

Momentum dependent mean-field dynamics for in-medium Υ -interactions

Th. Gaitanos, A. Violaris



ΤΜΗΜΑ ΦΥΣΙΚΗΣ

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ΘΕΣΣΑΛΟΝΙΚΗΣ

- Introduction
- Non-Linear Derivative (NLD) model of relativistic hadrodynamics
- RMF approach to infinite nuclear matter & in-medium hyperons
- Saturation properties, EoS, symmetry energy
- In-medium proton & antiproton optical potentials
- In-medium hyperon optical potentials
- Final remarks & outlook

Gaitanos & Kaskulov, Nuclear Physics A 899 (2013) 133

Gaitanos & Kaskulov, Nuclear Physics A 940 (2015) 181

Gaitanos & Violaris, in preparation

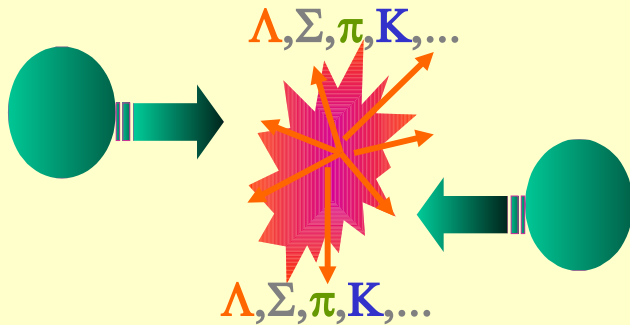
Introduction...

Important for astrophysics

explore EoS beyond saturation, e.g., at high densities

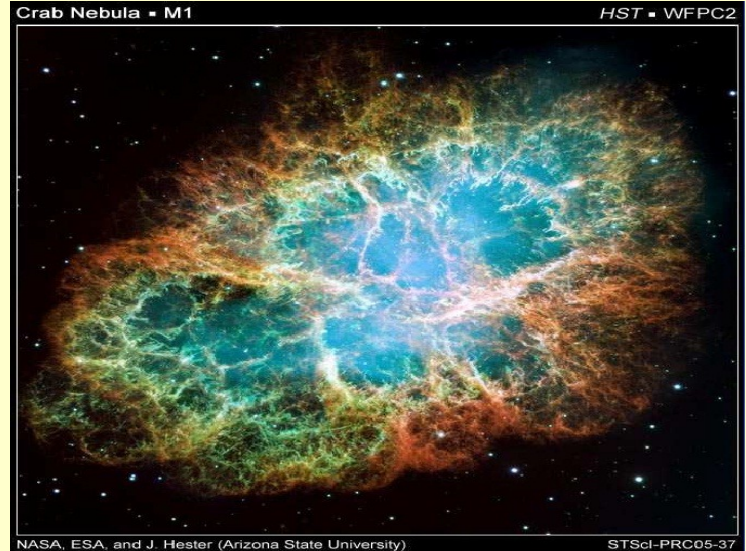
Heavy-ion collisions

(collective flow, meson production)



Densities of fireball for HIC@SIS:
 $\rho \sim (2-3)\rho_0$

Neutron stars (mass & radius)



Densities in static NS: $\rho \sim (8-10)\rho_0$

- ↳ In high-density matter (+kinematics) → particles with high-momenta p
- ↳ Not only density dependence, but also momentum dependence (MD) essential

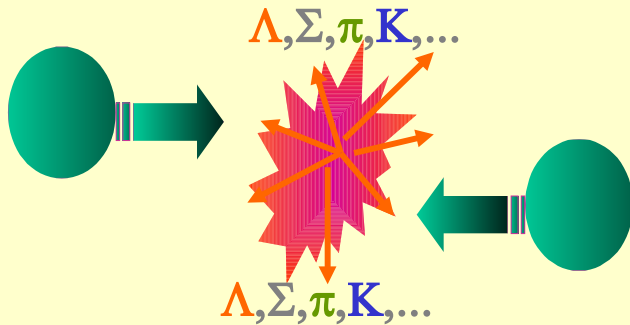
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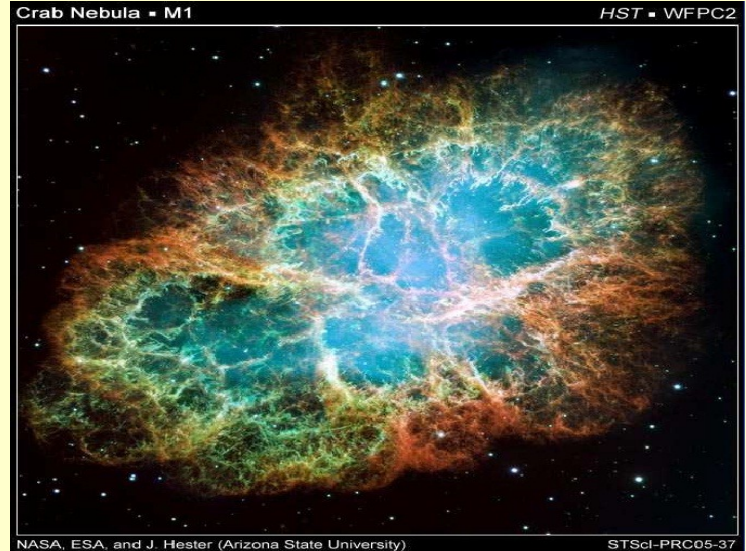
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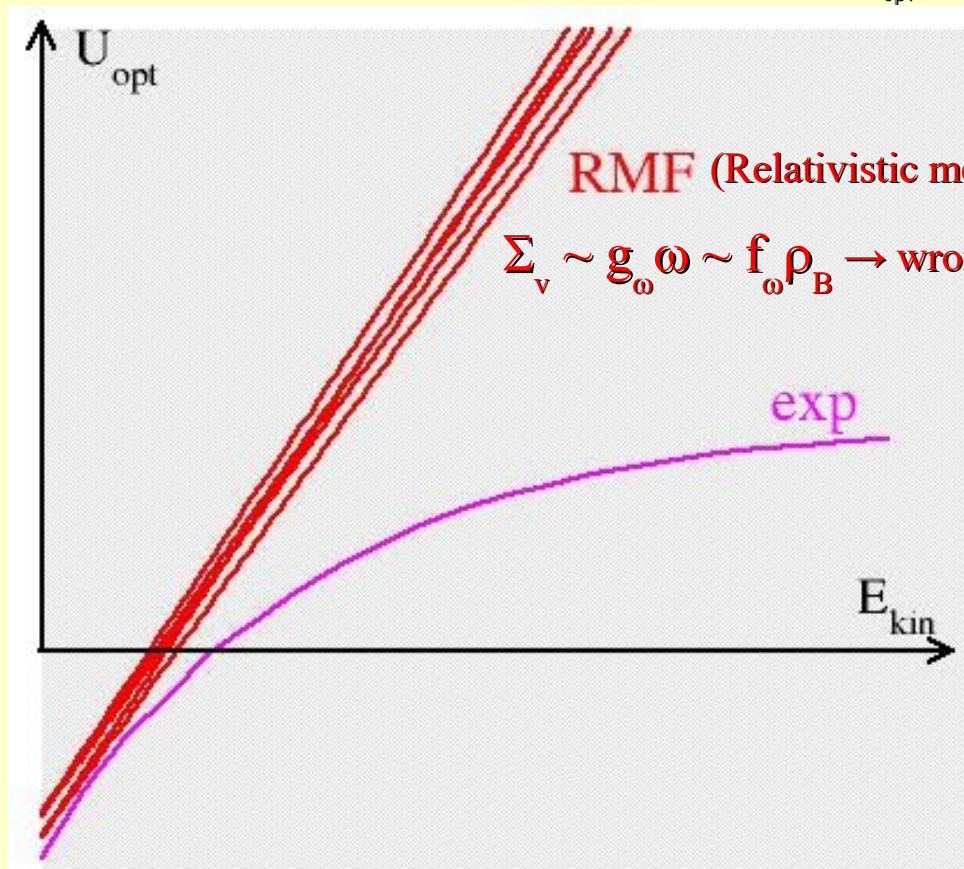
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Introduction...

In-medium proton Schrödinger-equivalent $Re(U_{opt})$

$$U_{opt} = \frac{E}{m} \Sigma_v - \Sigma_s + \frac{1}{2m} (\Sigma_s^2 - \Sigma_v^2)$$



DBHF & Dirac-phenomenology:
saturating fields (particular vector) with rising p

Solutions so far:

- non-local (Hartree-Fock) contributions to RMF (Hartree) mean-field
Weber, Blättel, Cassing et al., Nucl. Phys. A539 (1992) 713
- first-order derivative coupling terms into the interaction Lagrangian
S. Typel, Phys. Rev. C71, 064301 (2005)

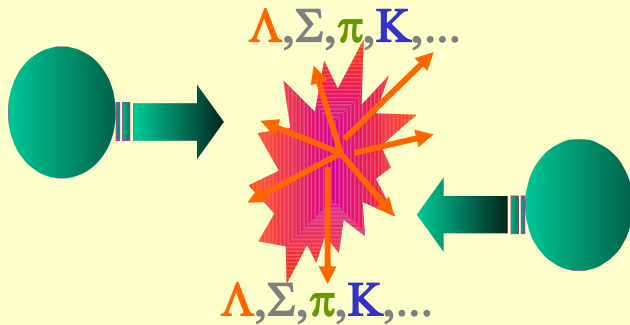
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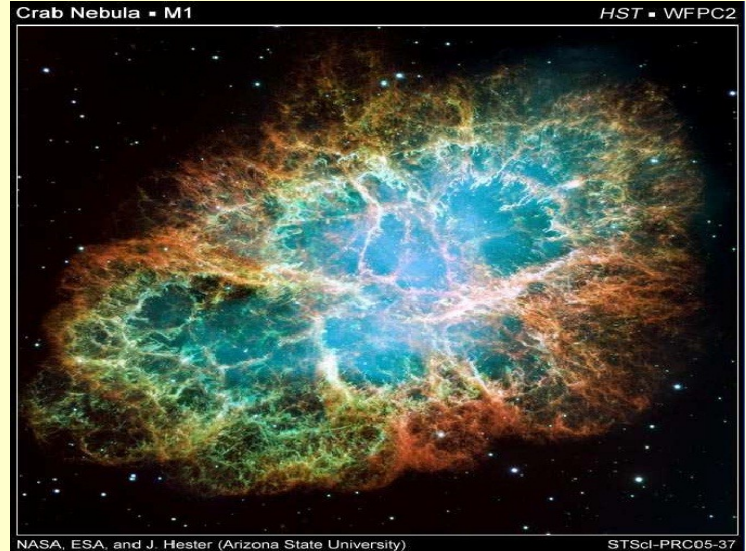
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▫ Not only density dependence, but also momentum dependence (MD) essential

▫ **Non-Linear Derivative (NLD) model:**

RMF-based model with regulators for the high-momentum tails of the interaction

The Non-Linear Derivative (NLD) model...

NLD Lagrangian : as in conventional QHD

$$\begin{aligned}\mathcal{L} = & \frac{1}{2} \left[\bar{\Psi} \gamma_{\mu} i \overrightarrow{\partial}^{\mu} \Psi - \bar{\Psi} i \overleftarrow{\partial}^{\mu} \gamma_{\mu} \Psi \right] - m \bar{\Psi} \Psi - \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma) \\ & + \frac{1}{2} m_{\omega}^2 \omega_{\mu} \omega^{\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \vec{\rho}_{\mu} \vec{\rho}^{\mu} - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu} \\ & + \mathcal{L}_{int}\end{aligned}$$

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Interaction Lagrangian : as in conventional QHD + non-linear derivative operators

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Non-linear derivative operators : Taylor expansion of partial derivatives ξ

$$\overrightarrow{\mathcal{D}} := \mathcal{D} \left(\overrightarrow{\xi} \right) = \sum_{j=0}^{n \rightarrow \infty} \frac{\partial^j}{\partial \overrightarrow{\xi}^j} \mathcal{D} \Big|_{\overrightarrow{\xi} \rightarrow 0} \frac{\overrightarrow{\xi}^j}{j!} \quad \overrightarrow{\xi} = -\frac{v^{\alpha} i \overrightarrow{\partial}_{\alpha}}{\Lambda}$$

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cut-off, will regulate the high-momentum tail of RMF fields

NLD: The formalism...

NLD Lagrangian: contains higher field derivatives: $\mathcal{L}(\varphi_r, \partial_{\alpha_1} \varphi_r, \partial_{\alpha_1 \alpha_2} \varphi_r, \dots, \partial_{\alpha_1 \dots \alpha_n} \varphi_r)$

→ Generalized Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \varphi_r} + \sum_{i=1}^n (-)^i \partial_{\alpha_1 \dots \alpha_i} \frac{\partial \mathcal{L}}{\partial (\partial_{\alpha_1 \dots \alpha_i} \varphi_r)} = 0$$

→ Generalized Noether-Theorem: conserved current

$$J^\mu = -i \left[\mathcal{K}_r^\mu \varphi_r + \mathcal{K}_r^{\mu \sigma_1} \partial_{\sigma_1} \varphi_r + \mathcal{K}_r^{\mu \sigma_1 \sigma_2} \partial_{\sigma_1 \sigma_2} \varphi_r + \dots + \mathcal{K}_r^{\mu \sigma_1 \dots \sigma_n} \partial_{\sigma_1 \dots \sigma_n} \varphi_r \right]$$

with the following tensors

$$\mathcal{K}_r^{\mu \sigma_1 \dots \sigma_m} = \sum_{i=1}^n (-)^{i+1} \prod_{j=1}^{i-1} \partial_{\alpha_j} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu \alpha_j \sigma_1 \dots \sigma_m} \varphi_r)}.$$

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infinite series rsp. to higher-order field derivatives, but...

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All infinite series can be resummed to compact expressions !

NLD field equations...

→ Dirac equation for nucleons $[\gamma_\mu(i\partial^\mu - \Sigma^\mu) - (m - \Sigma_s)] \Psi = 0$ with selfenergies

$$\Sigma^\mu = g_\omega \omega^\mu \overrightarrow{\mathcal{D}} + g_\rho \vec{\tau} \cdot \vec{\rho}^\mu \overrightarrow{\mathcal{D}} + \dots$$

$$\Sigma_s = g_\sigma \sigma \overrightarrow{\mathcal{D}} + \dots \quad (\text{up to terms containing derivatives of the meson fields})$$

→ Meson field equations:

$$\partial_\alpha \partial^\alpha \sigma + m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} = \frac{1}{2} g_\sigma [\bar{\Psi} \overleftarrow{\mathcal{D}} \Psi + \bar{\Psi} \overrightarrow{\mathcal{D}} \Psi],$$

$$\partial_\mu F^{\mu\nu} + m_\omega^2 \omega^\nu = \frac{1}{2} g_\omega [\bar{\Psi} \overleftarrow{\mathcal{D}} \gamma^\nu \Psi + \bar{\Psi} \gamma^\nu \overrightarrow{\mathcal{D}} \Psi],$$

$$\partial_\mu \vec{G}^{\mu\nu} + m_\rho^2 \vec{\rho}^\nu = \frac{1}{2} g_\rho [\bar{\Psi} \overleftarrow{\mathcal{D}} \gamma^\nu \vec{\tau} \Psi + \bar{\Psi} \vec{\tau} \gamma^\nu \overrightarrow{\mathcal{D}} \Psi]$$

→ Noether current (energy-momentum tensor):

$$J^\mu = \bar{\Psi} \gamma^\mu \Psi - \frac{1}{2} g_\sigma [\bar{\Psi} \overleftarrow{\mathcal{D}}^\mu \Psi - \bar{\Psi} \overrightarrow{\mathcal{D}}^\mu \Psi] \sigma + \frac{1}{2} g_\omega [\bar{\Psi} \overleftarrow{\mathcal{D}}^\mu \gamma^\alpha \Psi - \bar{\Psi} \gamma^\alpha \overrightarrow{\mathcal{D}}^\mu \Psi] \omega_\alpha \\ + \frac{1}{2} g_\rho [\bar{\Psi} \overleftarrow{\mathcal{D}}^\mu \gamma^\alpha \vec{\tau} \Psi - \bar{\Psi} \gamma^\alpha \overrightarrow{\mathcal{D}}^\mu \vec{\tau} \Psi] \vec{\rho}_\alpha + \dots$$

RMF approach to infinite asymmetric nuclear matter...

→ Plane wave Ansatz for Ψ and $\bar{\Psi}$ and $\mathcal{D} = \mathcal{D}(\xi)$ with $\xi = -\frac{v_\alpha p^\alpha}{\Lambda}$

$$\Sigma_{vi}^\mu = g_\omega \omega^\mu \mathcal{D} + g_\rho \tau_i \rho^\mu \mathcal{D}, \quad \Sigma_{si} = g_\sigma \sigma \mathcal{D}$$

$$m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} = g_\sigma \sum_{i=p,n} \langle \bar{\Psi}_i \mathcal{D} \Psi_i \rangle = g_\sigma \rho_s$$

$$m_\omega^2 \omega = g_\omega \sum_{i=p,n} \langle \bar{\Psi}_i \gamma^0 \mathcal{D} \Psi_i \rangle = g_\omega \rho_0$$

$$m_\rho^2 \rho = g_\rho \sum_{i=p,n} \tau_i \langle \bar{\Psi}_i \gamma^0 \mathcal{D} \Psi_i \rangle = g_\rho \rho_I .$$

$$T^{\mu\nu} = \sum_{i=p,n} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \leq p_{F_i}} d^3 p \frac{\Pi_i^\mu p^\nu}{\Pi_i^0} - g^{\mu\nu} \langle \mathcal{L} \rangle$$

$$\Pi_i^\mu = p_i^{*\mu} + m_i^* \left(\partial_p^\mu \Sigma_{si} \right) - \left(\partial_p^\mu \Sigma_{vi}^\beta \right) p_{i\beta}^*$$

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meson-field equations

$$m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} = g_\sigma \sum_{i=p,n} \langle \bar{\Psi}_i \mathcal{D} \Psi_i \rangle = g_\sigma \rho_s$$
$$m_\omega^2 \omega = g_\omega \sum_{i=p,n} \langle \bar{\Psi}_i \gamma^0 \mathcal{D} \Psi_i \rangle = g_\omega \rho_0$$
$$m_\rho^2 \rho = g_\rho \sum_{i=p,n} \tau_i \langle \bar{\Psi}_i \gamma^0 \mathcal{D} \Psi_i \rangle = g_\rho \rho_I$$

Equation of State (EoS)

$$\varepsilon = \sum_{i=p,n} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \leq p_{F_i}} d^3 p E(\vec{p}) - \langle \mathcal{L} \rangle$$
$$P = \frac{1}{3} \sum_{i=p,n} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \leq p_{F_i}} d^3 p \frac{\vec{\Pi}_i \cdot \vec{p}}{\Pi_i^0} + \langle \mathcal{L} \rangle$$

Features of NLD model...

cut-off Λ regulates

1) DD & MD of selfenergies

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1) DD & MD of selfenergies

2) DD of meson-field sources
(particularly for ω -field)

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cut-off Λ regulates

1) DD & MD of selfenergies

2) DD of meson-field sources
(particularly for ω -field)

3) fully thermodynamic consistent
(important for neutron stars)

NLD results: saturation...

Parameters

	\vec{D}	cut-off	Λ_s [GeV]	Λ_v [GeV]	g_σ	g_ω	g_ρ	b [fm ⁻¹]	c	m_σ [GeV]	m_ω [GeV]	m_ρ [GeV]
NLD	$\frac{1}{1 + \sum_{j=1}^4 (\zeta_j^\alpha i \vec{\partial}_\alpha)^2}$	$\frac{\Lambda^2}{\Lambda^2 + \vec{p}^2}$	0.95	1.125	10.08	10.13	3.50	15.341	-14.735	0.592	0.782	0.763

Comparison with other models

Model	ρ_{sat} [fm ⁻³]	E_b [MeV/A]	K [MeV]	a_{sym} [MeV]	L [MeV]	K_{sym} [MeV]	K_{asy} [MeV]
NLD	0.156	-15.30	251	30	81	-28	-514
NL3*	0.150	-16.31	258	38.68	125.7	104.08	-650.12
DD	0.149	-16.02	240	31.60	56	-95.30	-431.30
D ³ C	0.151	-15.98	232.5	31.90	59.30	-74.7	-430.50
DBHF	0.185	-15.60	290	33.35	71.10	-27.1	-453.70
	0.181	-16.15	230	34.20	71	87.36	-340
empirical	0.167 ± 0.019	-16 ± 1	230 ± 10	31.1 ± 1.9	88 ± 25	-	-550 ± 100

→ Lalazissis

→ Typel

→ Li, Machleidt, Brockmann

→ Fuchs

NLD results: saturation...

Parameters

	\vec{D}	cut-off	[GeV]		g_ρ	b [fm ⁻¹]	c	m_σ [GeV]	m_ω [GeV]	m_ρ [GeV]
NLD	$\frac{1}{1 + \sum_{j=1}^4 (\zeta_j^\alpha i \vec{\partial}_\alpha)^2}$	$\frac{\Lambda^2}{\Lambda^2 + \vec{p}^2}$	0.95	1.125	10.08	10.13	3.50	15.341	-14.735	0.592 0.782 0.763

monopole form

Comparison with other models

Model	ρ_{sat} [fm ⁻³]	E_b [MeV/A]	K [MeV]	a_{sym} [MeV]	L [MeV]	K_{sym} [MeV]	K_{asy} [MeV]
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NLD results: saturation...

Parameters

	\vec{D}	cut-off	Λ_s [GeV]	Λ_v [GeV]	g_σ	g_ω	g_ρ	b [fm ⁻¹]	c	m_σ [GeV]	m_ω [GeV]	m_ρ [GeV]
NLD	$\frac{1}{1 + \sum_{j=1}^4 (\zeta_j^\alpha i \vec{\partial}_\alpha)^2}$	$\frac{\Lambda^2}{\Lambda^2 + \vec{p}^2}$	0.95	1.125	10.08	10.13	3.50	15.341	-14.735	0.592	0.782	0.763

Comparison with other models

Model	ρ_{sat} [fm ⁻³]	E_b [MeV/A]	K [MeV]	a [MeV]	b [MeV]	c [MeV]	d [MeV]
NLD	0.156	-15.30	251	30	81	-28	-514
NL3*	0.150	-16.31	258	38.68	125.7	104.08	-650.12
DD	0.149	-16.02	240	31.60	56	-95.30	-431.30
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empirical	0.167 ± 0.019	-16 ± 1	230 ± 10	31.1 ± 1.9	88 ± 25	-	-550 ± 100

soft EoS at ρ_{sat}
but stiff at high ρ relevant for NS!

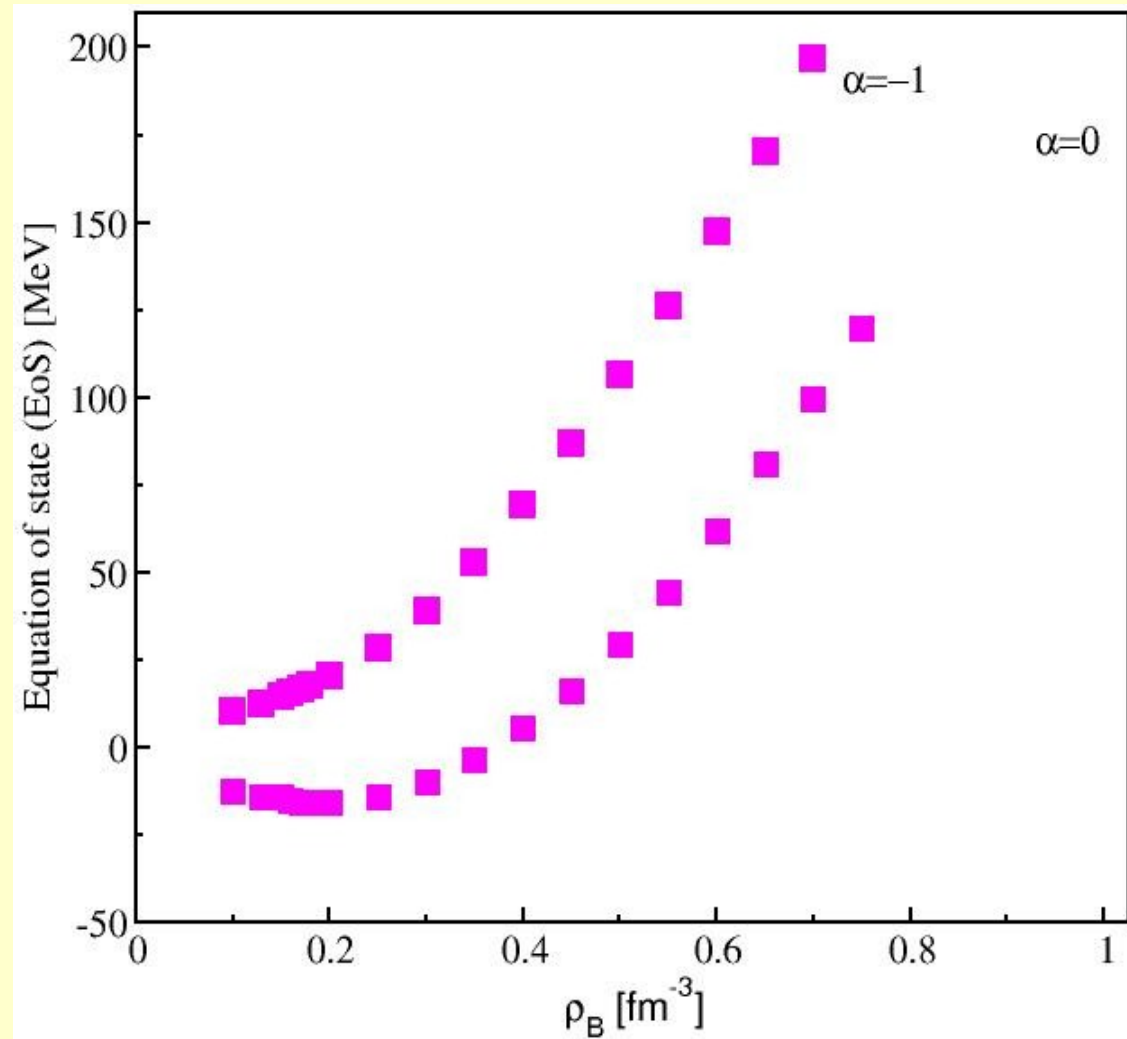
→ Lalazissis

→ Typel

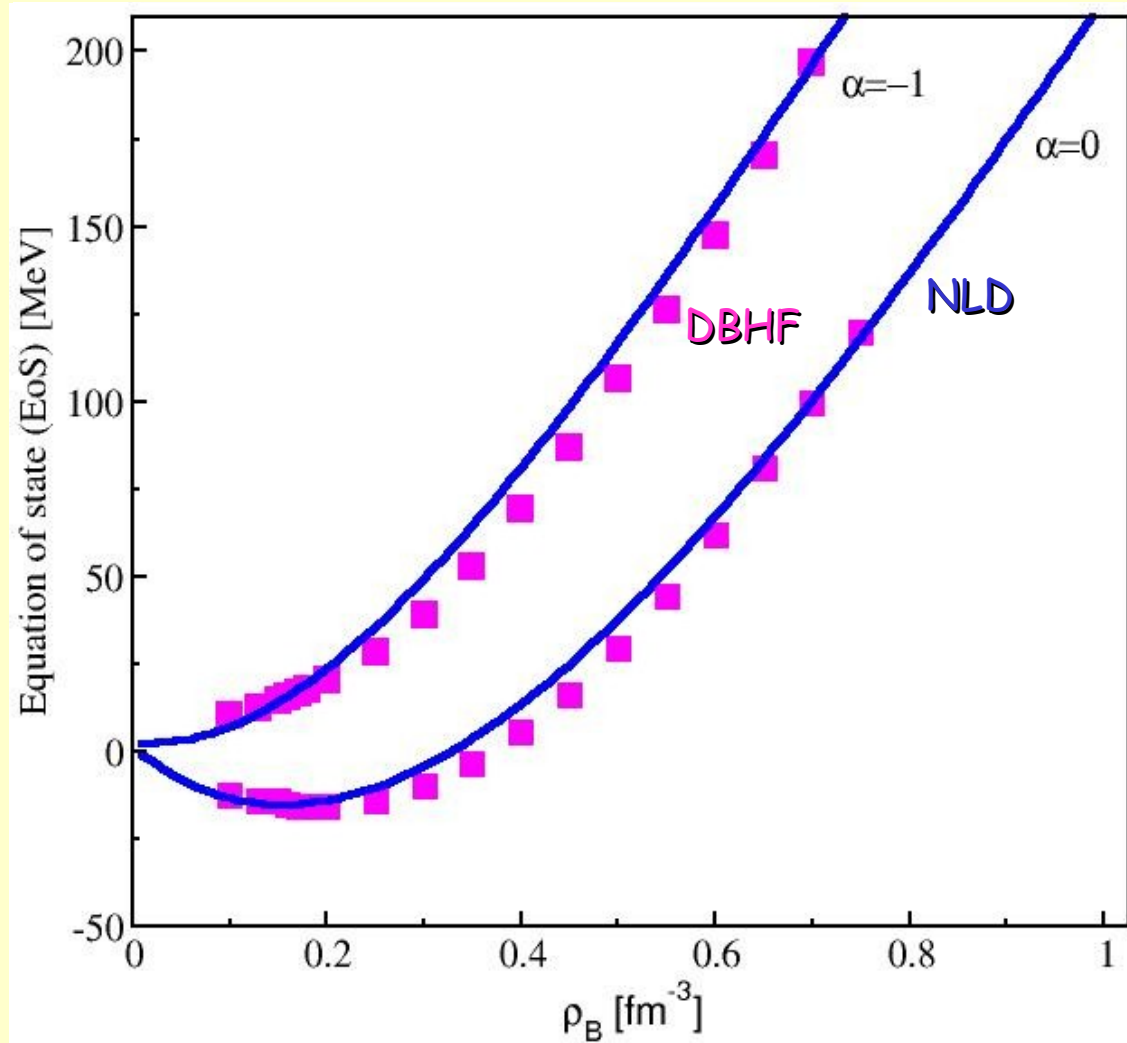
→ Li, Machleidt, Brockmann

→ Fuchs

NLD results: EoS...

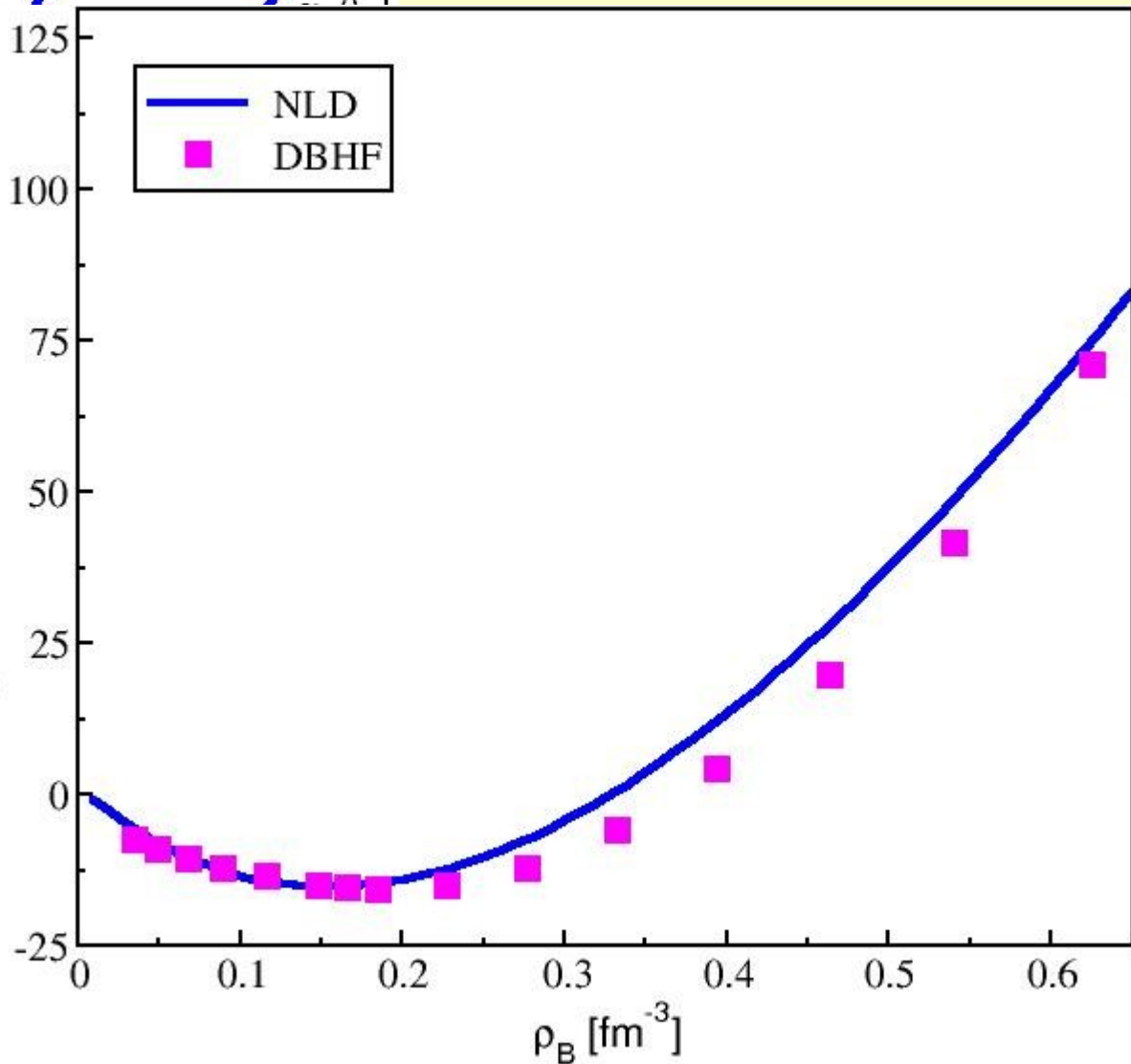
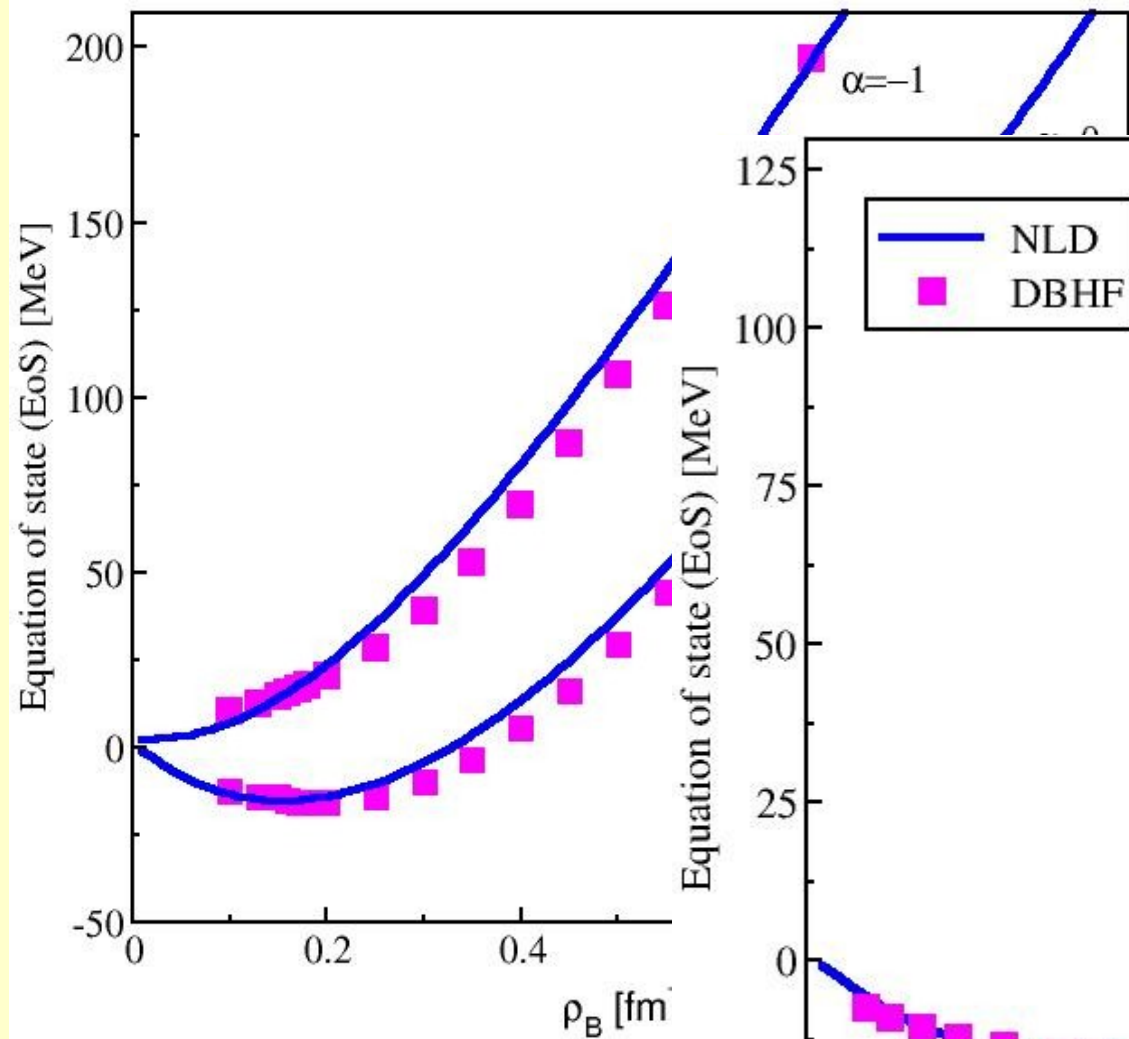


NLD results: EoS...

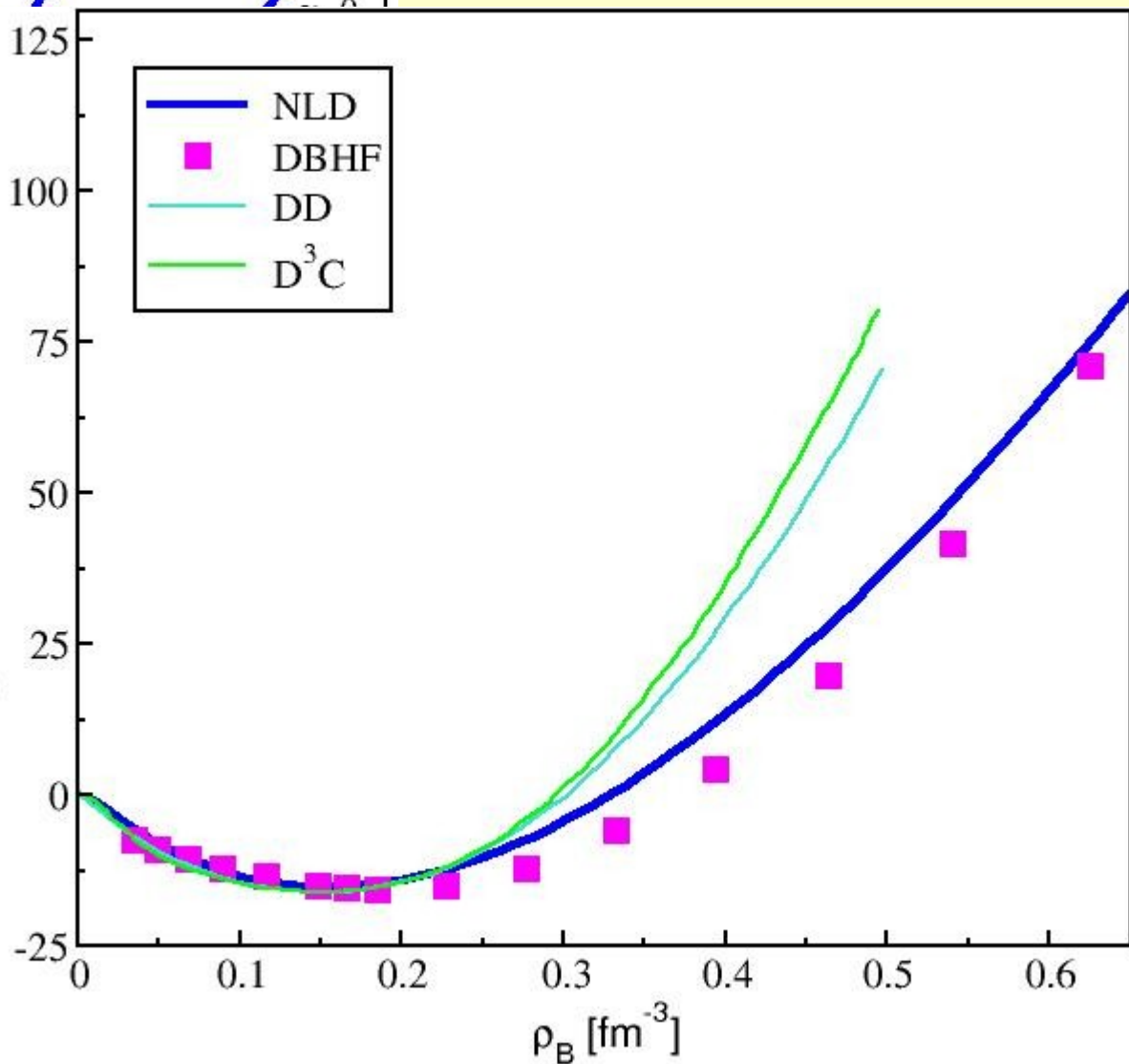
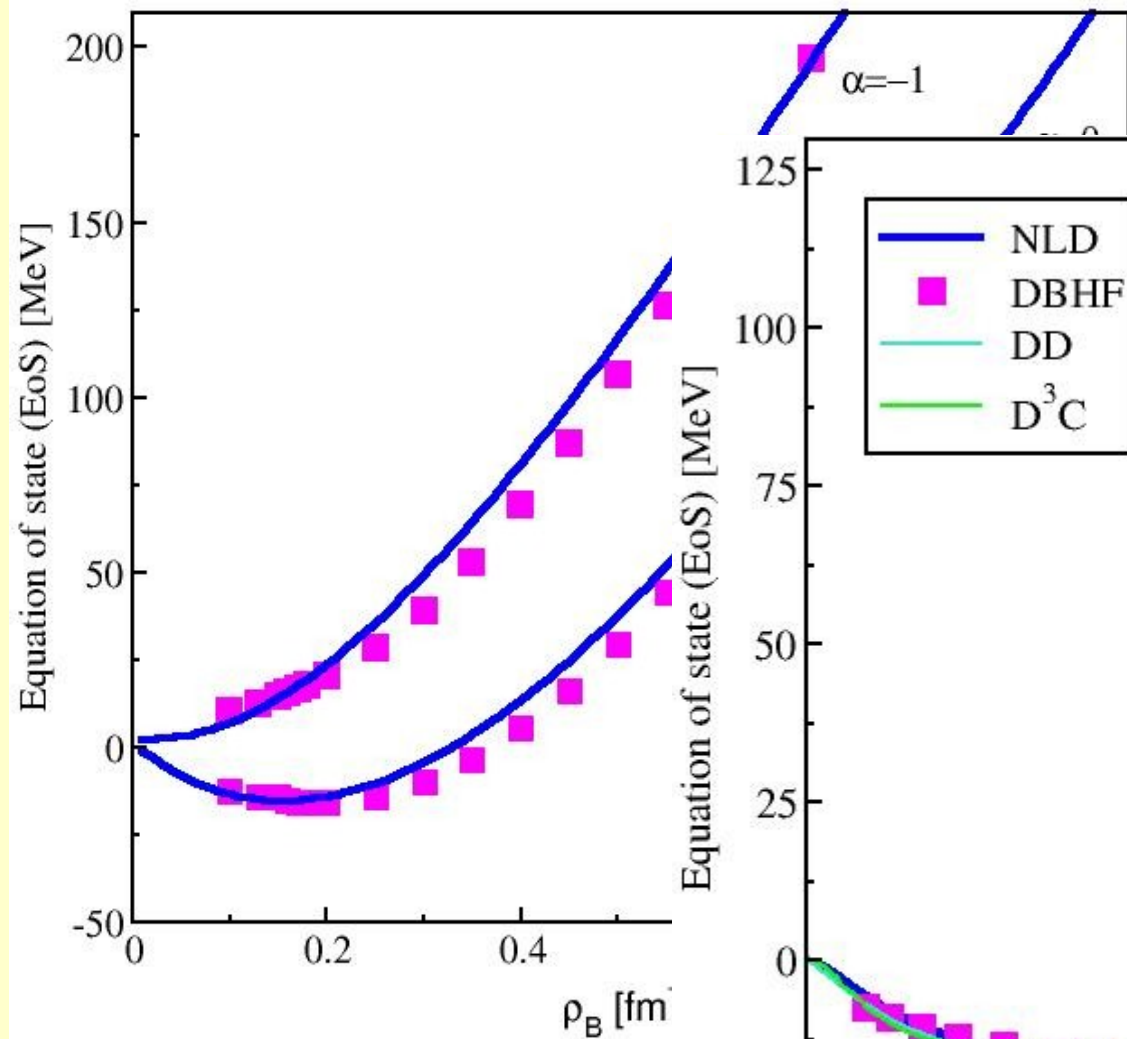


Remarkable comparison with microscopic DBHF !

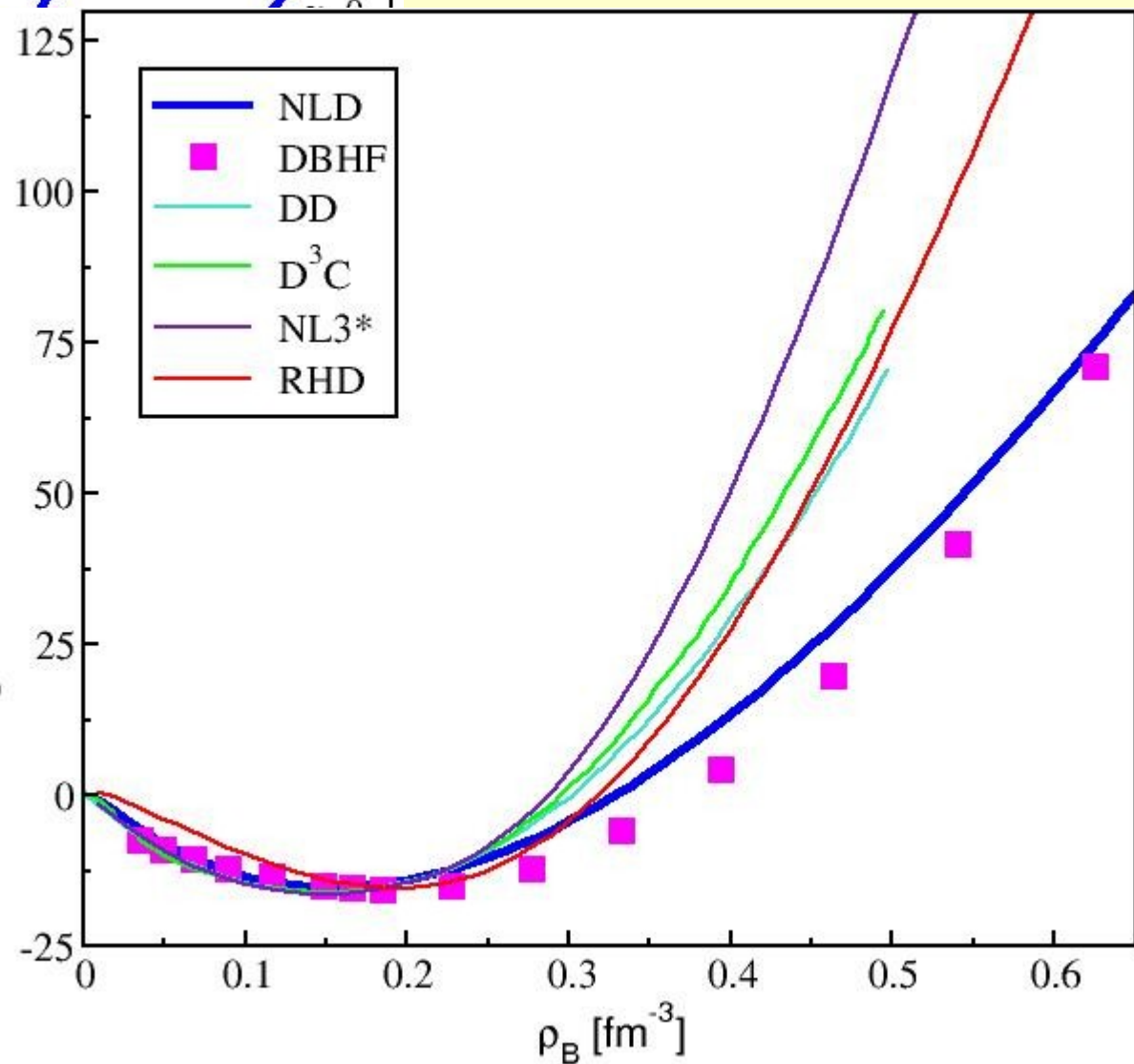
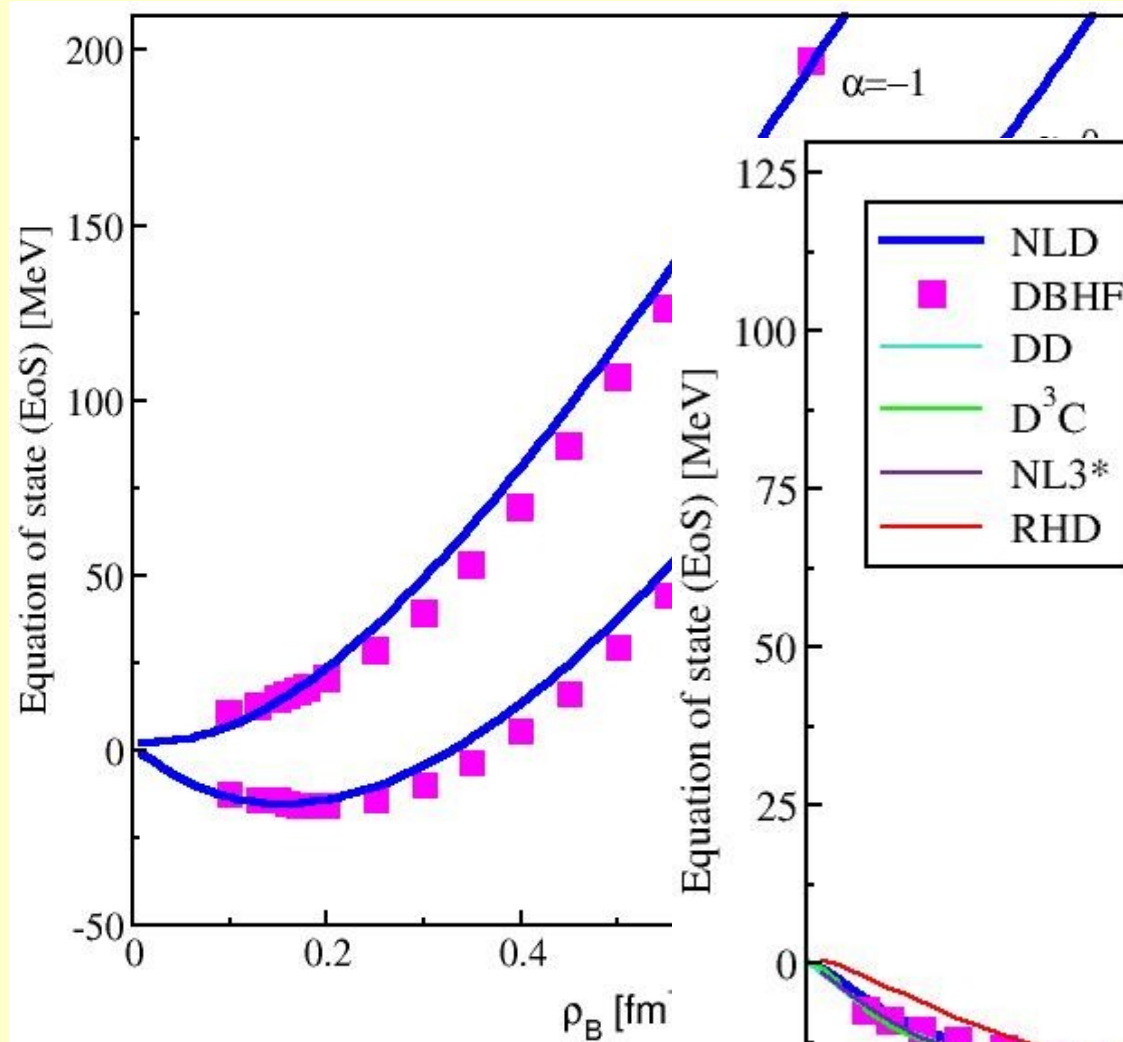
NLD results: EoS...



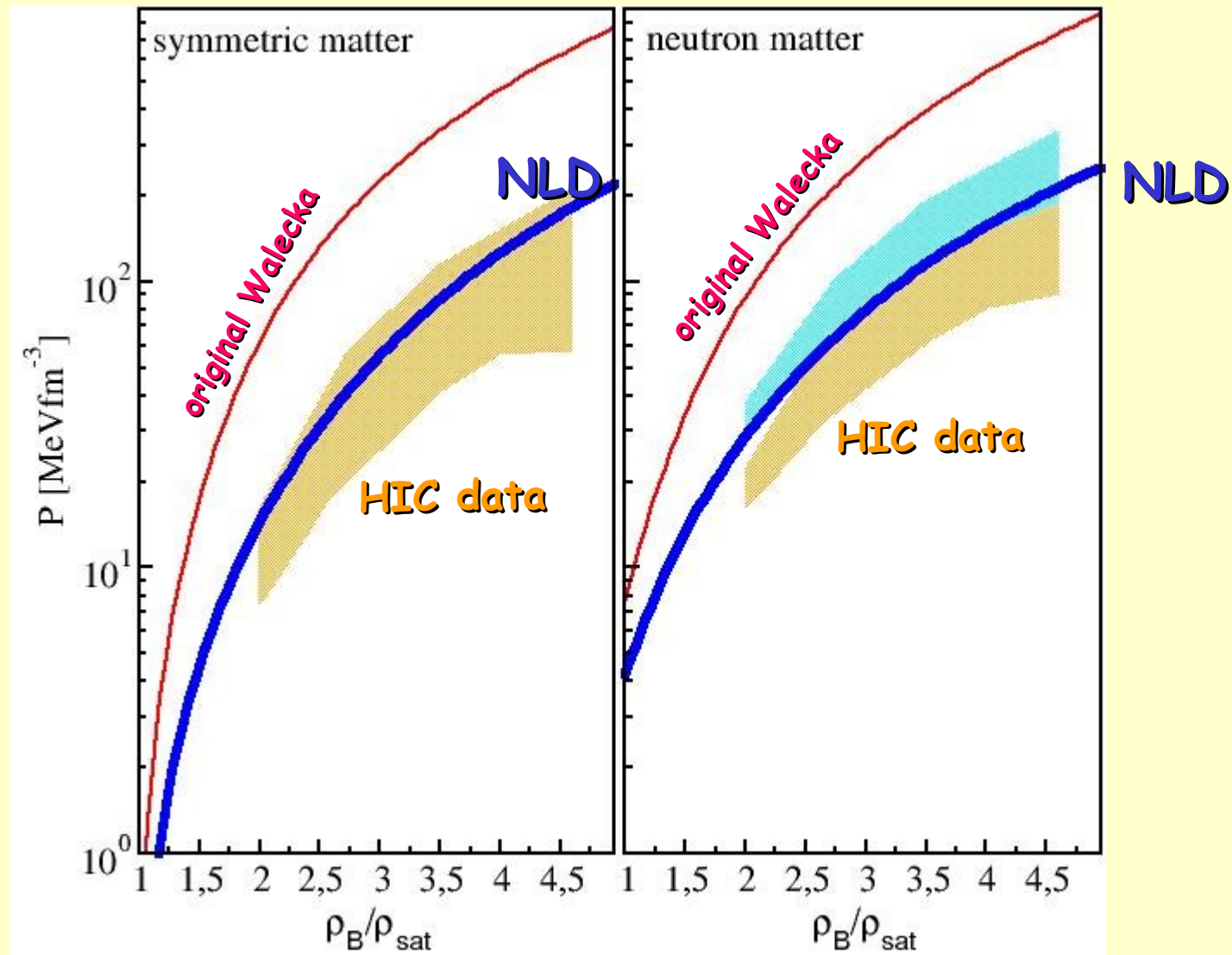
NLD results: EoS...



NLD results: EoS...

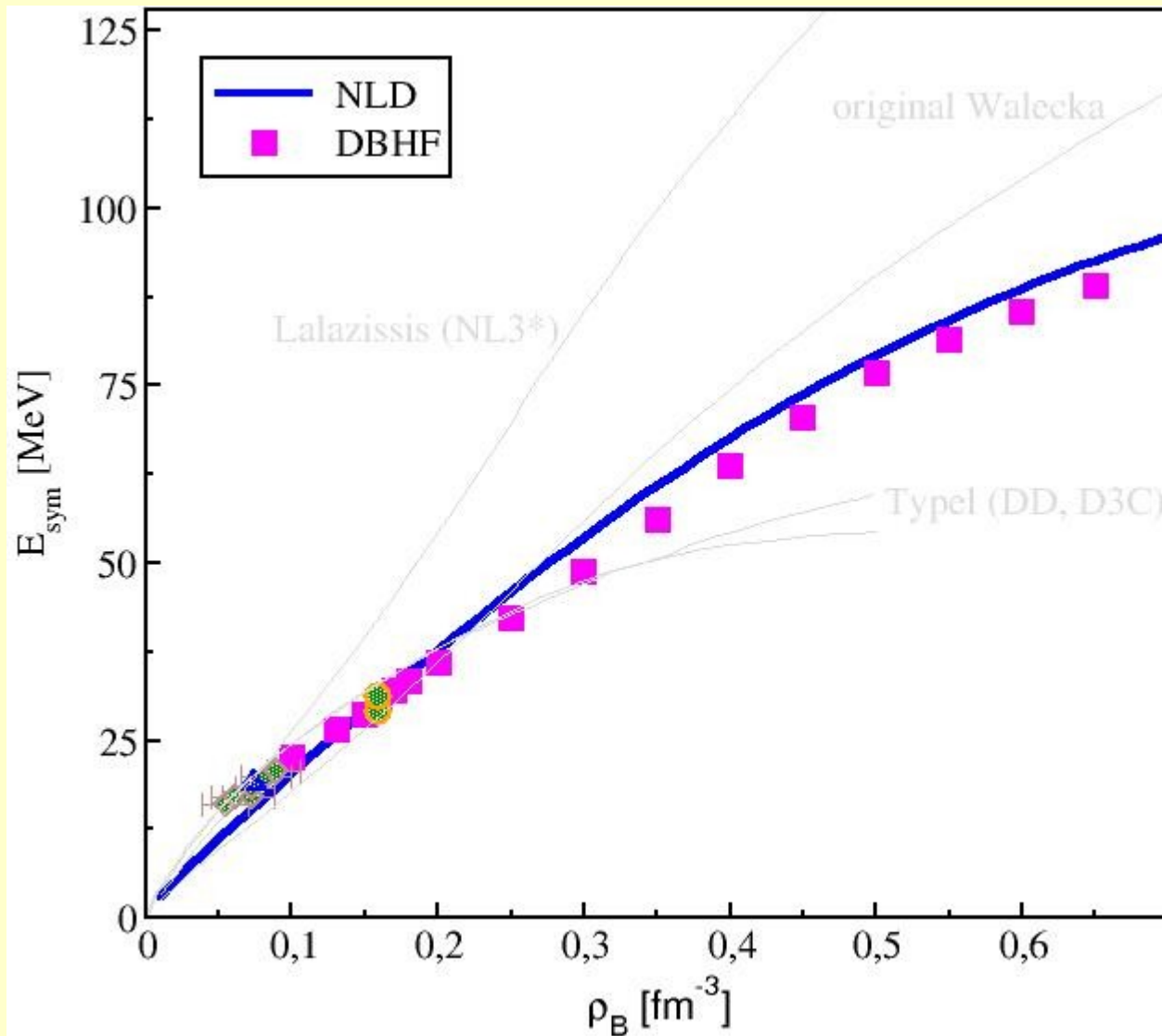


NLD results: EoS...



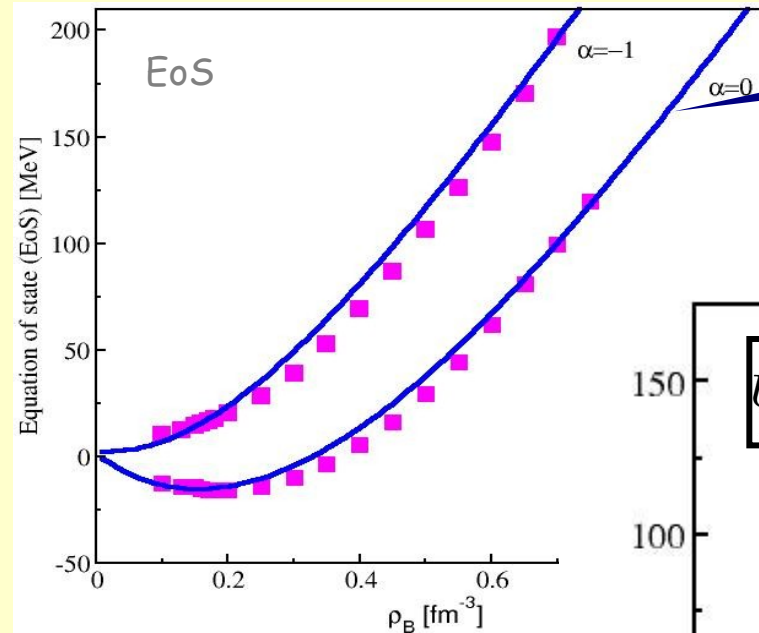
NLD model is consistent with HIC phenomenology

NLD results: symmetry energy...



Remarkable agreement with microscopic DBHF !

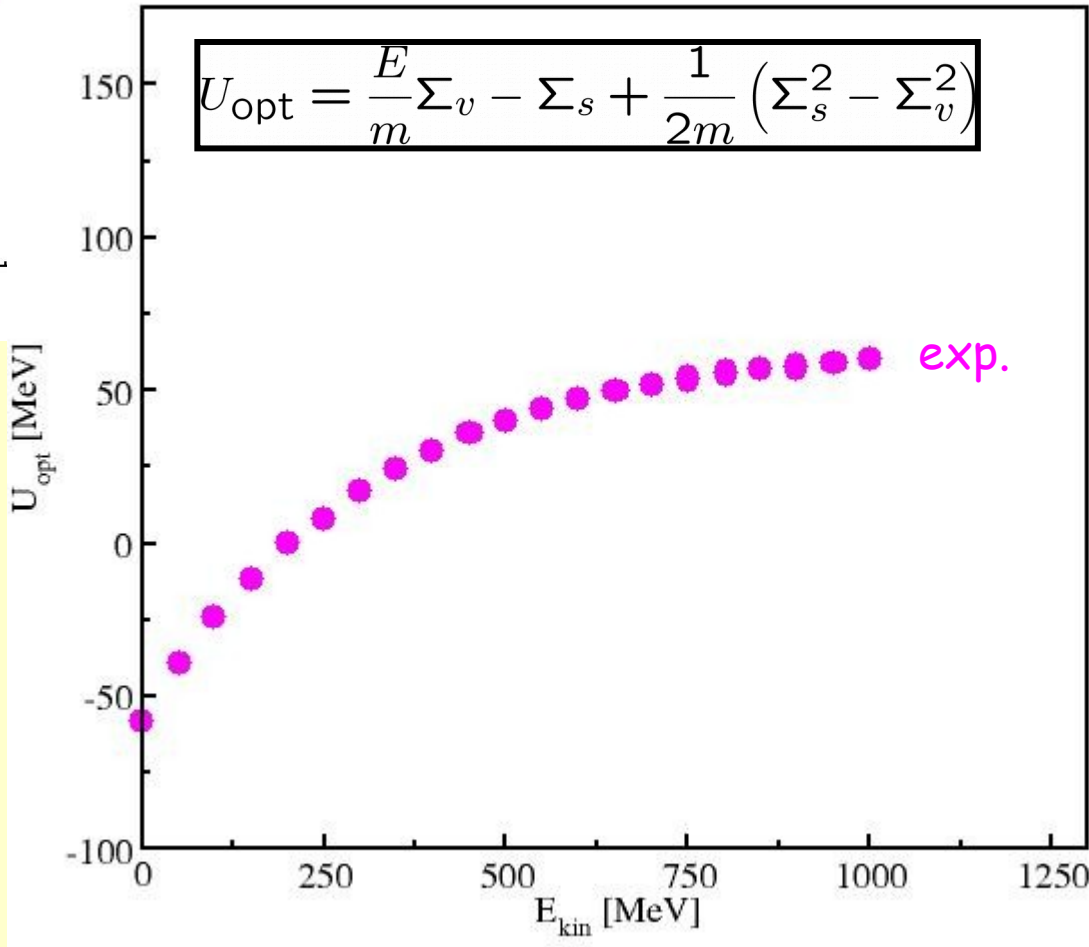
NLD results: MD & optical potentials...



high $\rho \rightarrow$ high momenta

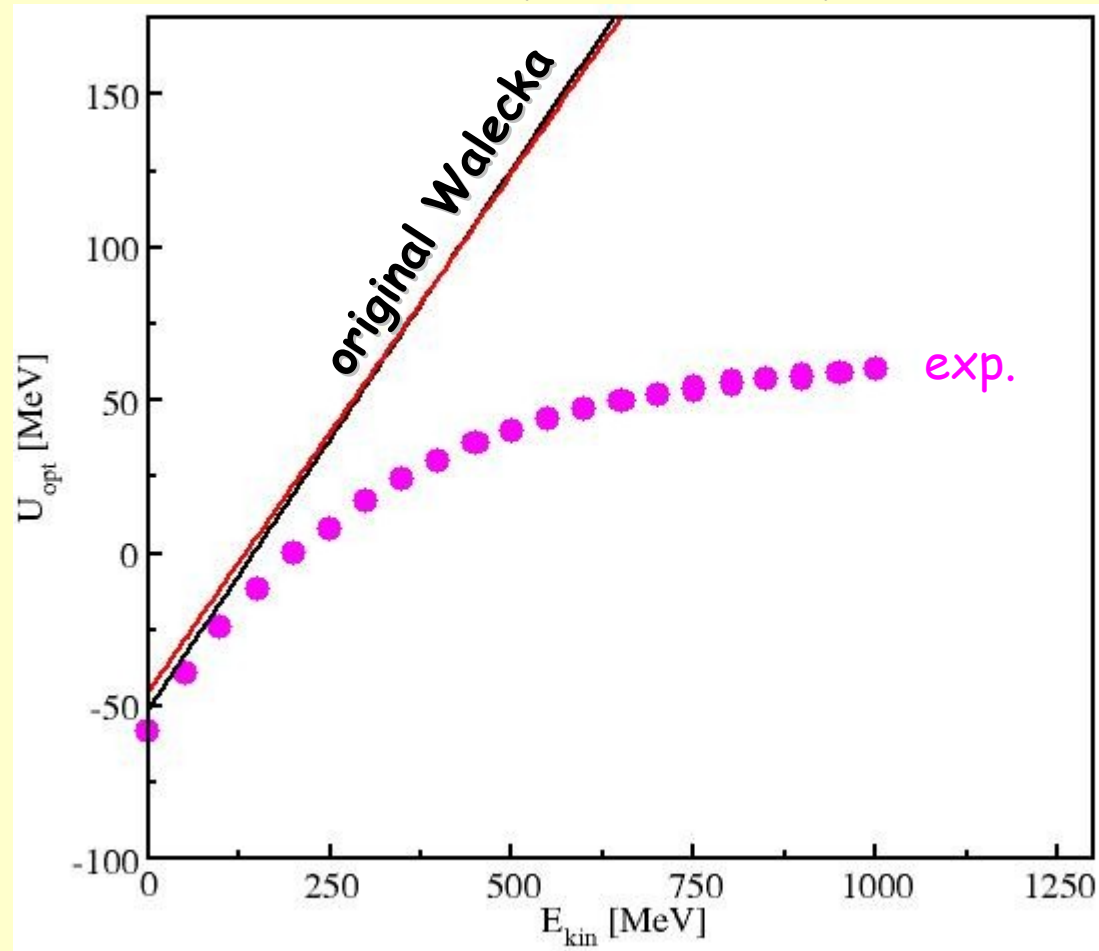
In-medium proton SEP (real part)

$$U_{\text{opt}} = \frac{E}{m} \Sigma_v - \Sigma_s + \frac{1}{2m} (\Sigma_s^2 - \Sigma_v^2)$$



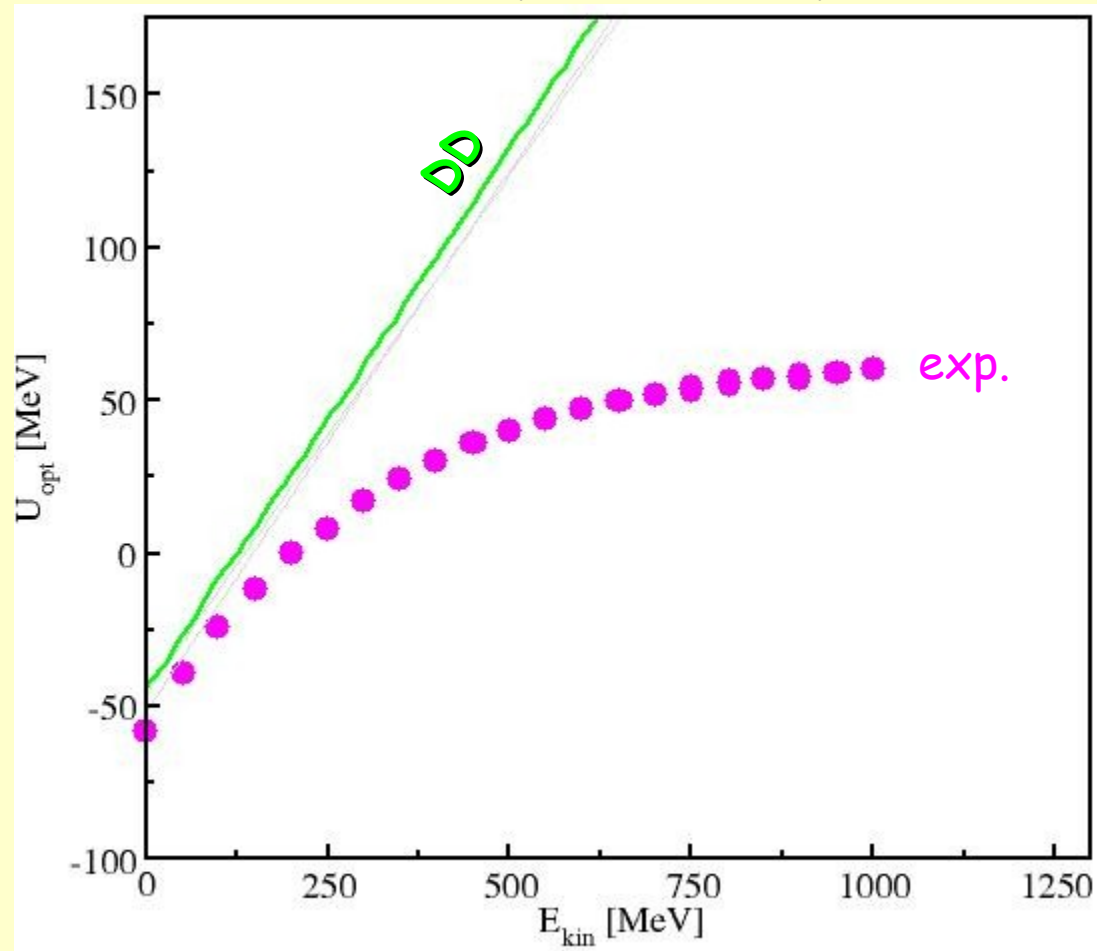
NLD results: MD & optical potentials...

In-medium proton SEP (real part)



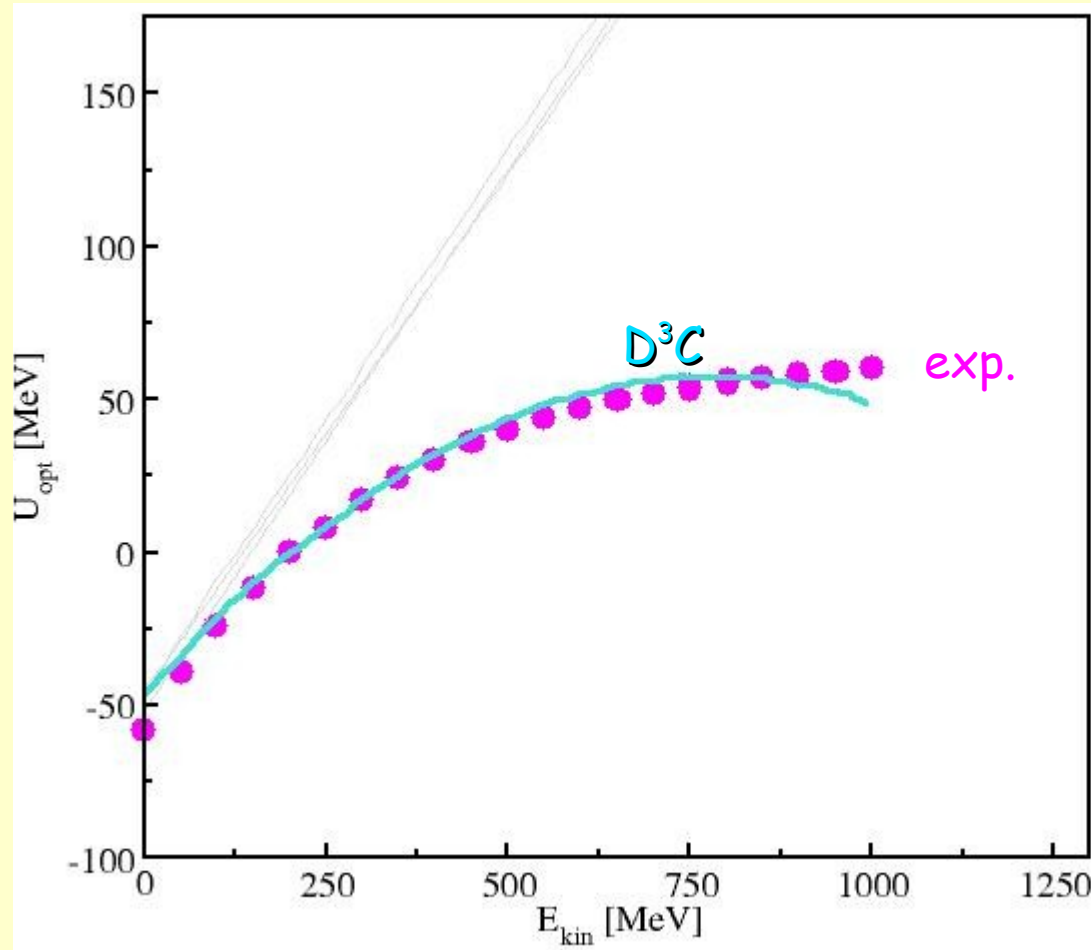
NLD results: MD & optical potentials...

In-medium proton SEP (real part)

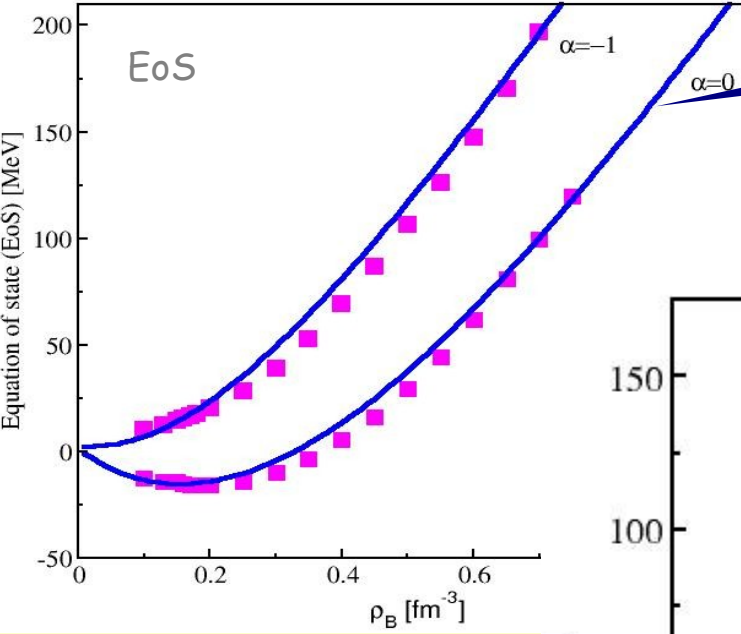


NLD results: MD & optical potentials...

In-medium proton SEP (real part)

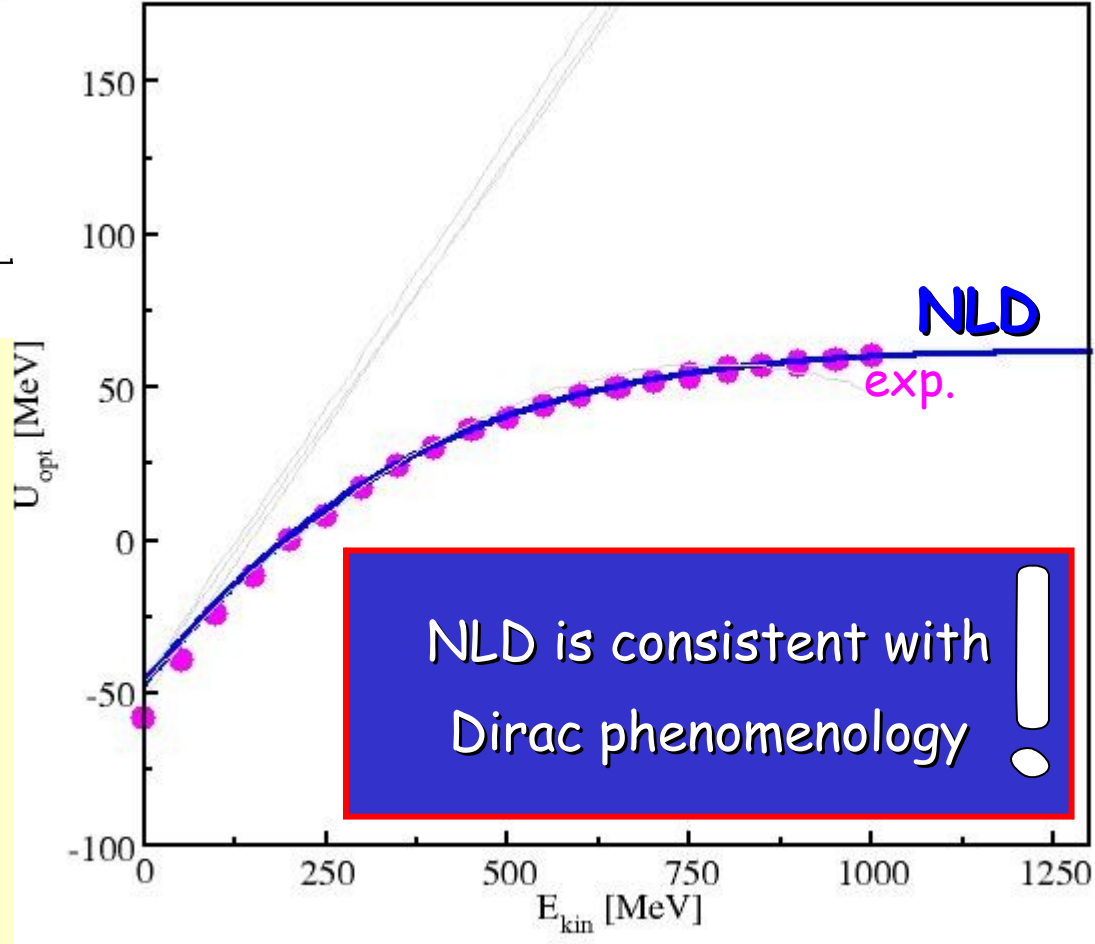


NLD results: MD & optical potentials...



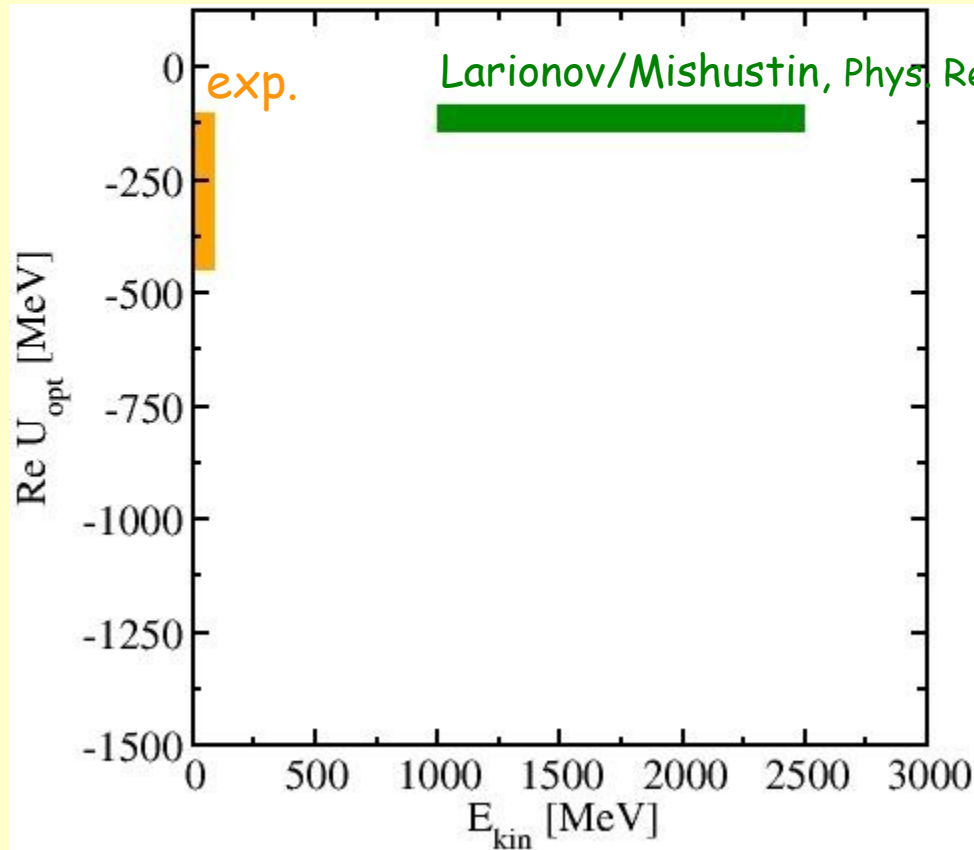
high $\rho \rightarrow$ high momenta

In-medium proton SEP (real part)



NLD results: MD & optical potentials (anti-proton)...

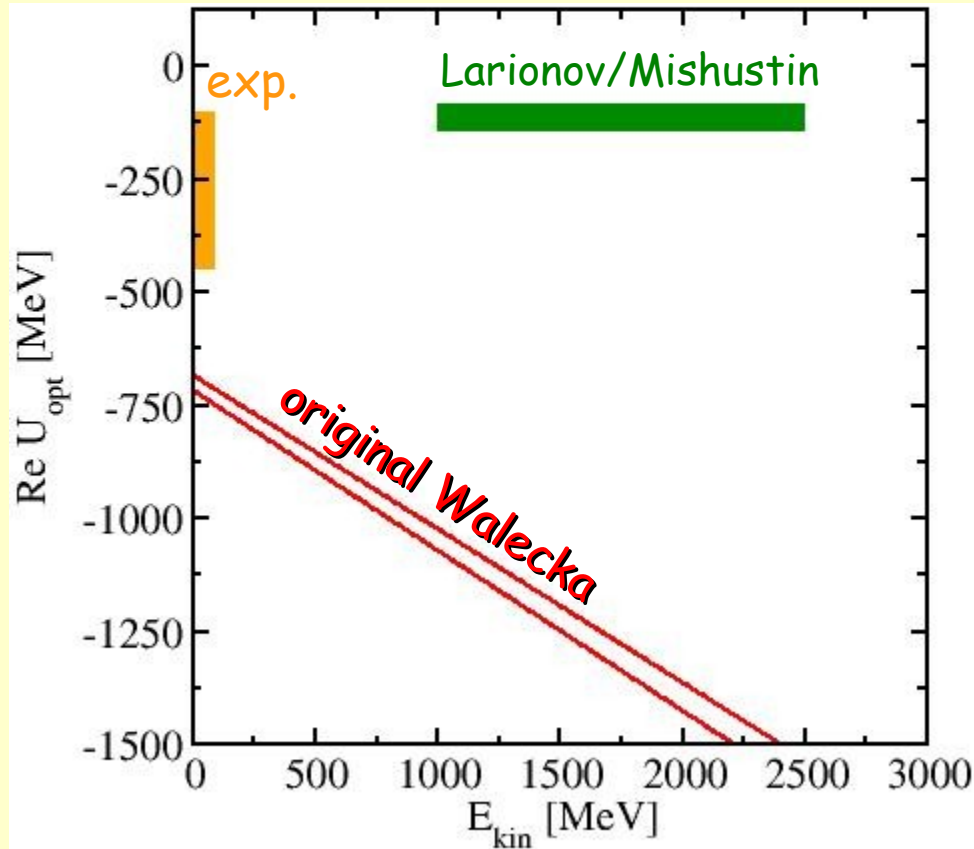
In-medium anti-proton SEP (real part)



Larionov/Mishustin, Phys. Rev. C80 ('09) 021601(R).

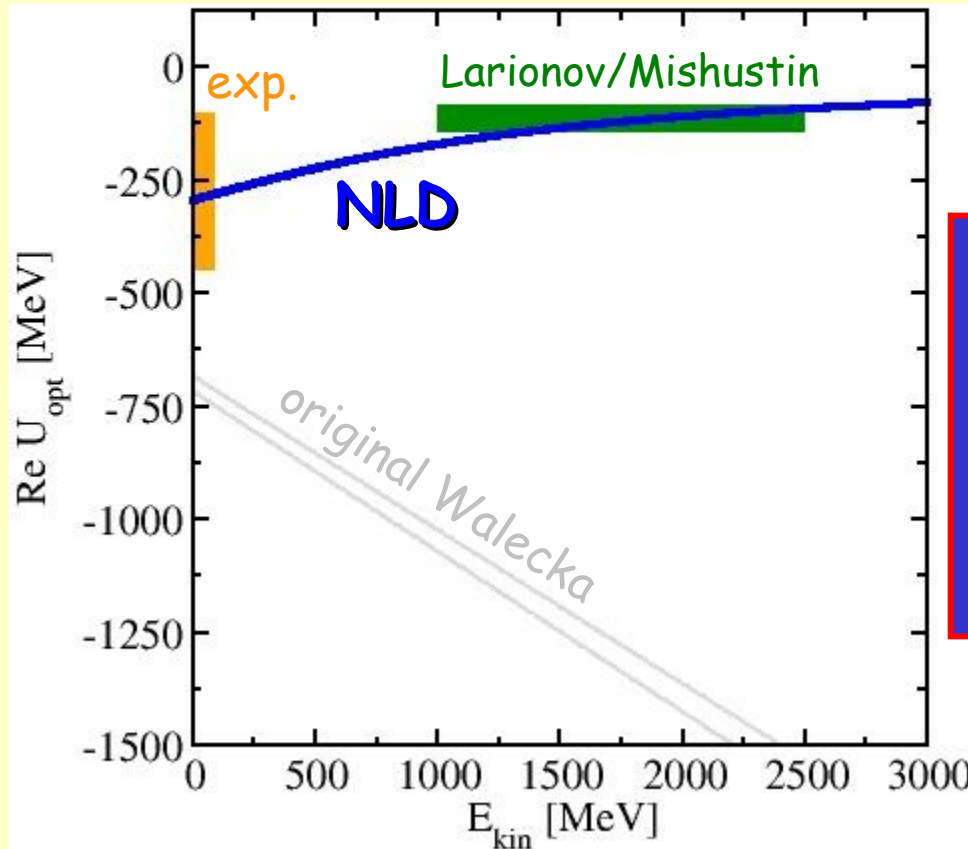
NLD results: MD & optical potentials (anti-proton)...

In-medium anti-proton SEP (real part)



NLD results: MD & optical potentials (anti-proton)...

In-medium anti-proton SEP (real part)

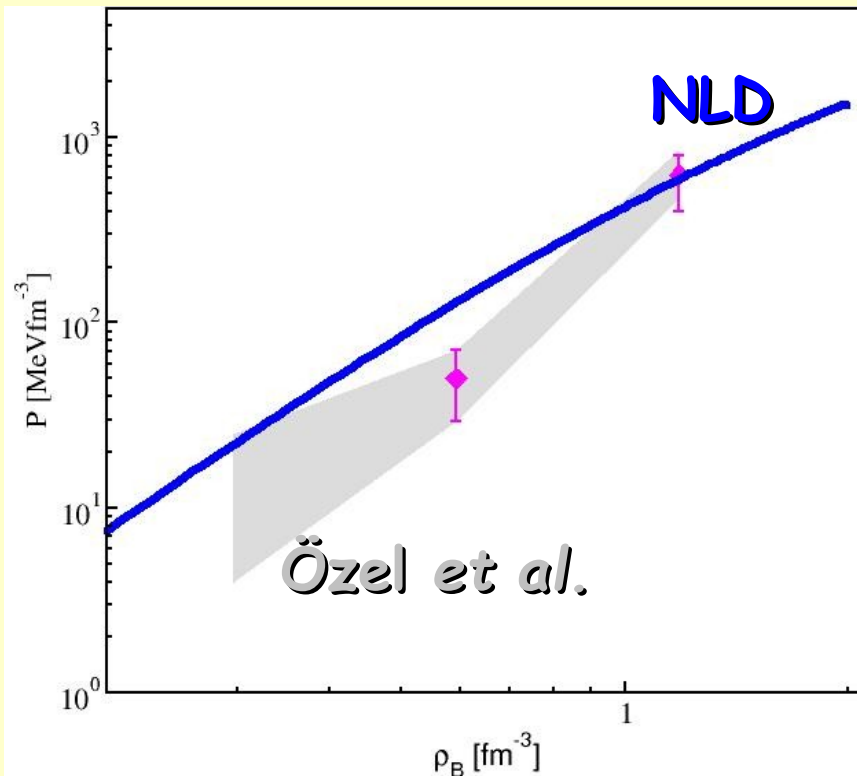


good description of
phenomenology
also for in-medium
anti-proton interaction

Also: NLD provides the imaginary part of SEP for anti-proton in-medium interactions
using dispersion relation (without subtractions) →

Phys. Lett. B703, ('11) 193

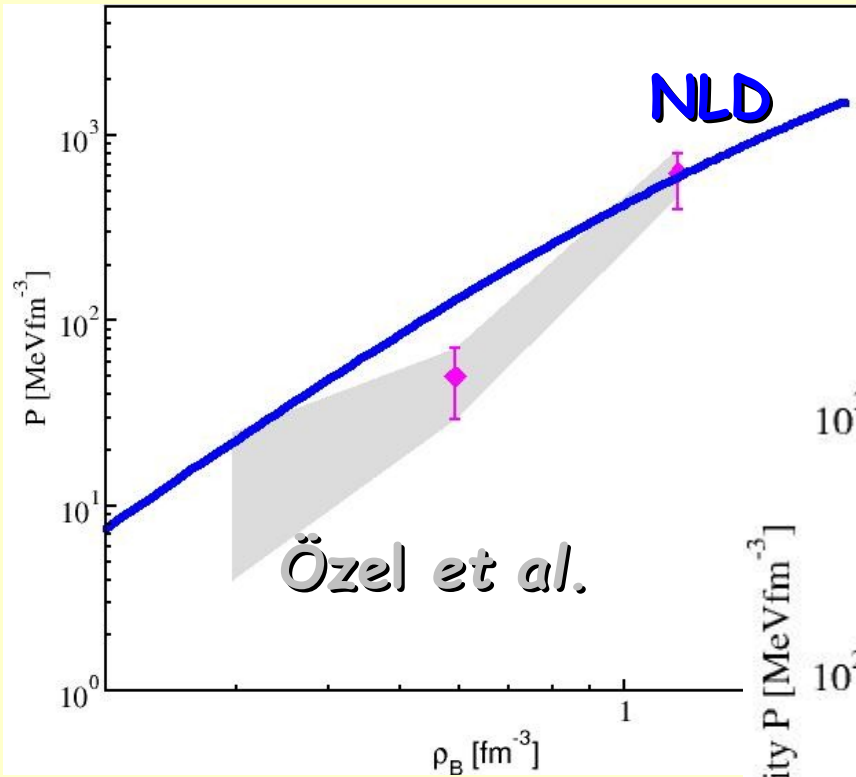
NLD results: high-density EoS at β -equilibrium...



Consistent with analyses of F. Özel..!

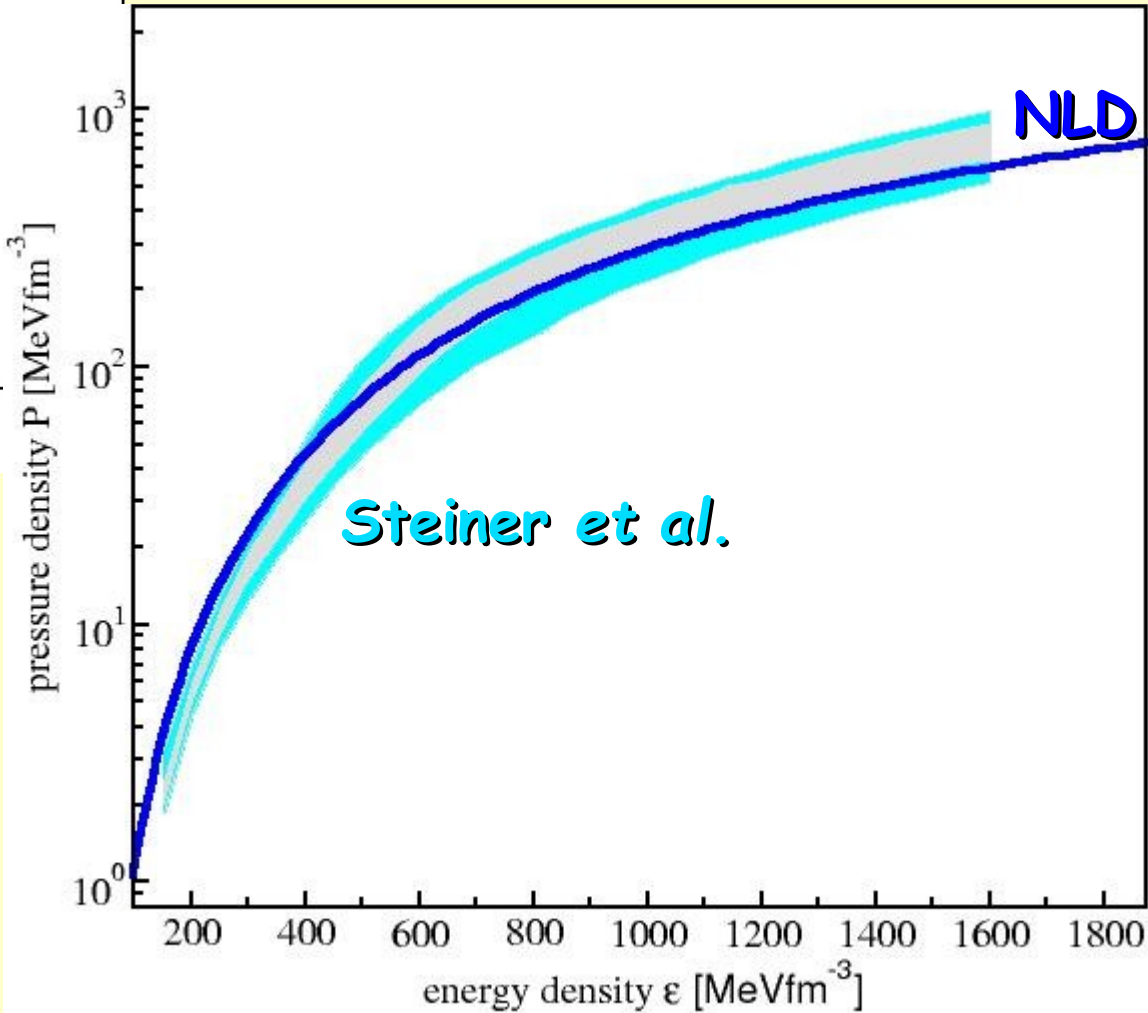
Phys. Rev. D82, 101301 (2010).

NLD results: high-density EoS at β -equilibrium...



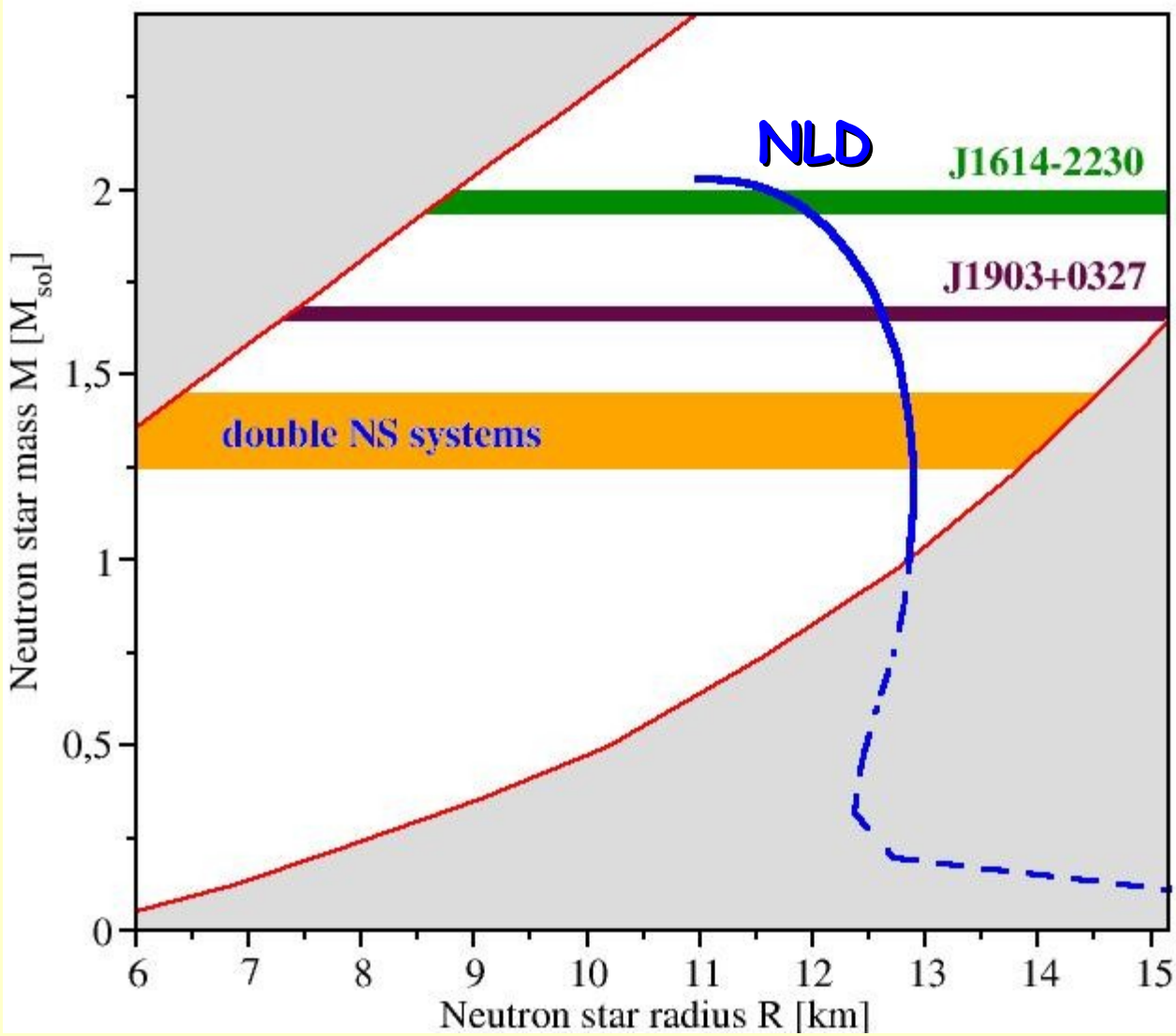
Consistent with analyses of F. Özel..!
Phys. Rev. D82, 101301 (2010).

... and A.W. Steiner
Astrophys. J. 722, 33 (2010).



NLD results: NS mass...

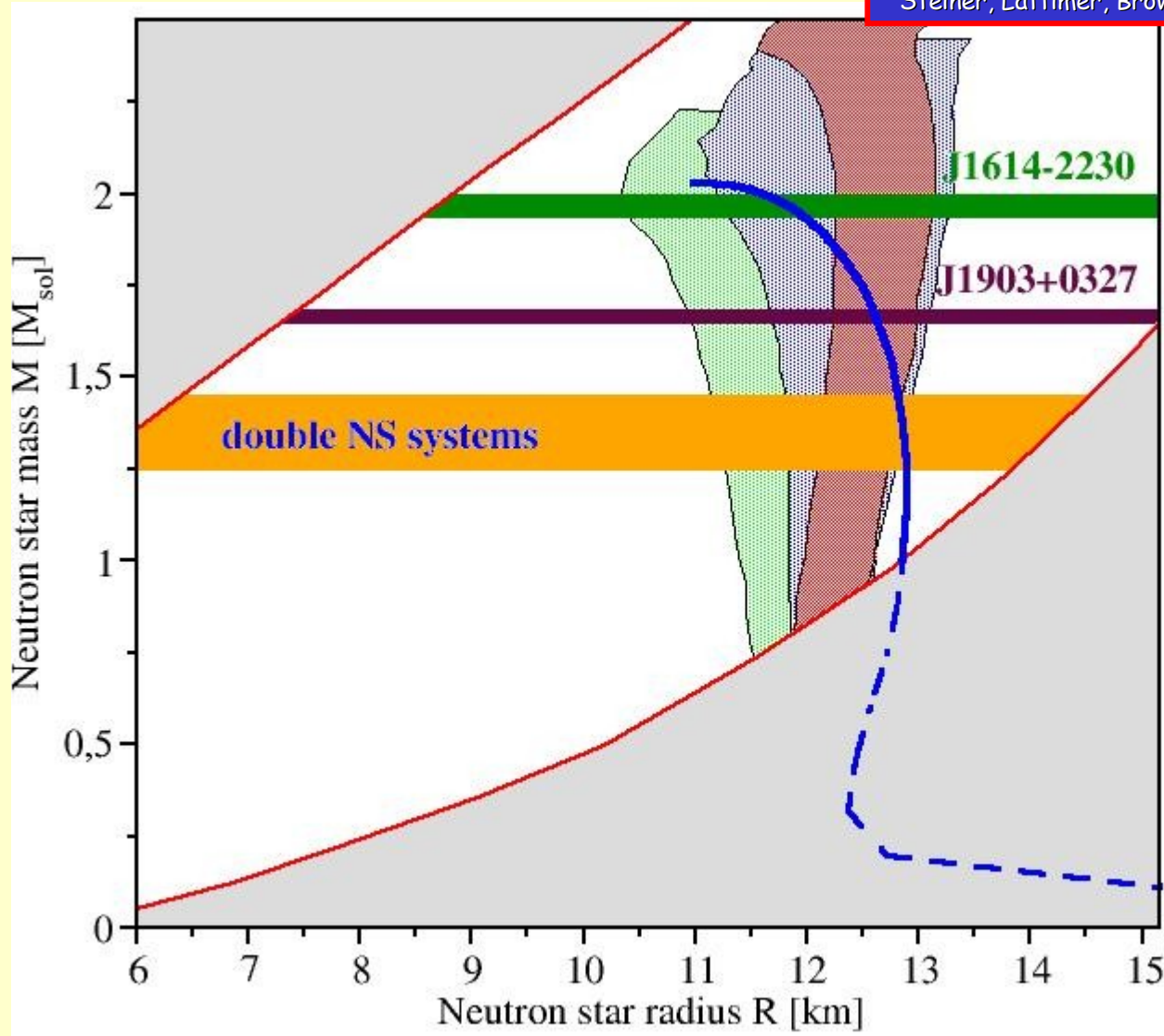
Compatible with ...
Lattimer/Prakash, Science 304 ('04) 536, Phys. Rep. 442 ('07) 109



NLD results: NS mass...

Compatible with all observations

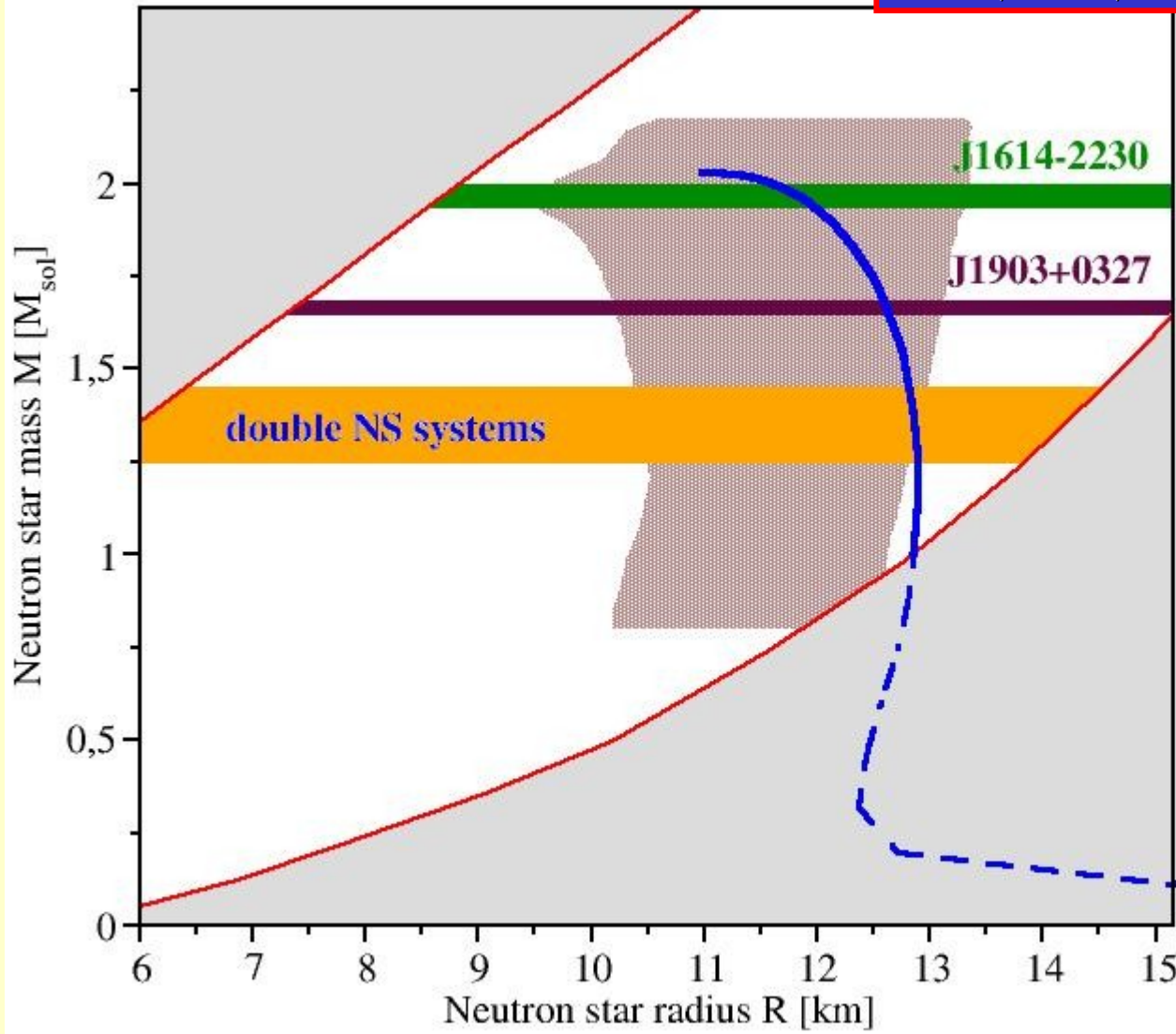
Lattimer/Prakash, Science 304 ('04) 536, Phys. Rep. 442 ('07) 109
Steiner, Lattimer, Brown, arXiv: 1205.6871



NLD results: NS mass...

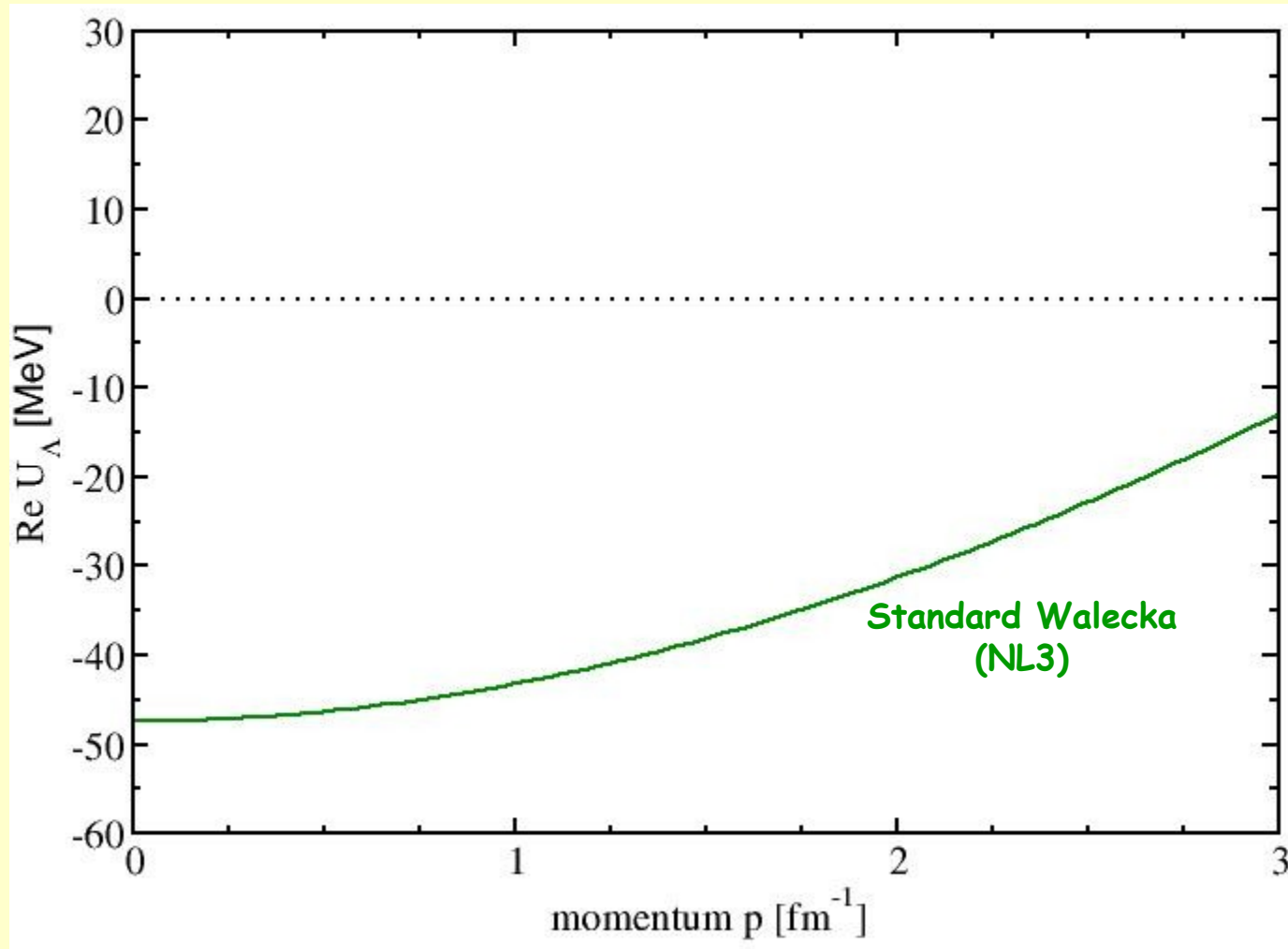
Compatible with all observations !

Lattimer/Prakash, Science 304 ('04) 536, Phys. Rep. 442 ('07) 109
Steiner, Lattimer, Brown, arXiv: 1205.6871



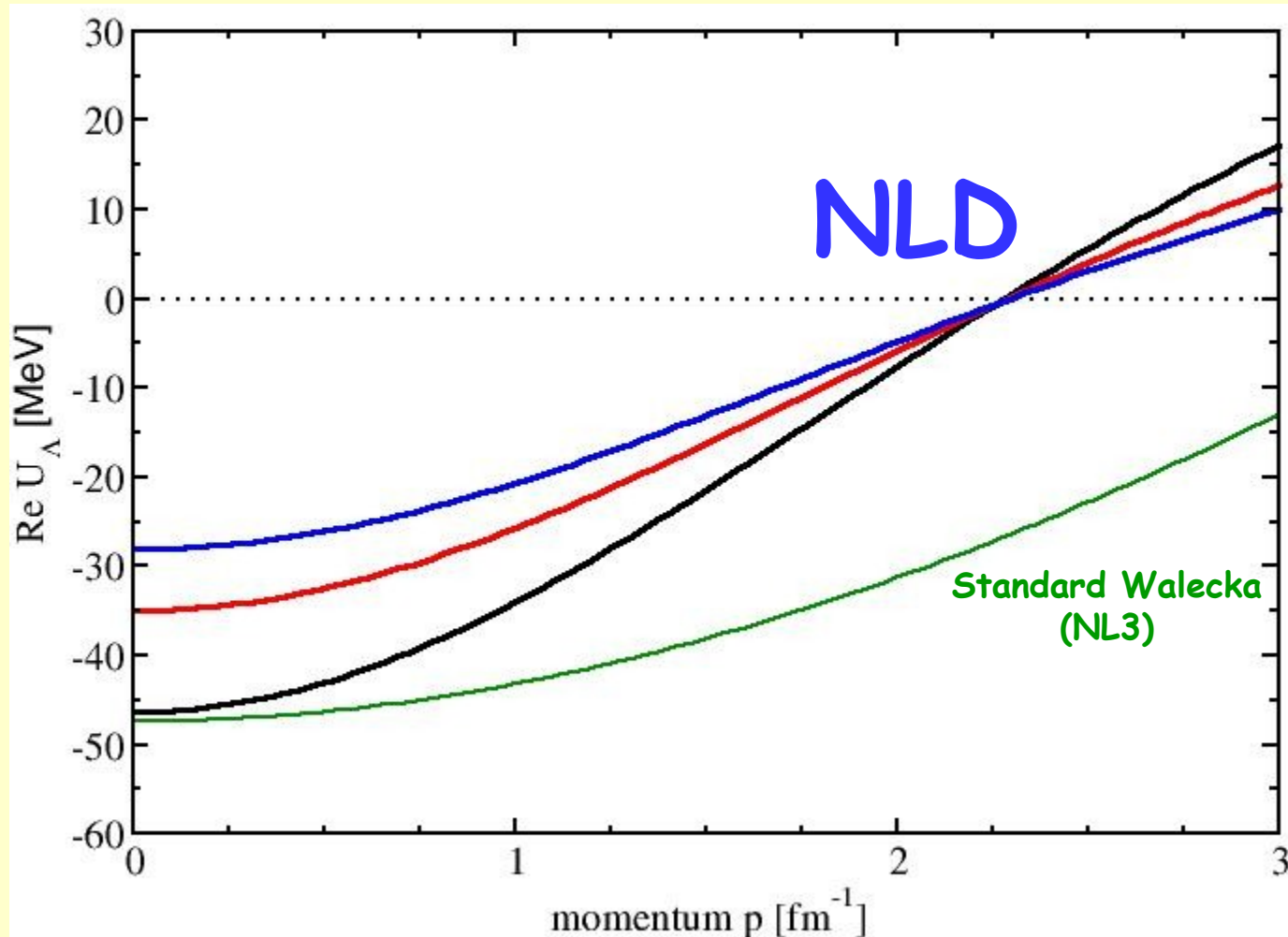
NLD results: in-medium Λ -opt. potential...

NLD + SU(3) for standard meson-nucleon couplings
Hyperon cut-off regulates MDI

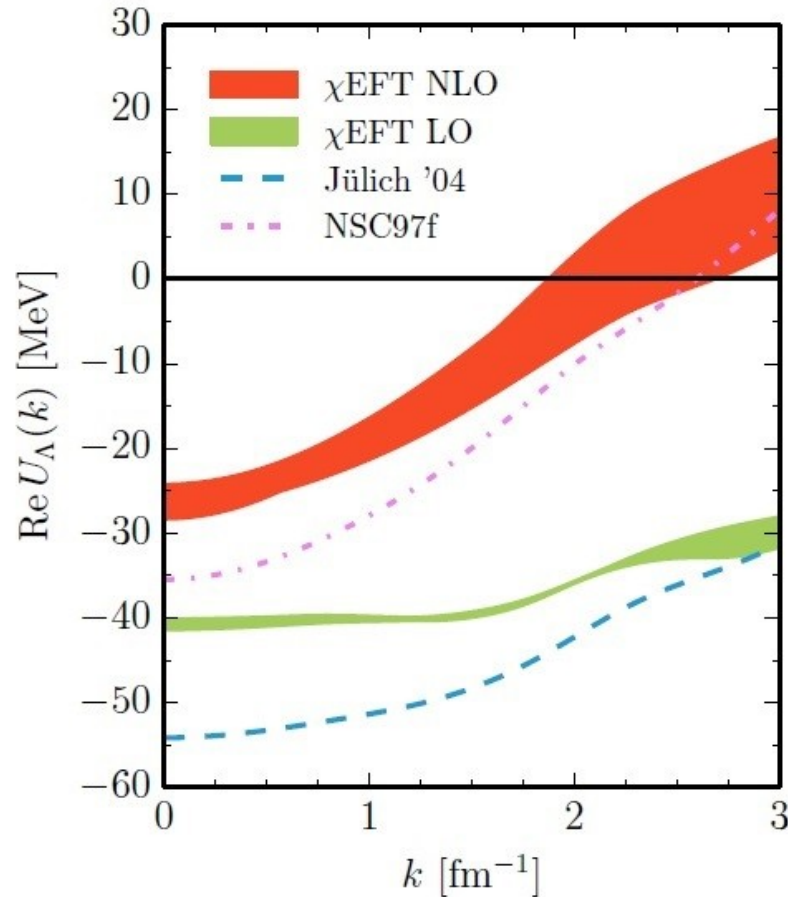


NLD results: in-medium Λ -opt. potential...

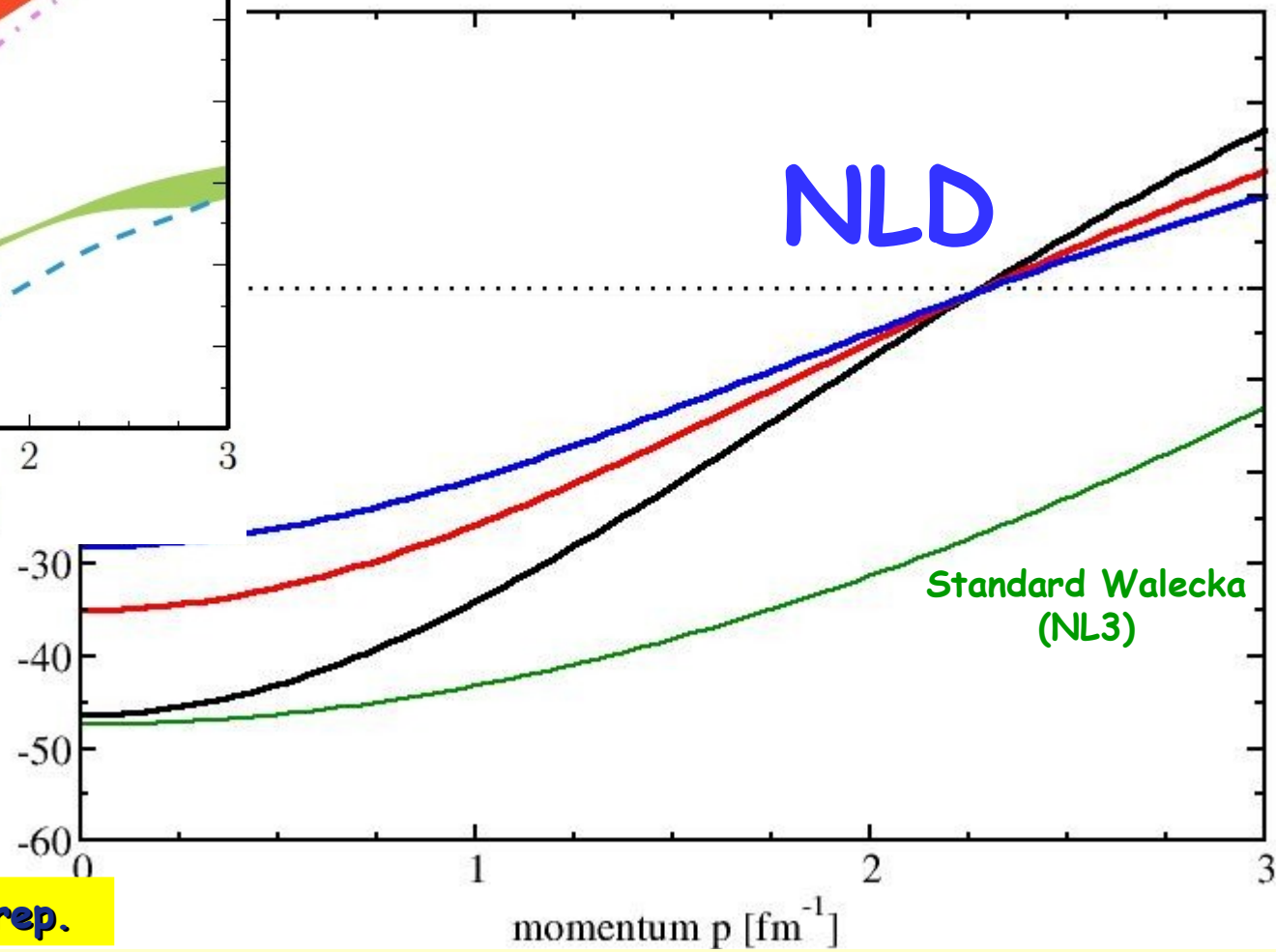
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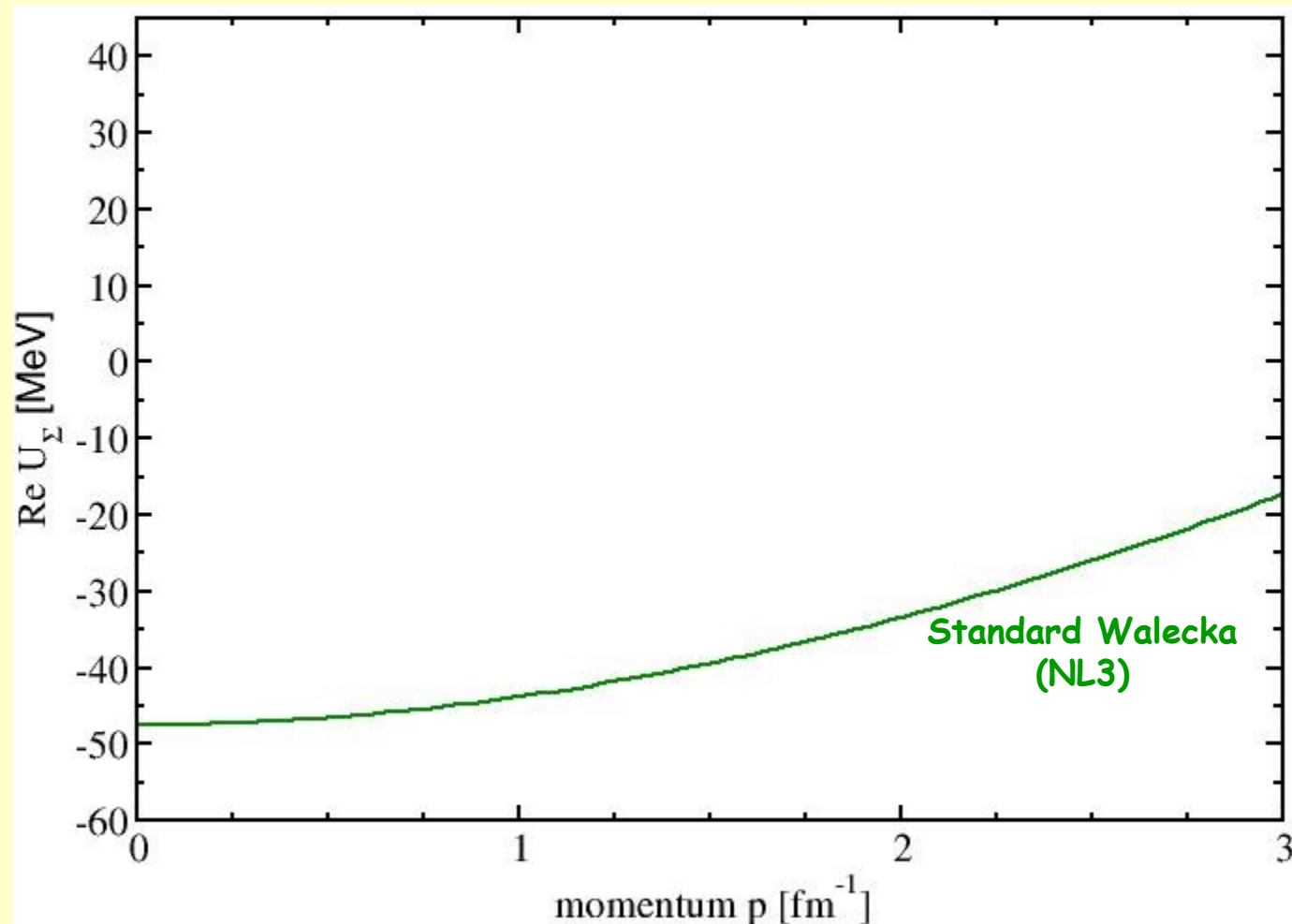


NLD versus chiral-EFT (Haidenbauer, et al., EPJA52 (16) 15)
Compares well with NLO-calculations
(cut-off $\Lambda=0.7$ & 1 GeV for σ & w)



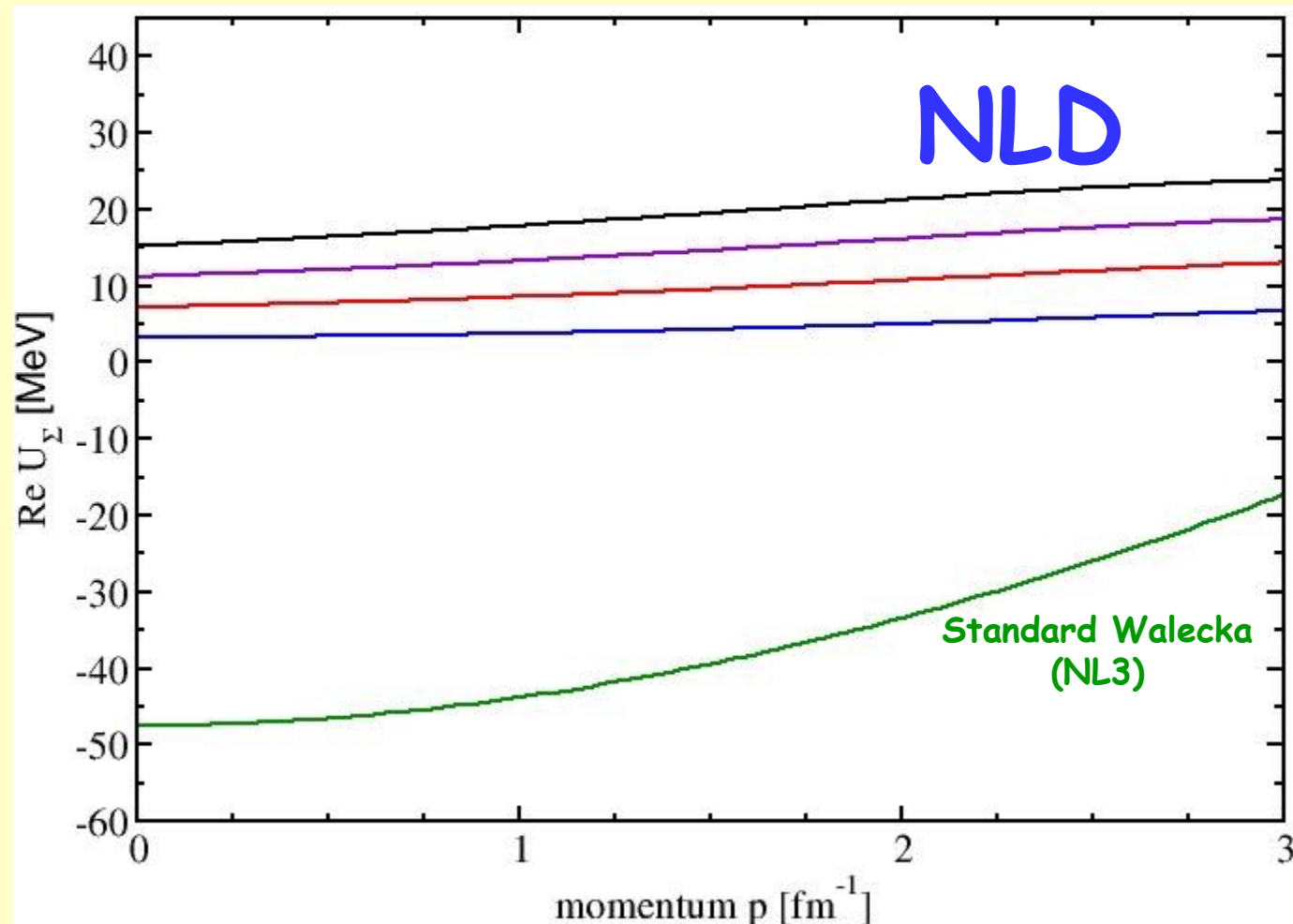
NLD results: in-medium Σ -opt. potential...

Σ -opt. potential attractive in conventional RMF+SU(3)



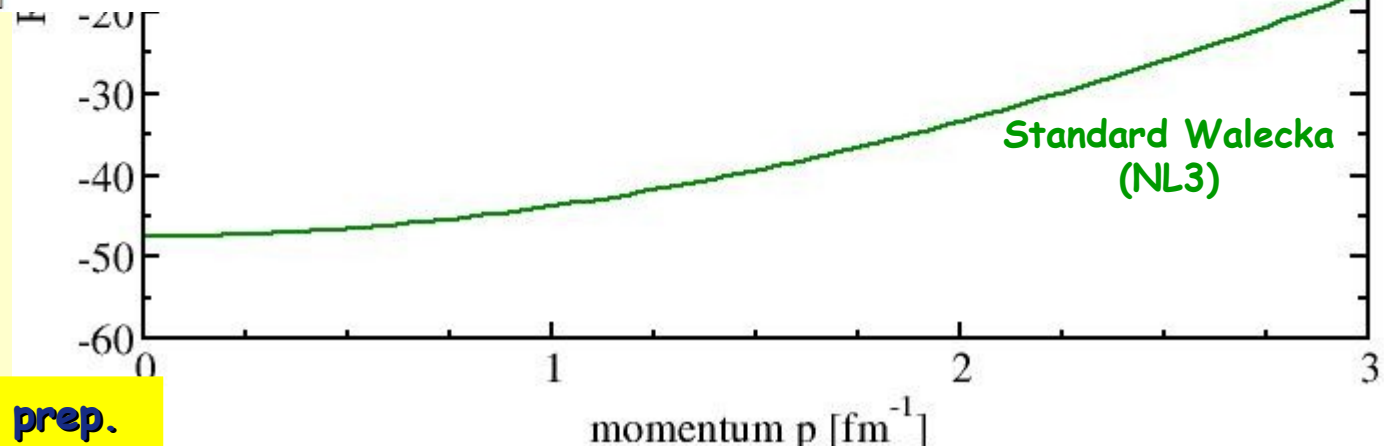
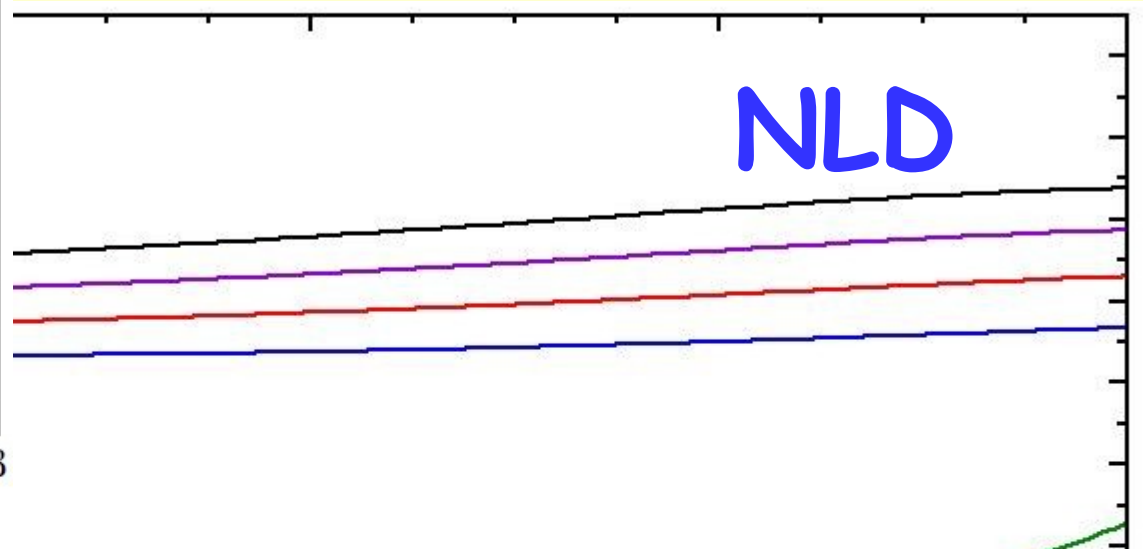
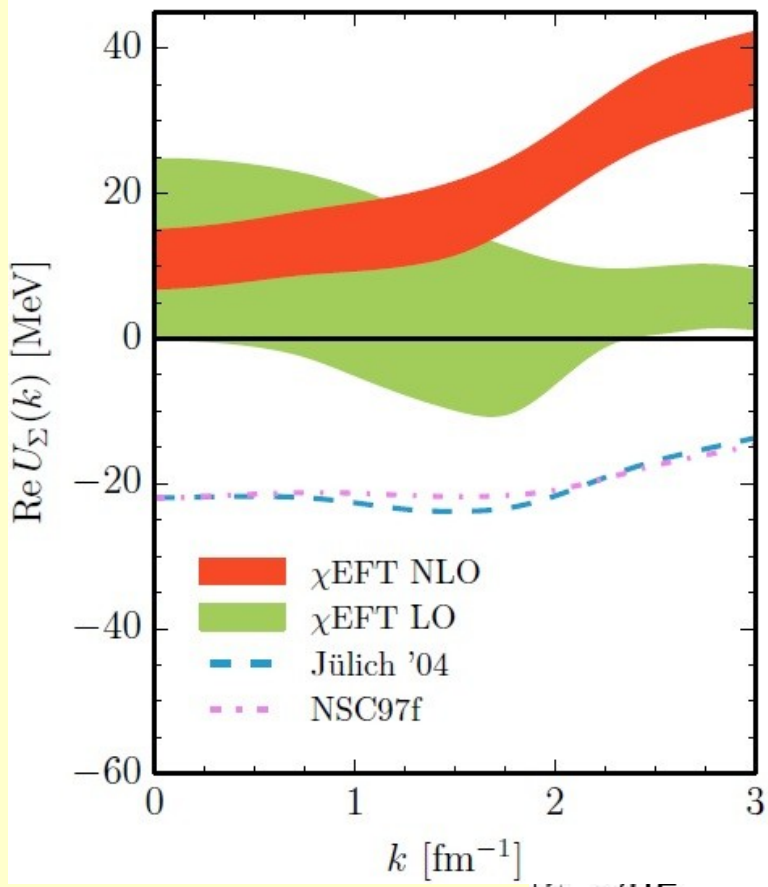
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Σ -opt. potential attractive in conventional RMF+SU(3)
Hyperon (Σ) cut-off Λ in NLD induces repulsion!



NLD results: in-medium Σ -opt. potential...

Σ -opt. potential attractive in conventional RMF+SU(3)
Hyperon (Σ) cut-off Λ in NLD induces repulsion
in good agreement with chiral-EFT



Final remarks & outlook...

➤ NLD model

- keeping simplicity (RMF) to describe complexity (non-linear ρ & p dependences)
- realized by covariant introduction of regulators on a Lagrangian level
- in RMF: cut-off Λ regulates high ρ - & p -components of mean-fields
- cut-off Λ regulates also p -dependence of hyperon opt. pot.!

➤ NLD Results

- EoS soft at low ρ ($K \sim 250$ MeV), but stiff at high ρ
remarkable agreement with microscopic DBHF
- Correct MD for in-medium proton (!) and (!) antiproton interactions
- compatible with all recent observations of high- ρ EoS & NS
- compatible with recent results from chiral-EFT for hyperons in matter

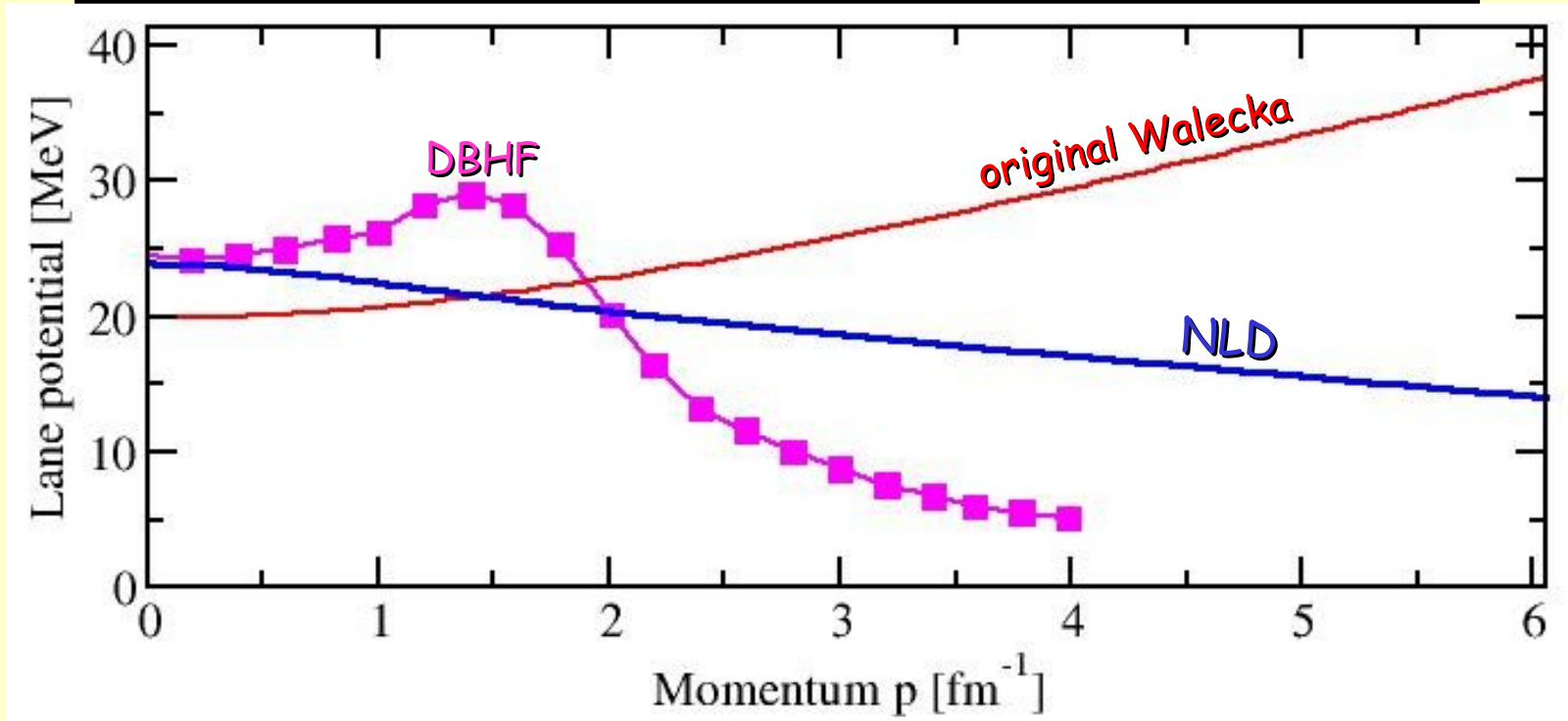
➤ Future/under progress developments

- apply NLD to finite nuclei (done by others...)
- apply NLD to p -nucleus scattering in Eikonal approx. (done, it works)
- apply NLD to heavy-ion collisions (under progress)

Back up slides

Asymmetric nuclear matter: (Lane) optical potential...

$$U_{\text{lane}} = (U_{\text{opt},n} - U_{\text{opt},p}) / (2\alpha), \quad U_{\text{opt}} = \frac{E}{m} \Sigma_v - \Sigma_s + \frac{1}{2m} (\Sigma_s^2 - \Sigma_v^2)$$

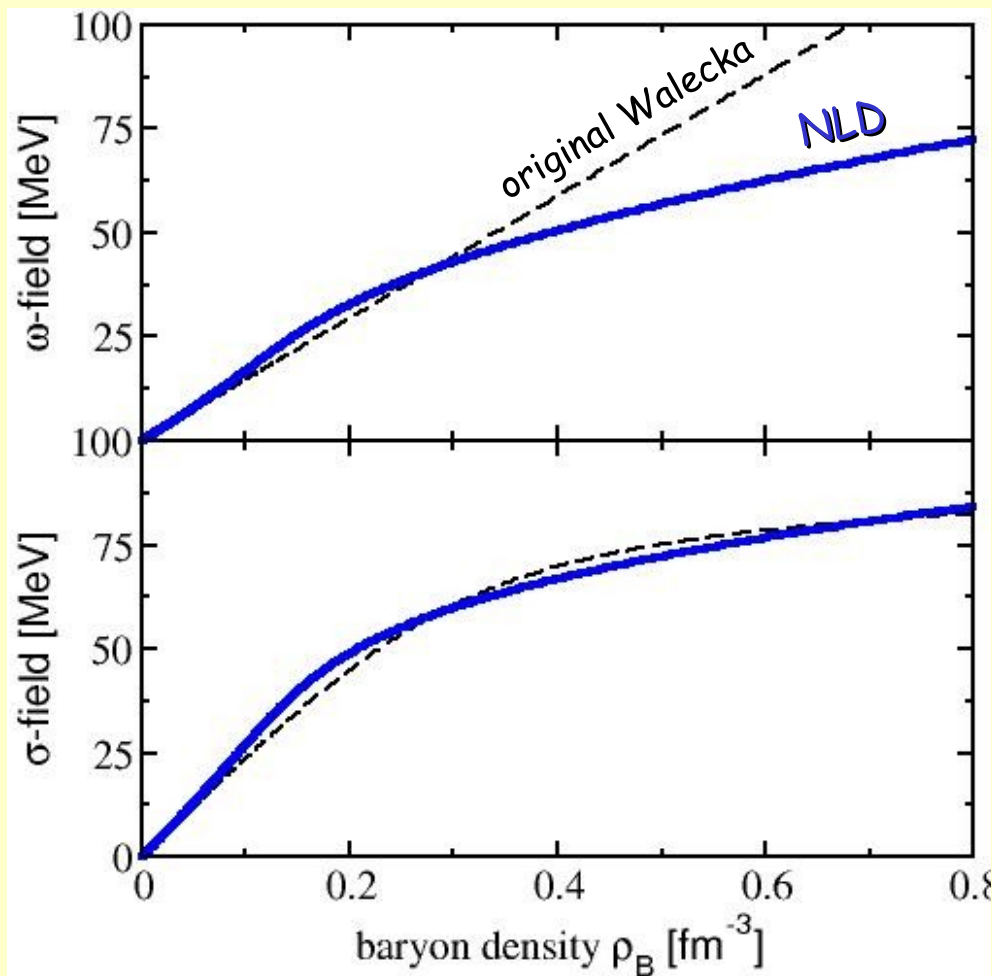


NLD ($\Lambda=0.770$ GeV)

Consistent with microscopic DBHF



Nuclear matter: NLD meson fields (density dependence)...



$$\omega \sim \rho_0 = \frac{\kappa}{(2\pi)^3} \int_{p < p_F} d^3p e^{-\frac{E-m}{\Lambda}}$$



NLD ω field

increasing NL damping with density

$$\sigma \sim \rho_s = \frac{\kappa}{(2\pi)^3} \int_{p < p_F} d^3p \left(\frac{m^*}{E^*} \right) e^{-\frac{E-m}{\Lambda}}$$



NLD σ field

Scalar damping dominates...

NLD effect

Strong vector-field suppression with increasing baryon density
convergent behavior at $\rho_B \rightarrow \infty$