[642]	5/2[633]	7/2[624]	9/2[615]	1/2[640]	3/2[631]	5/2[622]	7/2[613]	9/2[604]	11/2[606]	1/2[660]	3/2[651]	5/2[642]	7/2[633]	9/2[624]	11/2[615]	13/2[606]
1/2[611]	8.05	−0.16	0	0.25	0	0	0.22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/2[600]		8.53	0	0	0	0	0.17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3/2[602]			8.58	0	0	0	0	0.18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/2[631]				7.24	0	0	−0.22	0	0	0	0.26	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0	0
3/2[622]					7.61	0	0	−0.18	0	0	0	0	0	0	0	0	0.27	0	0	0	0	0	0	0	0	0	0	0
5/2[613]						8.01	0	0	−0.13	0	0	0	0.20	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0
1/2[620]							7.56	0	0	0	0	0	0	0	0	0.29	0	0	0	0	0	0	0	0	0	0	0	0
3/2[611]								7.92	0	0	0	0	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0
5/2[602]									8.32	0	0	0	0	0	0	0	0	0.16	0	0	0	0	0	0	0	0	0	0
7/2[604]										8.46	0	0	0	0.13	0	0	0	0	0.20	0	0	0	0	0	0	0	0	0
1/2[651]											6.77	0	0	0	0	−0.20	0	0	0	0	0	**0.22**	0	0	0	0	0	0
3/2[642]												6.97	0	0	0	0	−0.18	0	0	0	0	0	**0.28**	0	0	0	0	0
5/2[633]													7.21	0	0	0	0	−0.16	0	0	0	0	0	**0.31**	0	0	0	0
7/2[624]														7.49	0	0	0	0	−0.13	0	0	0	0	0	**0.31**	0	0	0
9/2[615]															7.81	0	0	0	0	−0.09	0	0	0	0	0	**0.28**	0	0
1/2[640]																6.92	0	0	0	0	0	**0.23**	0	0	0	0	0	0
3/2[631]																	7.12	0	0	0	0	0	**0.26**	0	0	0	0	0
5/2[622]																		7.35	0	0	0	0	0	**0.25**	0	0	0	0
7/2[613]																			7.63	0	0	0	0	0	**0.20**	0	0	0
9/2[604]																				7.95	0	0	0	0	0	**0.13**	0	0
11/2[606]																					8.17	0	0	0	0	0	**0.22**	0
1/2[660]																						**7.40**	0	0	0	0	0	0
3/2[651]																							**7.44**	0	0	0	0	0
5/2[642]																								**7.51**	0	0	0	0
7/2[633]																									**7.62**	0	0	0
9/2[624]																										**7.78**	0	0
11/2[615]																											**7.97**	0
13/2[606]																												**8.21**

- The changes inflicted on the Nilsson diagrams by the replacement of the parity orbitals with their 0[110] counterparts do not change the main features of the diagrams

- This is because the "intruder" parity orbitals have the same orbital angular momentum , spin and total angular momentum

- It is expected that several physical properties of the relevant heavy deformed nuclei can be correctly determined by the new proxy SU(3) scheme

THANK YOU FOR YOUR TIME!!!