AMIAS: A Model Independent Analysis Scheme

From Hadronics to Medical Imaging

Efstathios Stiliaris, C.N. Papanicolas

Section of Nuclear and Particle Physics, Department of Physics
National & Kapodistrian University of Athens
&
The Cyprus Institute, Nicosia, Cyprus

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H(e,e′p)π⁰ Measurements at the Δ(1232) Resonance

OOPS Collaboration, \( Q^2 = 0.127 \text{ (GeV/c)}^2 \)

Begin of the story
MIT 2004
Workshop on The Shape of Hadrons
2004 MIT, USA
2006 Athens, GR

Overview: The Shape of Hadrons

A. M. Bernstein* and C.N. Papanicolas†

*Department of Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139, USA

†Institute of Accelerating Systems and Applications, Athens, Greece
and Department of Physics, University of Athens, Greece

N.F. Sparveris et al. PRL 94 (2005) 022003
How to extract model independent amplitudes from experimental data?
The signal for deformation in the $\gamma^* N \rightarrow \Delta$ transition

$p(qqq)$

$I = \frac{1}{2}$  \hspace{1cm} $J = \frac{1}{2}$

938 MeV

$\Delta(qqq)$

$I = \frac{3}{2}$  \hspace{1cm} $J = \frac{3}{2}$

1232 MeV

Spherical  $\Rightarrow$  M1

Deformed  $\Rightarrow$  M1, E2, C2  \hspace{1cm} Deformation signal

\[
\begin{align*}
\text{CMR} &= \text{Re} \left( \frac{S_{1+}^{3/2}}{M_{1+}^{3/2}} \right) \\
\text{EMR} &= \text{Re} \left( \frac{E_{1+}^{3/2}}{M_{1+}^{3/2}} \right)
\end{align*}
\]
Out of Plane Spectroscopy

\[ \sigma = J_\Omega \Gamma_v \frac{p_{cm}}{k_{cm}} ( R_T + \epsilon_L R_L + \epsilon R_{TT} \cos 2\phi_{X\gamma} - \nu_{LT} R_{LT} \cos \phi_{X\gamma} - \hbar \nu_{LT}' R_{LT}' \sin \phi_{X\gamma} ) \]

\[ R_T + \epsilon L R_L = \frac{\sigma_0 + \sigma_{\pi/2} + \sigma_\pi + \sigma_{3\pi/2}}{4K} \]

\[ R_{TT} = \frac{\sigma_0 - \sigma_{\pi/2} + \sigma_\pi - \sigma_{3\pi/2}}{4K\epsilon} \]

\[ R_{LT} = \frac{\sigma_\pi - \sigma_0}{2K\nu_{LT}} \]

\[ R_{LT}' = \frac{\sigma_{3\pi/2} - \sigma_{\pi/2}}{2K\hbar\nu_{LT}'} \]
Out of Plane Spectroscopy

\[ H(e,e'p)\pi^0 \]

\[
\sigma = J_\Omega \Gamma_v \frac{p_{cm}}{k_{cm}} \left( R_T + \epsilon_L R_L + \epsilon R_{TT} \cos 2\phi_{X\gamma} \right.
\]
\[
- \nu_{LT} R_{LT} \cos \phi_{X\gamma} - h \nu'_{LT} R'_{LT} \sin \phi_{X\gamma} \left. \right) \]

\[
R_T + \epsilon_L R_L = \frac{\sigma_{\pi/4} + \sigma_{5\pi/4}}{2K}
\]
\[
R_{LT} = \frac{\sigma_{3\pi/4} - \sigma_{\pi/4}}{\sqrt{2K} \nu_{LT}}
\]
\[
R'_{LT} = \frac{\sigma_{5\pi/4} - \sigma_{3\pi/4}}{\sqrt{2K} \nu'_{LT}}
\]
OOPS Spectrometer
MIT-Bates Linear Accelerator

A1 Spectrometer
MAMI – Mainz, Germany

Proposed and designed by C.N. Papanicolas
Model Errors

- Extracted amplitudes and their ratios (EMR, CMR) are characterized by statistical, systematic and model error.
- Model error often dominates.
- So far we have only guestimates, at best!

A Model Independent Analysis Scheme

AMIAS

Based on statistical concepts and Monte Carlo techniques

A Simple AMIAS Example: Fitting a Straight Line

• Theory: \( y = A_0 + A_1 x \)
  \( \{A_{\mu}\} = (A_0, A_1) \)

• Pseudodata

Generated with:

\( A_0 = 7.0 \pm 0.8 \)
\( A_1 = 1.0 \pm 0.2 \)

How to analyze them with AMIAS?
A Simple AMIAS Example: Fitting a Straight Line

\[ \chi^2 = 2783 \]

\[ \exp\{-\chi^2/2\} \]

\[ Y_i = \text{YY}_i \]

\[ N = 100 \]
Multipole Expansion

\[ F_1 (W, z) = \sum_{l=0}^{\infty} \left[ \left( M_{l+1}(W) + E_{l+1}(W) \right) P_{l+1}^i(z) + \left( (l+1) M_{l-1}(W) + E_{l-1}(W) \right) P_{l-1}^i(z) \right] \]

\[ F_2 (W, z) = \sum_{l=1}^{\infty} \left[ \left( (l+1) M_{l+1}(W) + M_{l-1}(W) \right) P_{l}^i(z) \right] \]

\[ F_3 (W, z) = \sum_{l=1}^{\infty} \left[ \left( (E_{l+1}(W) - M_{l+1}(W)) P_{l+1}^n(z) + \left( E_{l-1}(W) + M_{l-1}(W) \right) P_{l-1}^n(z) \right) \right] \]

\[ F_4 (W, z) = \sum_{l=2}^{\infty} \left[ \left( M_{l+1}(W) - E_{l+1}(W) - M_{l-1}(W) - E_{l-1}(W) \right) P_{l}^n(z) \right] \]

\[ F_5 (W, z) = \sum_{l=0}^{\infty} \left[ \left( (l+1) L_{l+1}(W) P_{l+1}^i(z) - l L_{l-1}(W) P_{l-1}^i(z) \right) \right] \]

\[ F_6 (W, z) = \sum_{l=1}^{\infty} \left[ \left( l L_{l-1}(W) - (l+1) L_{l+1}(W) \right) P_{l}^i(z) \right]. \]

**Chew-Goldberger-Low-Nambu (CGLN) Amplitudes**

E. Amaldi, S. Fubini and G. Furlan: *Pion-Electroproduction* (1979) Springer Verlag
$E_{L^+}, E_{L^-}, M_{L^+}, M_{L^-}, L_{L^+}, L_{L^-} \quad 0 \leq L \leq L_{cut}$

F1, F2, F3, F4, F5, F6 \ (CGLN)

Response Functions: $R_T, R_L, R_{TT}, R_{LT}, \ldots$

OBSERVABLES
L = 0...5
Total = (36-5) Complex Multipoles \{A_1, A_2, ... A_{31}\}

Random Variation of ALL Amplitudes \(A_i\)
(uniformly \(\pm 1\sigma, \pm 2\sigma, \ldots\))
- Unitarization -

Experimental Data

Calculation of Cross Sections

Calculation of \(\chi^2\)

Analysis
- Sensitivity
- Correlations

Ensemble of Solutions \(\{A_1, A_2, ... A_{31}\}, \chi^2\)

Results
\(A_i\) Central Value
\(\delta A_i\) Uncertainty
Sensitivity on Amplitude $A_i$

$A_1, \ldots, A_i, \ldots, A_{31}, \chi^2$

$A_i$ is uniformly distributed

$\chi^2$ versus $A_i$ Plot

$A_i$ is uniformly distributed
Applying $\chi^2$ Cut on **SENSITIVE** Amplitude $A_i$

$A_i$ Distribution

![Graph showing $A_i$ distribution with projections for different $\chi^2$ cuts: $\chi^2 < 200$, $\chi^2 < 120$, $\chi^2 < 80$, $\chi^2 < 40$.](image)
Applying $\chi^2$ Cut on NON SENSITIVE Amplitude $A_i$

$\chi^2 < 200$

$\chi^2 < 80$

$\chi^2 < 120$

$\chi^2 < 40$

$A_i$ Distribution
<table>
<thead>
<tr>
<th>E0+</th>
<th>E0-</th>
<th>M0+</th>
<th>M0-</th>
<th>L0+</th>
<th>L0-</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1+</td>
<td>E1-</td>
<td>M1+</td>
<td>M1-</td>
<td>L1+</td>
<td>L1-</td>
</tr>
<tr>
<td>E2+</td>
<td>E2-</td>
<td>M2+</td>
<td>M2-</td>
<td>L2+</td>
<td>L2-</td>
</tr>
<tr>
<td>E3+</td>
<td>E3-</td>
<td>M3+</td>
<td>M3-</td>
<td>L3+</td>
<td>L3-</td>
</tr>
<tr>
<td>E4+</td>
<td>E4-</td>
<td>M4+</td>
<td>M4-</td>
<td>L4+</td>
<td>L4-</td>
</tr>
<tr>
<td>E5+</td>
<td>E5-</td>
<td>M5+</td>
<td>M5-</td>
<td>L5+</td>
<td>L5-</td>
</tr>
</tbody>
</table>

**FREE PARAMETER**

- \( N \)
- \( m \): fitted
- \( n \): fixed
- \( N-m-n \): randomly varying

**MODEL DEPENDENT FIT**

- \( m = 3 \)
- \( n = N - 3 \)
- 0 randomly varying

**MODEL INDEPENDENT ANALYSIS**

- \( m = 0 \)
- \( n = 0 \)
- \( N \): randomly varying
AMIAS: Sensitivity Analysis
Bates-Mainz Data \((Q^2=0.127 \text{ (GeV/c)}^2, W=1232 \text{ MeV})\)

### Extracted Values

<table>
<thead>
<tr>
<th>Multipole</th>
<th>Extracted Value</th>
<th>Relative Error</th>
<th>MAID-2003</th>
<th>Sato &amp; Lee</th>
<th>DMT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{1+})</td>
<td>27.24 ± 0.20</td>
<td>0.73 %</td>
<td>27.464</td>
<td>27.661</td>
<td>27.489</td>
</tr>
<tr>
<td>(L_{1+})</td>
<td>0.82 ± 0.09</td>
<td>17.7 %</td>
<td>1.000</td>
<td>0.672</td>
<td>0.986</td>
</tr>
<tr>
<td>(L_{0+})</td>
<td>2.23 ± 0.41</td>
<td>18.4 %</td>
<td>2.345</td>
<td>1.008</td>
<td>1.994</td>
</tr>
<tr>
<td>(E_{0+})</td>
<td>3.44 ± 0.70</td>
<td>20.3 %</td>
<td>2.873</td>
<td>2.213</td>
<td>3.206</td>
</tr>
<tr>
<td>(E_{1+})</td>
<td>1.16 ± 0.32</td>
<td>24.1 %</td>
<td>1.294</td>
<td>1.288</td>
<td>1.401</td>
</tr>
</tbody>
</table>

### Probability Distributions

- \(L_0^+ \text{ vs } L_1^+\)
- \(E_0^+ \text{ vs } E_1^+\)
- \(M_1^+ \text{ vs } L_1^+\)

### Correlations

- \(L_0^+ \text{ vs } L_1^+\)
- \(E_0^+ \text{ vs } E_1^+\)
Bates-Mainz Data ($Q^2=0.127\ (GeV/c)^2, W=1232\ MeV)$
Mainz $W=1232$ Photoproduction Data

Model Independent Analysis
Lefteris Markou (CyI)

Model Predictions:

- **SAID (PR15):** -2.2
- **SAID (CM12):** -1.9
- **MAID07:** -2.1

Recoil polarization measurements for neutral pion electroproduction at $Q^2 = 1$ (GeV/$c$)$^2$ near the $\Delta$ resonance

Jlab Hall A Data $\rightarrow$ Vasileios Hantzikos (UoA)
AMIAS and Lattice QCD

Spectral decomposition of the hadron propagator → QCD Correlators

\[
\bar{C}(t_j) = \sum_{n=0}^{\infty} A_n e^{-E_n t_j}
\]

Conventional Fit

How many terms?

Correlations?

AMIAS has been successfully tested in the determination of hadron excited states in Lattice QCD applied to the nucleon.

AMIAS has been successfully tested in the determination of hadron excited states in Lattice QCD applied to the nucleon.

AMIAS in the Emission Tomography

\[ I_\theta(r) = I_0 \cdot e^{-\int_{L_r,\theta} \mu(x,y) \, ds} \]

The usual solution: Radon Transform

\[ p(r, \theta) = \mathcal{R}\{f(x, y)\} \]

\[ \int_{-\infty}^{\infty} f(r \cdot \cos \theta - s \cdot \sin \theta, r \cdot \sin \theta + s \cdot \cos \theta) \, ds. \]
Tomographic Reconstruction

Algebraic Reconstruction Methods

\[ R_i = \sum_{j=1}^{N^2} P_{ij} \times Q_j \]

NP: Number of Projections (Angles)
NR: Number of Rays

\[
R(NP \times NR) = P(NP \times NR, N^2) \times Q(N^2)
\]

Calculated Rays  Projection Matrix  Reconstructed Matrix
AMIAS in the Emission Tomography

PHANTOM

AMIAS

ART

MLEM

Background = 0.5 %

Loizos Koutsantonis (Cyl)
Nuclear Imaging Technologies in Tomography

1\textsuperscript{st} \(\gamma\)-Camera System (Resistive Chain Technique) Lab-Prototype

WP1 Mechanical Construction & Electronics

Clinical Prototype (Breast Imaging)

- PhD Thesis (Zioga)
- Patent Application

2\textsuperscript{nd} \(\gamma\)-Camera System (PSPMT Individual Anode Readout Scheme)

WP2 Improve Sensitivity

WP3 Add Optical Modality

Coincidence Electronics Interface / Development Readout and DAQ System

Prototype Compton Camera

- PhD Thesis (Mikeli)
- Lab Prototype

Dual-Modality SPECT and Optical Computed Tomography

- PhD Thesis (Rapsomanikis)
- Lab Prototype

WP4 Construction Two-Head PET Detector

Dual-Panel PET Detector

Clinical Applications

- Planar Breast Imaging (LAIKO & IATRIKO Hospital of Athens)
- Positron Emission Mammography (PEM)

SPECT Lab Athens
Conclusions

The AMIAS Method is demonstrated to:

- Be model independent.
- Extract maximum information for all available Multipoles, without any bias; it is capable of ranking them in order of significance.
- Account for the correlations amongst the contributing Multipoles, and to provide an easy visualization of them.
- Yield uncertainties which have a precise meaning, in terms of confidence levels.
- Be numerically robust, regardless of the data base.
Thank You!

One-Day Workshop on New Aspects and Perspectives in Nuclear Physics
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University of Ioannina, Greece