

# *Spin-orbit splittings of neutron states in $N = 20$ isotones from covariant density functionals (CDF) and their extensions.*

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# Motivation: history

Main goal:

Examine isospin/density dependence of **spin-orbit** (SO) in CDFT

The **spin orbit** (SO) force introduced ~ 60 years ago corrects the **shell gaps**.

## Non-Rel. MF

Skyrme: zero range

Gogny: finite range

Both use 2-body SO term - has to be adjusted

## RMF

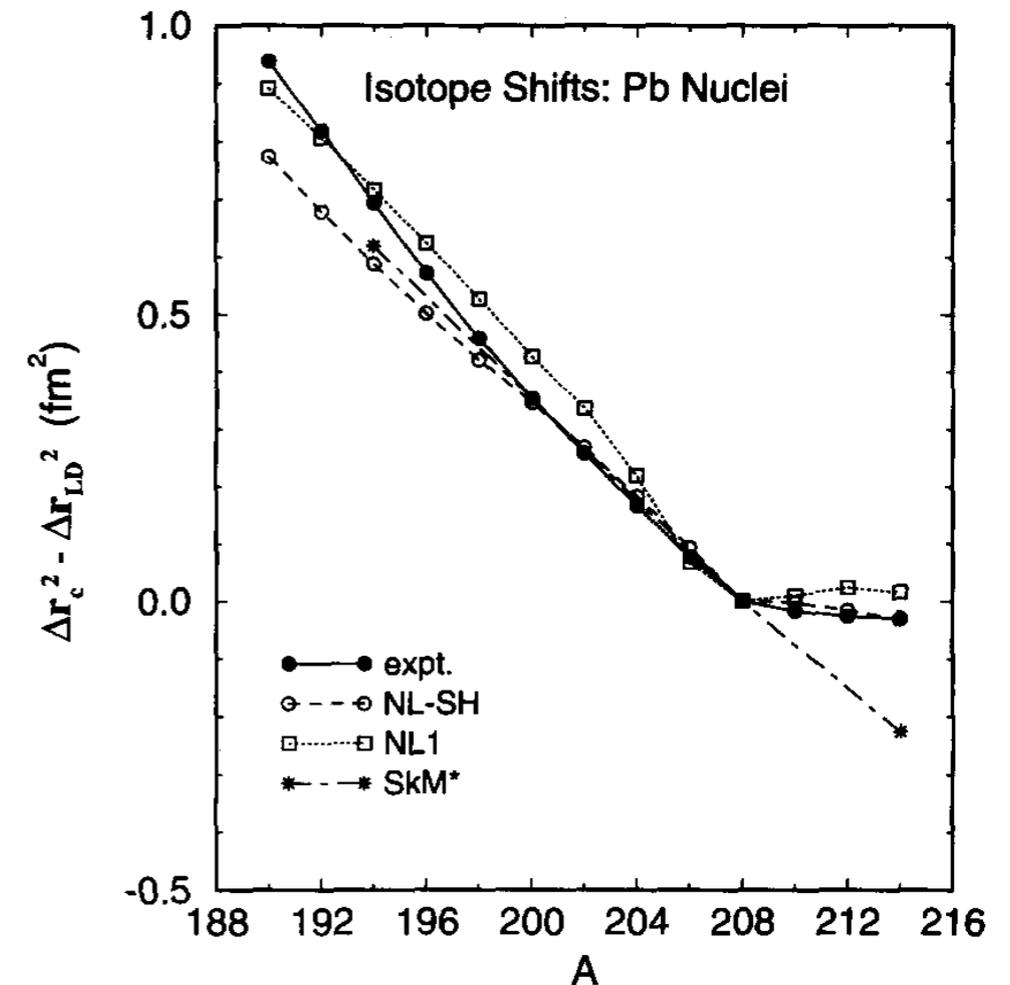
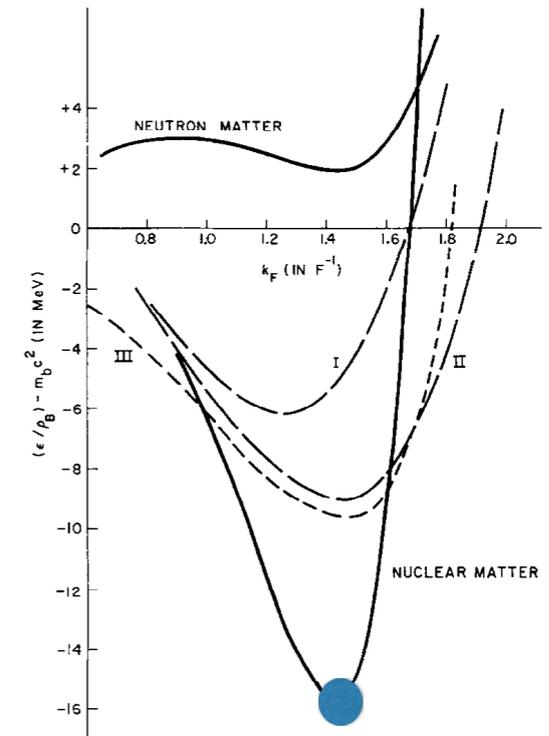
Based on the Walecka meson exchange model

Major advantage SO term naturally from Dirac eq.

Both approaches successful along stability line - similar results

SO difference appear in extreme isospin cases.

Well known example: isotopic shifts in charge radii in Pb



# Motivation: the $^{34}\text{Si}$ bubble experiment

recent experiment effort - constraint on isospin dep. of SO

Study  $N = 20$  isotones  $^{40}\text{Ca}$ ,  $^{36}\text{S}$ ,  $^{34}\text{Si}$

Concentrate on neutron states  $2p = 2p_{1/2} - 2p_{3/2}$   $1f = 1f_{5/2} - 1f_{7/2}$

1st Adv.:  $A \approx 40$   $2p$  splitting around 2MeV and both SO partners accessible

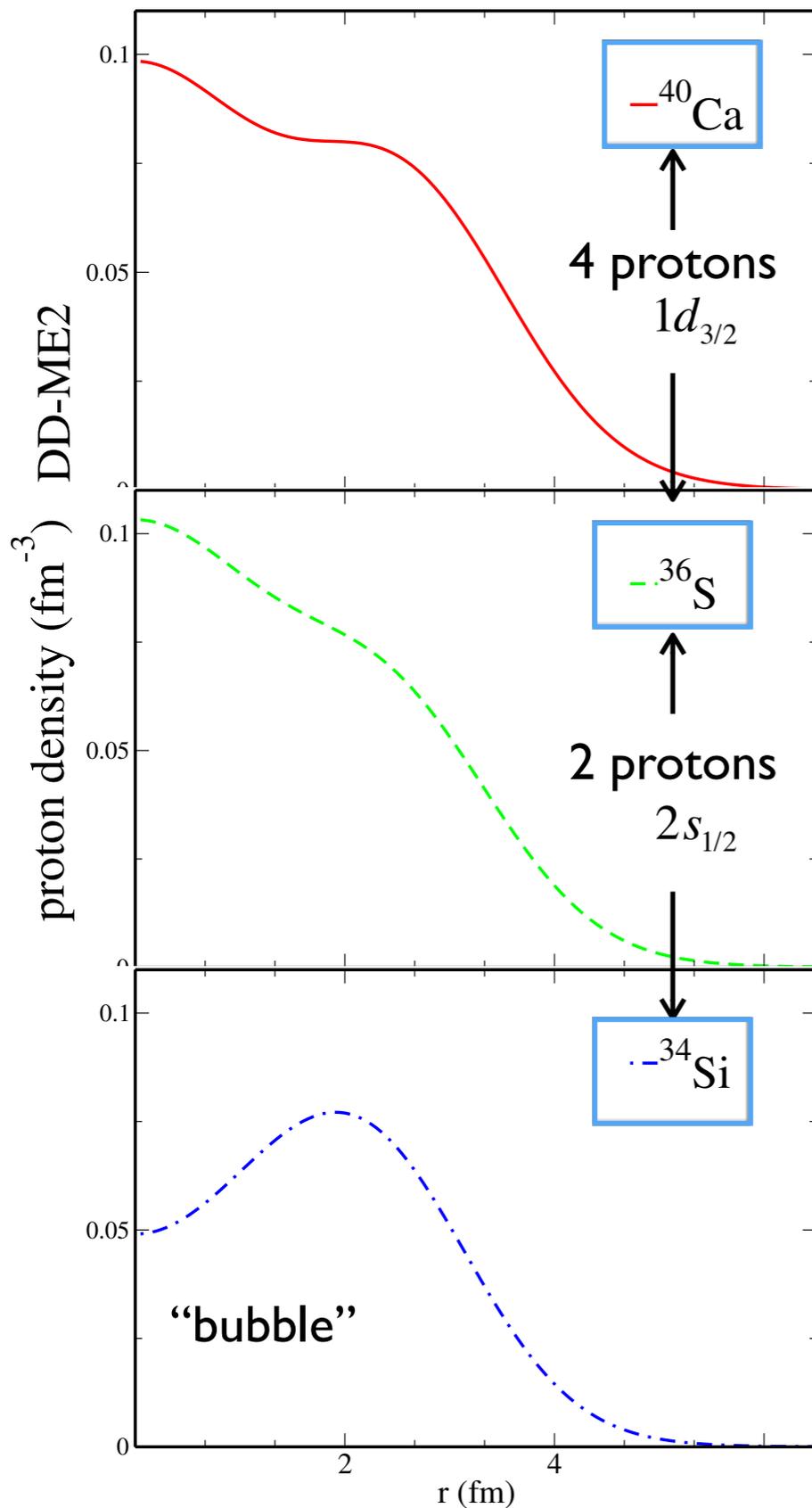
2n Adv.: Occurrence of ‘Bubble’ in  $^{34}\text{Si}$  proton dens.

Theory: [M.Grasso *et.al.*, PRC 79,034318 (2009)]

Expt: [A.Mutschler, O. Sorlin *et.al.* NPHYS3916 (2016)]  
knock-out reaction  $^{34}\text{Si}(-1p)^{33}\text{Al}$

Deduce Occupancy 0.17(3) of  $2s_{1/2}$  only 10% cp to  $^{36}\text{S}$

Since  $\text{SO} \propto \nabla\rho$  see bubble structure influence SO splitting



# Motivation: the $^{34}\text{Si}$ bubble experiment

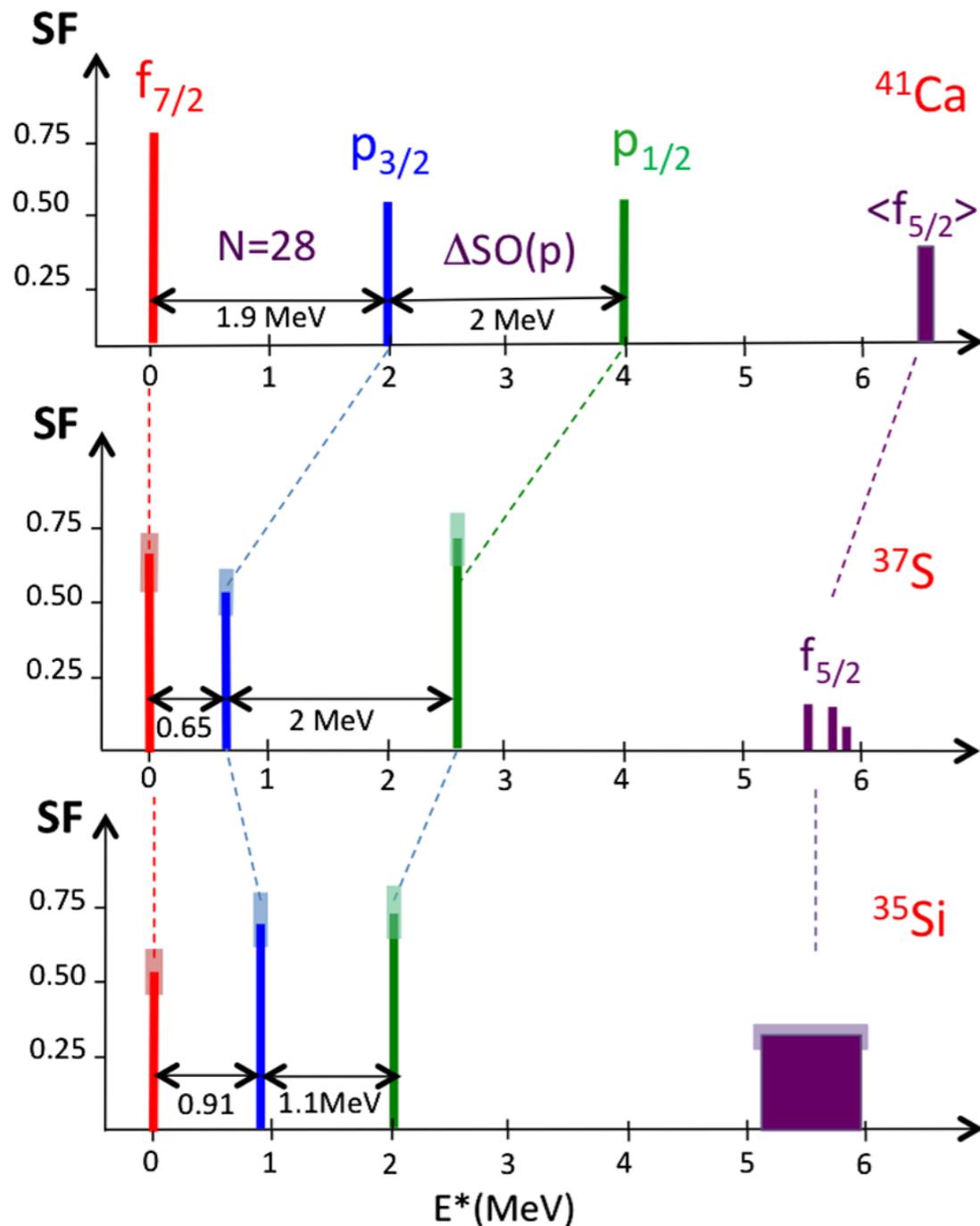


FIG. 3 (color online). Distribution of the *major* fragments of the single particle strength in  $^{41}\text{Ca}$  (top),  $^{37}\text{S}$  (middle), and in  $^{35}\text{Si}$  (bottom). SF values in  $^{41}\text{Ca}$  are taken from Ref. [29]. The centroid of the  $5/2^-$  strength, obtained from a summed SF strength of 0.32, is indicated as  $\langle f_{5/2} \rangle$ . The SF of the  $5/2^-$  components in  $^{37}\text{S}$  are taken from [24], while all others SF are derived from the present work with error bars due to statistics and fit distributions.

## Proposal:

Use the bubble structure to constrain the SO force

Experiment [Burgunder et. al. PRL **112**, 042502

(2014):

transfer reaction  $^{34}\text{Si}(d,p)^{35}\text{Si}$   
Measure neutron s.p. energy with

## Results:

No change in 2p splitting between  $^{41}\text{Ca}$  and  $^{37}\text{S}$   
Large reduction of 2p between  $^{37}\text{S}$  and  $^{35}\text{Si}$   
No significant change for 1f splitting

## Non-Rel. calculations

[M. Grasso *et. al.*, Phys. Rev. **C 92**, 054316  
(2015).]

Using skyrme-SLy5 and Gogny-D1S functionals  
in the MF-HF level

# 1. Pure Mean-Field

## SO term in Mean-Field

**RMF** nucleons 4-cmpt Dirac spinors  $\psi$   
 virtual mesons  $\sigma, \omega, \rho, (\delta)$   
 Nucleons obey Dirac equation

$$(\boldsymbol{\alpha} \cdot \mathbf{p} + \beta(M + S) + V)\psi_i = \varepsilon_i \psi_i,$$

Relativistic fields scalar  $S$  and vector  $V$ . (Time-rev.)  $S = g_\sigma \sigma (+g_\delta \delta) \quad V = g_\omega \omega^0 + g_\rho \tau_3 \rho_3^0 + eA^0.$

In non-Rel. expansion:  $V_{S.O.} = \mathbf{W} \cdot (\mathbf{p} \times \boldsymbol{\sigma})$

$$\mathbf{W} = \frac{1}{2\tilde{M}^2} \nabla(V - S) \quad \text{with} \quad \text{effective mass} \quad \tilde{M} = M - \frac{1}{2}(V - S)$$

Large S.O. term  
 $V - S \approx 750 \text{ MeV}$

In the spherical case:  $V_{S.O.} = \frac{1}{4\tilde{M}^2} \frac{1}{r} \frac{d(V - S)}{dr} \boldsymbol{\ell} \cdot \mathbf{s}$

Under approximation- zero range limit  $\mathbf{W}_\tau = W_1 \nabla \rho_\tau + W_2 \nabla \rho_{\tau' \neq \tau}$

RMF **very small** isospin dep.

Non-Rel. **strong** isospin dep.

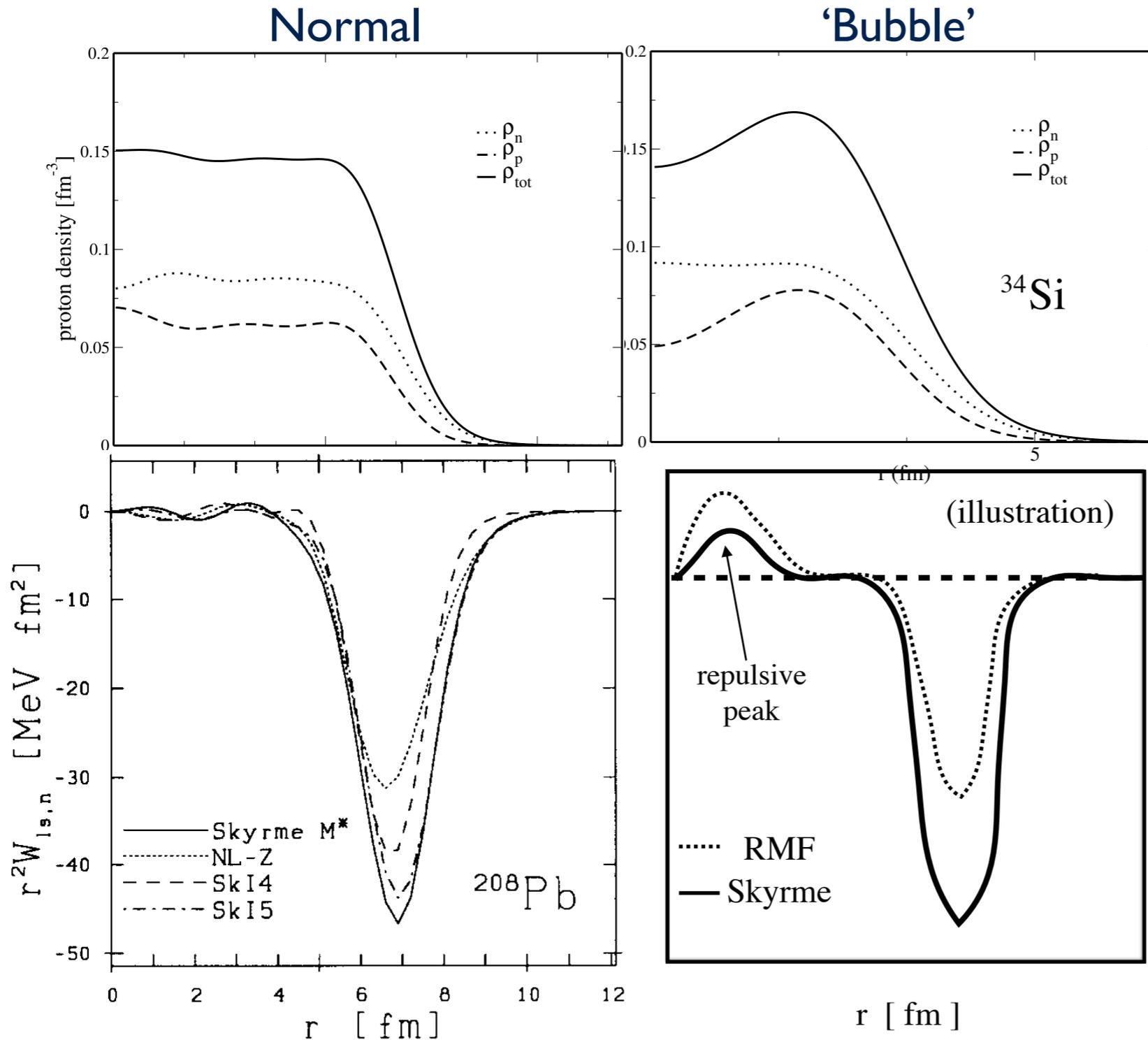
$$\frac{W_1}{W_2} \approx 1 + 2 \frac{C_\rho (+C_\delta)}{C_\omega + C_\sigma} \approx 1$$

$$\frac{W_1}{W_2} = 2.$$

$$C_i = g_i^2 / m_i^2 \quad (i = \sigma, \omega, \delta, \rho)$$

# 1. Pure Mean-Field

## SO term in Mean-Field



Gradient  $\nabla \rho$  determines the SO

### Most nuclei

- Attractive well around the surf.
- States with large  $l$  most affected

Non-Rel. deeper than RMF  $\longleftrightarrow$  Larger SO splittings

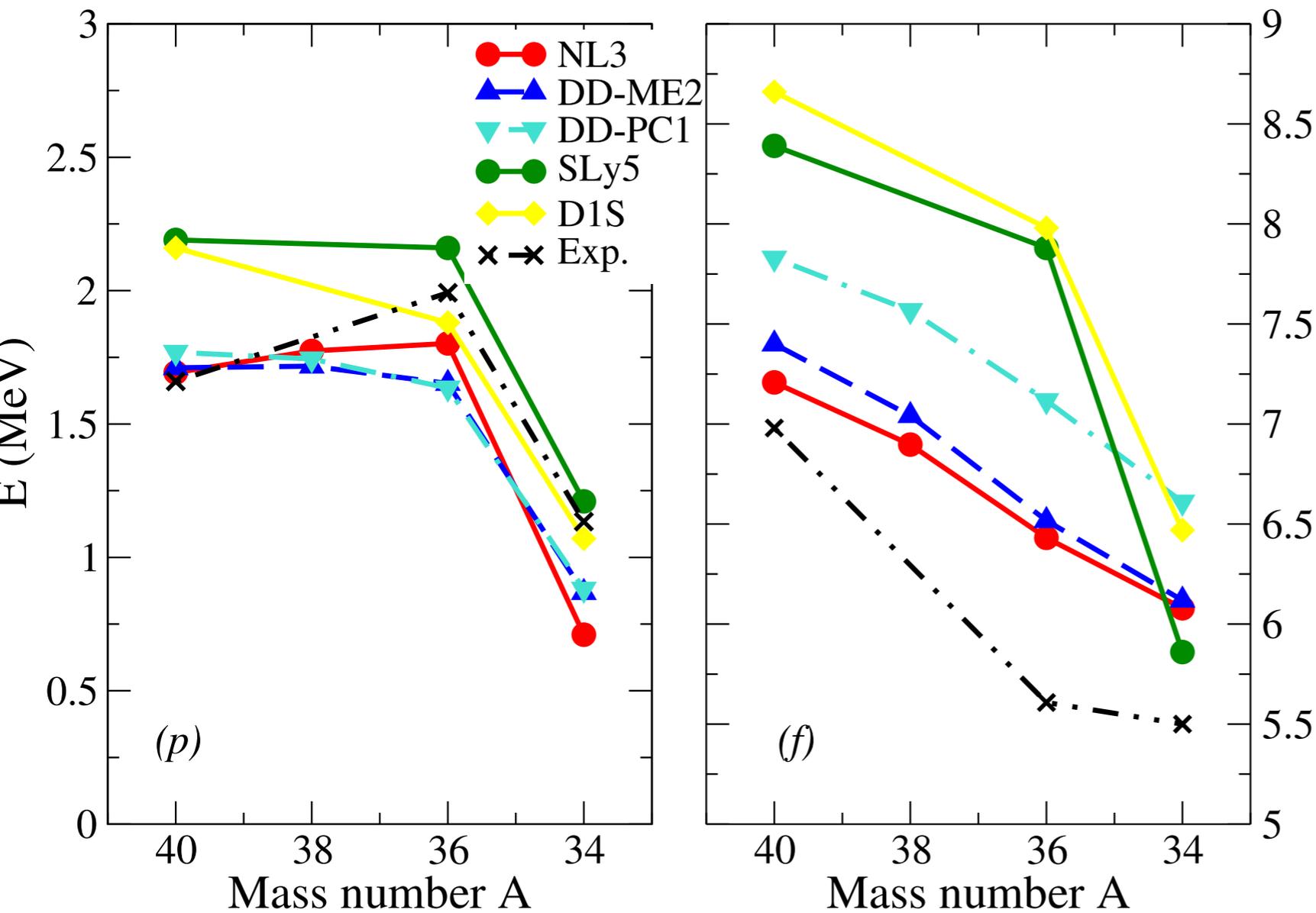
### 'Bubble' nuclei

- Additional repulsive peak at center.
- States with small  $l$  most affected-  
Reduces the size of the 2p splitting

RMF larger peak  $\longleftrightarrow$  Smaller 2p splitting  
Greater reduction

# 1. Pure Mean-Field Numerical results

Evolution of the neutron SO splittings with A



## RMF results in general

- No change in  $2p$  from  $40\text{Ca} \rightarrow 36\text{S}$
- Small, gradual reduction in  $1f$  split.
- Large sudden reduction in  $36\text{S} \rightarrow 34\text{Si}$  'bubble'

## Differences with non-Rel. and expt.

- Smaller size in all SO splittings
- $f$  split. similar in  $34\text{Si}$  because of the rel. reduction in SLy5-D1S
- Larger relative reduction of the  $2p$  splitting in the  $36\text{S} \rightarrow 34\text{Si}$  transition

# 2. The effect of pairing

## *RHB framework*

Pairing correlation in proton channel of  $^{38}\text{Ar}$ ,  $^{36}\text{S}$  and  $^{34}\text{Si}$   
Generalised Hartree-Bogolyubov framework of quasi-particles

$$\begin{pmatrix} \hat{h} - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h} + m + \lambda \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}$$

Single particle energies obtained in canonical basis  
RHB equivalent to RMF + BCS

$$v_\mu^2 = \frac{1}{2} \left[ 1 - \frac{\varepsilon_\mu - \lambda}{\sqrt{(\varepsilon_\mu - \lambda)^2 + \Delta_\mu^2}} \right]$$

Use TMR pairing force - equivalent to Gogny finite range - avoid cutoff

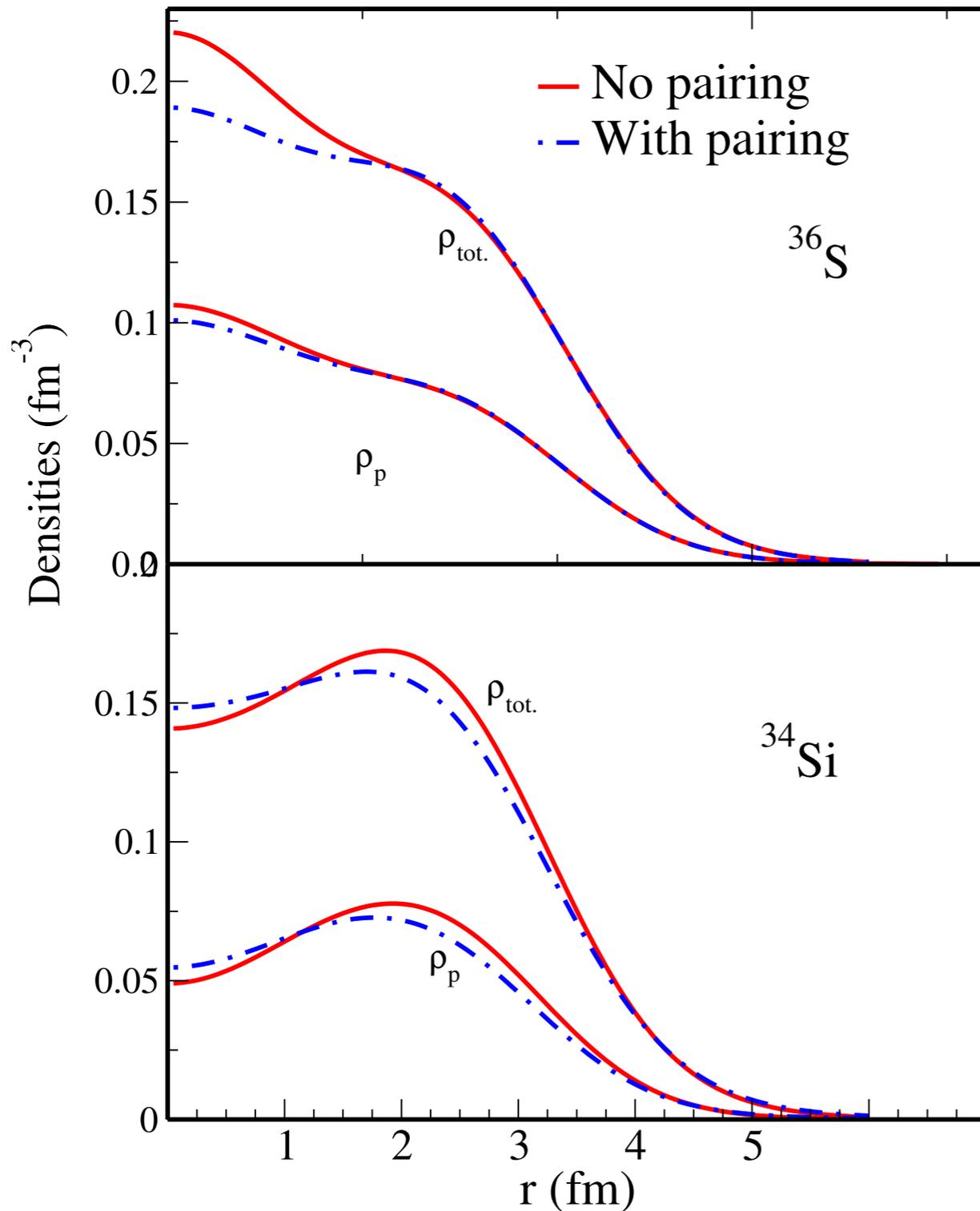
Pairing strength adjusted to OES

	$^{38}\text{Ar}$	$^{36}\text{S}$	$^{34}\text{Si}$
$\Delta_C^{(3)}$ (MeV)	0.93	0.45	1.95

# 2. The effect of pairing

## *Proton densities*

Pairing effect on densities

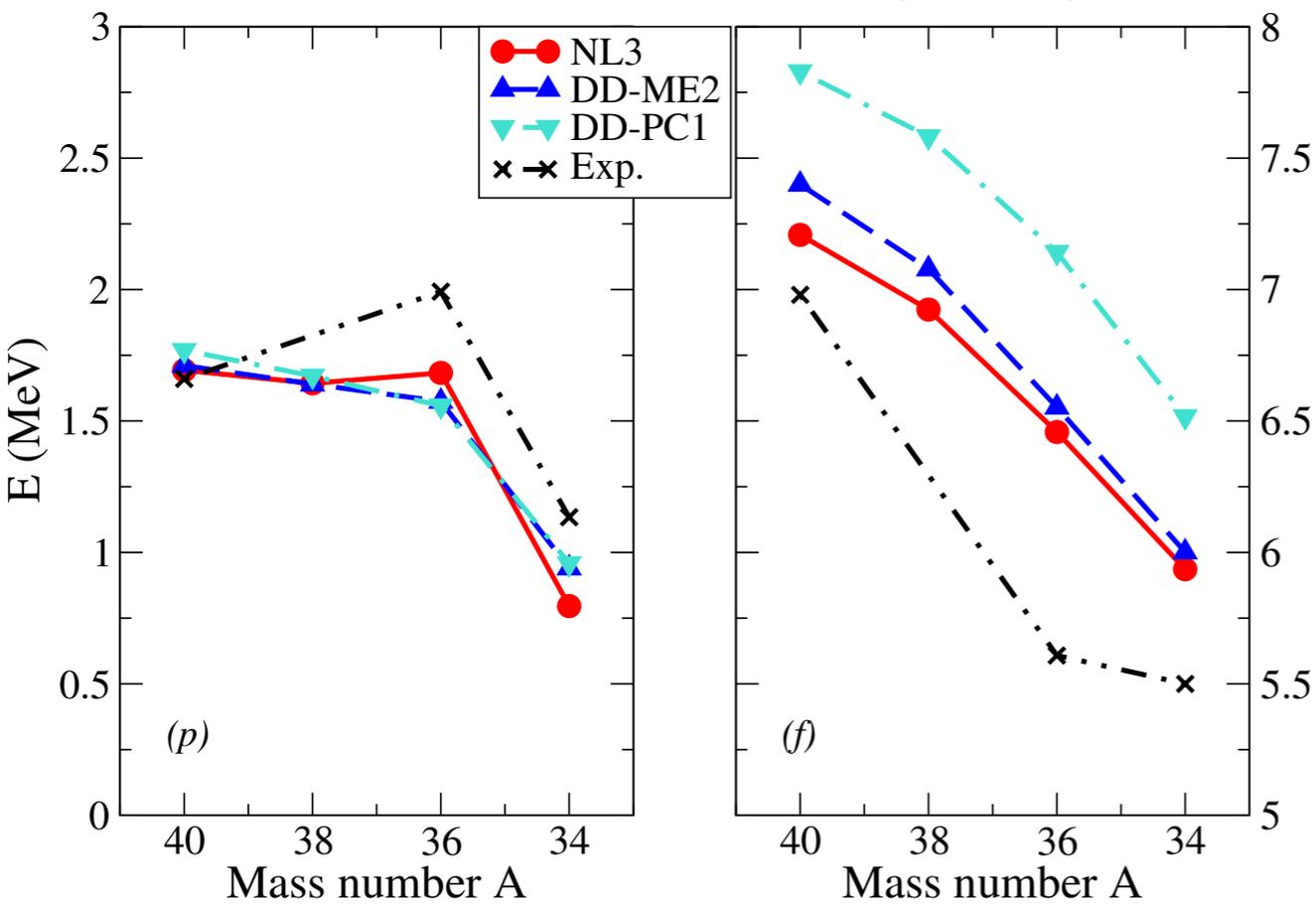


- Occupancy of  $2s1/2$ : decreased in  $^{36}\text{S}$   
increased in  $^{34}\text{Si}$
- Larger surface diffusion
- Smoothing of the Bubble structure

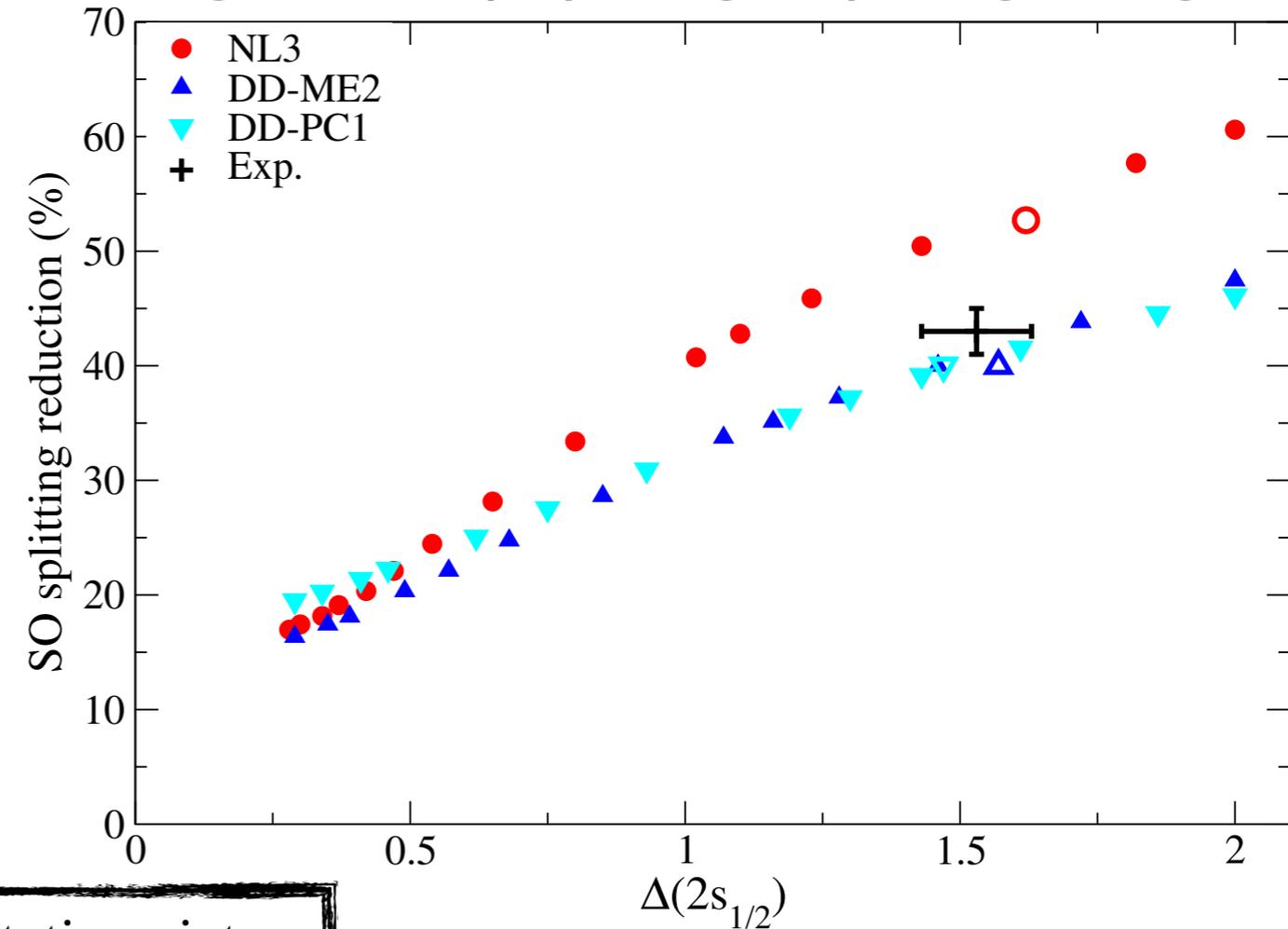
# 2. The effect of pairing

## Numerical results

Evolution of the neutron SO splittings with A



Change of the 2p splitting wt pairing strength



Same qualitative picture

### The inclusion of pairing

- In  $^{38}\text{Ar}$  and  $^{36}\text{S}$  increases  $f$  and reduces  $p$  splitting
- In  $^{34}\text{Si}$  reduces  $f$  and increases  $p$ , due to smaller central depletion

### Connection bt relative $p$ reduction and $\Delta(2s_{1/2})$

- Stronger pairing, smaller occupancy change, smoothening of 'bubble' -> Smaller relative  $p$  reduction  $^{36}\text{S} \rightarrow ^{34}\text{Si}$

# 3. The effect of the Tensor

## One pion exchange (OPE)

Rel. Hartree → Relativistic Hartree-Fock

G.A Lalazissis *et. al.* PRC80, 041301(R)(2009)

Add two terms corresponding to  
(OPE) dynamics

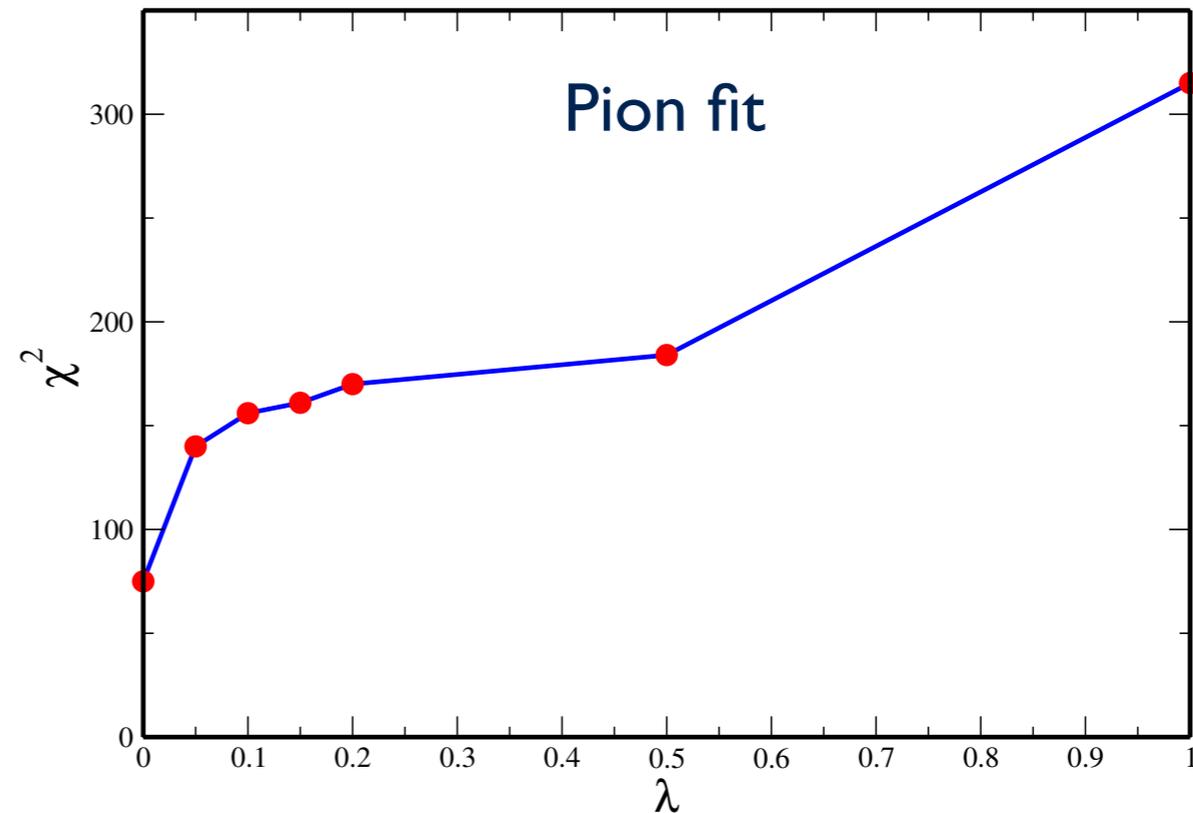
$$\mathcal{L}_\pi = \frac{1}{2} (\partial_\mu \vec{\pi} \partial^\mu \vec{\pi} - m_\pi^2) \vec{\pi}^2$$

$$\mathcal{L}_{pv} = -\frac{f_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \partial^\mu \vec{\pi} \vec{\tau} \psi$$

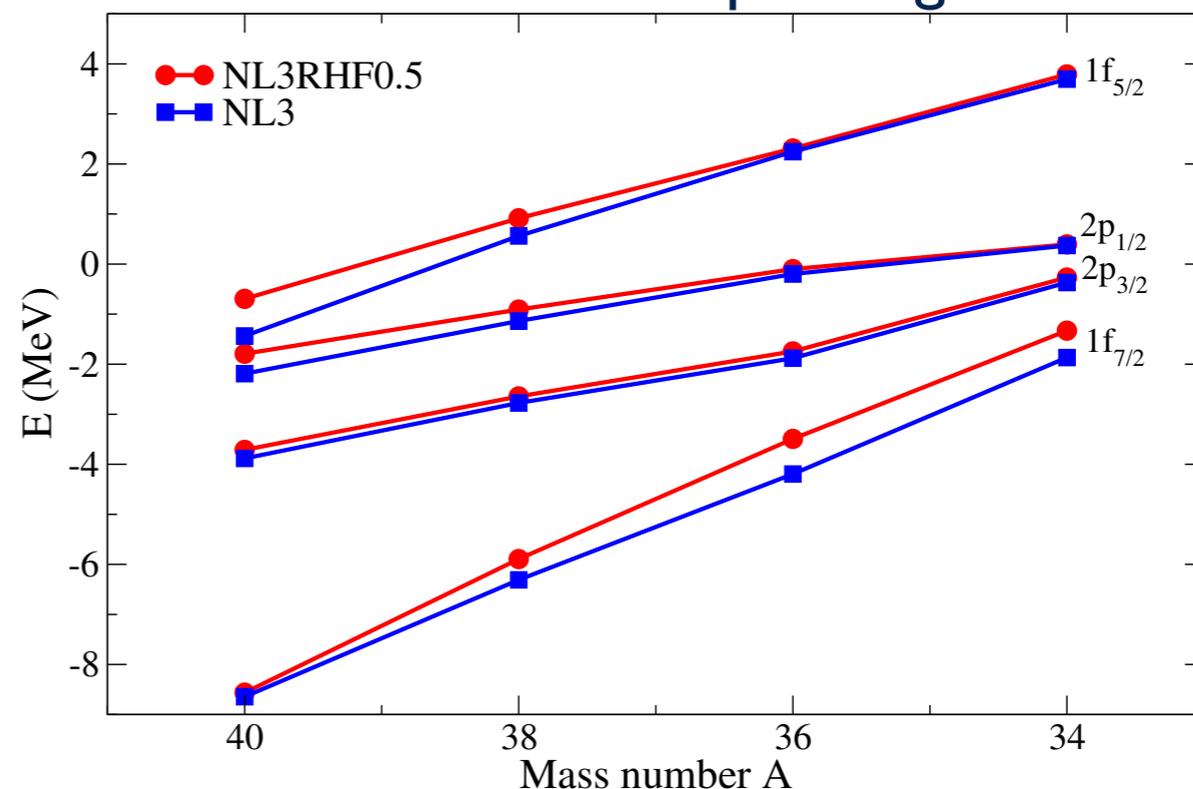
$$f_\pi^2 = \lambda f_\pi^2{}^{free}$$

- $\lambda = 0$ : Optimal refit incl. OPE of NL3
- $\lambda = 0.5$ : reproduces certain expt data of s.p. structure in Sn isotopes
- effect of tensor force bt neutrons and protons

determined by spin-orbit alignment  
antiparallel spins → attraction  
parallel spins → repulsion



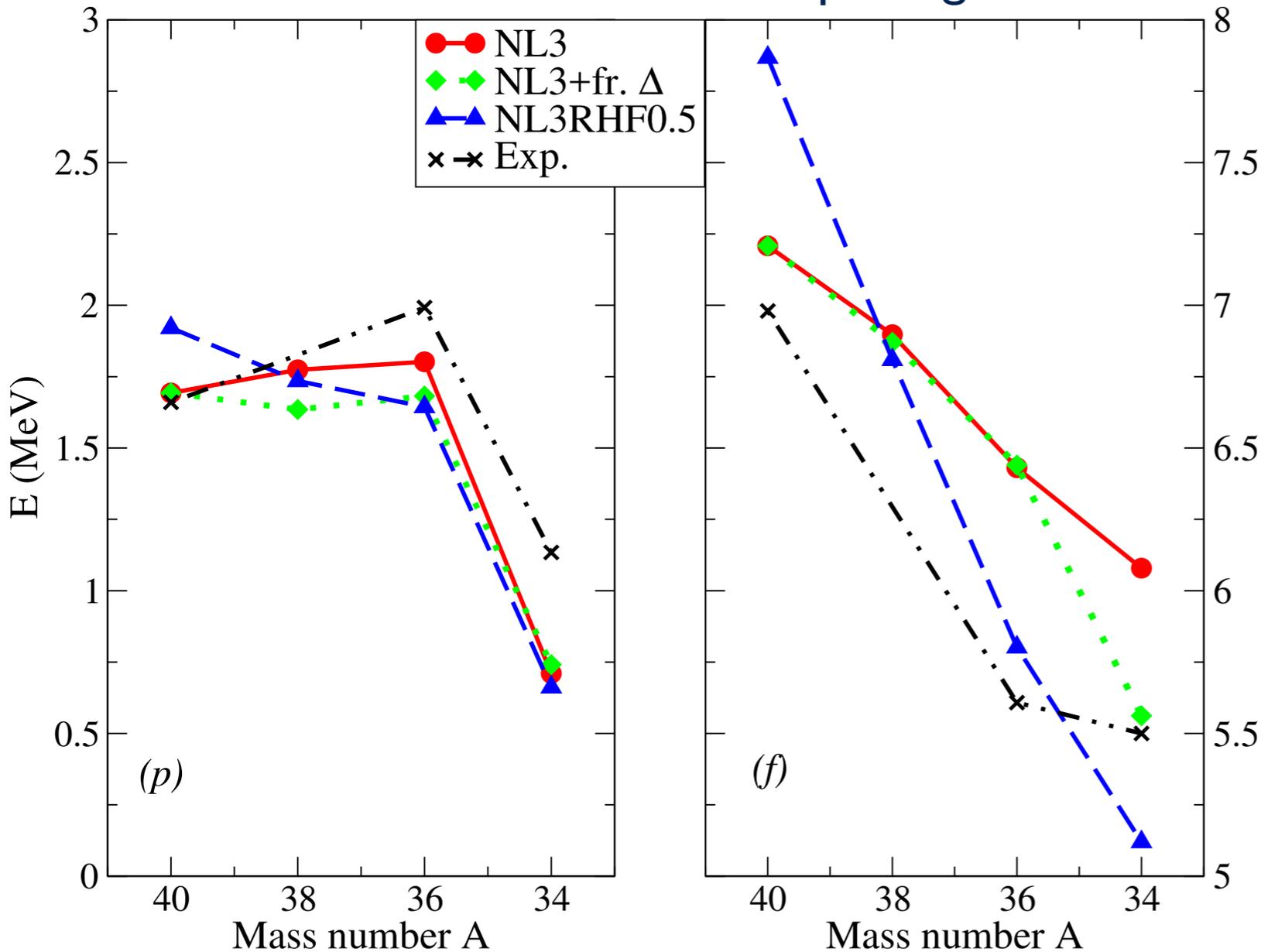
Tensor effect on s.p. energies



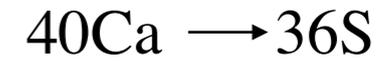
# 3. The effect of the Tensor

## Numerical results

Evolution of the neutron SO splittings with A



### Comparison with RH



Enhanced quenching of the *f* and *p* spl.



*f* splitting much smaller  
*p* spl. slightly decreased - Pure SO eff.

A frozen gap approximation for pairing  
 Occupancy of  $2s_{1/2}$  in  $34\text{Si}$  g.t. RH  
 Opposite effect than pairing

# 4. The effect of Particle Vibrational coupling-(PVC)

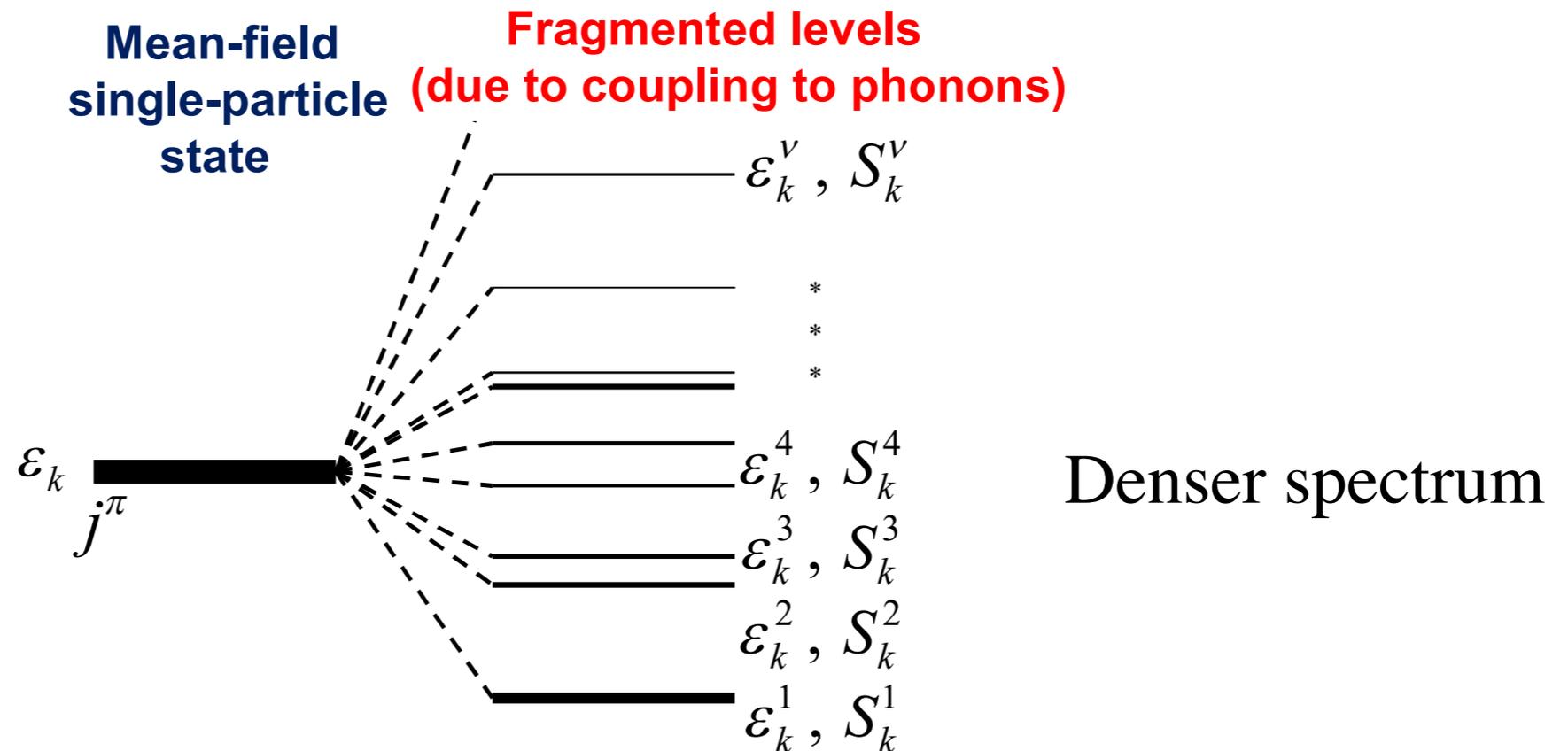
1st step static DFT 2nd step collective vibrations

PVC induces energy dependence on eff. potential

$$\Sigma = S + V + \Sigma(\omega) \quad \omega \text{ phonon frequency}$$

Mean field

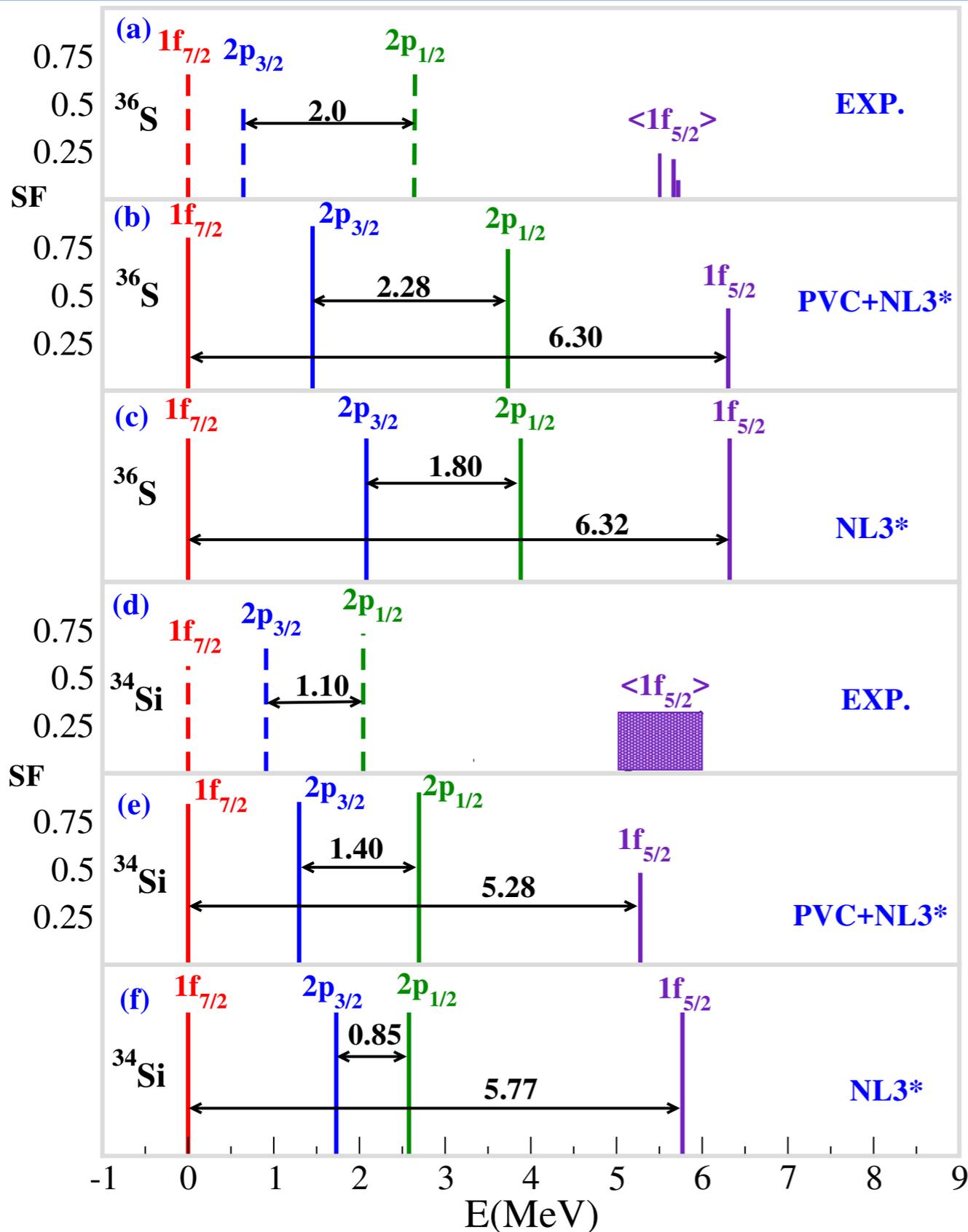
Pole part



$S_k^\nu$  Single particle strength, sum rule:  $\sum_{\nu} S_k^\nu = 1$

# 4. The effect of Particle Vibrational coupling

## Numerical Results



Direct comparison with expt.  
major fragments

No big effect on  $1f$  splitting

$2p$ -orbits shifted downwards  
more for orbits close to fermi  
increase in the size

rel. reduction closer to expt.

# Conclusions

- In general the observed qualitative picture is reproduced
  - Large sudden reduction on  $2p$  splitting going to the 'bubble'
  - No significant change otherwise
- Smaller isospin dependence leads to smaller SO splitting than non-Rel.
- Quantitatively relative reduction larger than expt.
- Pairing correlations correct for that
- Tensor effects mainly  $1f$ , showing the pure SO character of the  $2p$  reduction, acts opposite than pairing
- PVC improves the qualitative picture.
  - Shifts the  $2p$  orbits in the correct direction
  - $2p$  Relative reduction in  $36\text{S} \rightarrow 34\text{Si}$  closer to experiment