On proton deformation: Multipole amplitude extraction from photoproduction data with the AMIAS

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Nucleon Resonances, Partial waves, Multipoles ...

Resonance	Partial Wave	l_π*	I	J	M [MeV]	Multipoles
Δ(1232)	P_{33}	1	3/2	3/2	1232	$E_{1+}^{3/2}$, $M_{1+}^{3/2}$
N(1440)	P_{11}	1	1/2	1/2	1440	$M_{1-}^{1/2}$
N(1520)	D_{13}	2	1/2	3/2	1520	$E_{2-}^{1/2}$, $M_{2-}^{1/2}$
N(1535)	S_{11}	0	1/2	1/2	1535	$E_{0+}^{1/2}$
Δ(1620)	S_{31}	0	3/2	1/2	1620	$E_{0+}^{3/2}$

- L = 0, 1, 2, 3, … correspond to S, P, D, F, …
- Partial wave notation $(l_\pi)_{2I, 2J}$. I will be using L instead of l.
- Multipoles to which a resonance can give a resonance contribution.
- Important quantity $EMR = \frac{E_{1+}^{3/2}}{M_{1+}^{3/2}}$
- EMR indicates the amount of deformation of the nucleon.

Connect multipoles to observables

Experimentally accessible

Observables: $d\sigma_0(f(F_i)),\ \Sigma(f(F_i)),\ T(f(F_i)),\ P(f(F_i))\ldots$

Intermediate step: Links parameters to data

CGLN amplitudes*: $F_i(W,\theta_{cm}), i=1,4 \rightarrow \sum_{L=L_{min}}^{\infty} f(L, A_L, \theta_{cm})$

Parameters of the problem

Multipoles*: $A_L:\quad E_{L+},\ E_{L-},\ M_{L+},\ M_{L-},\qquad L\leqslant L_{cut}$

* Multipoles and CGLN amplitudes are complex quantities

Isospin decomposition

Multipoles: $A_L: \quad E_{L+}, \quad E_{L-}, \quad M_{L+}, \quad M_{L-}, \qquad L \leqslant L_{cut}$

$A_L^I: \quad E_{L+}^I, \quad E_{L-}^I, \quad M_{L+}^I, \quad M_{L-}^I, \qquad I = \frac{1}{2}, \frac{3}{2}, \qquad L \leqslant L_{cut}$

For full Isospin decomposition data from two channels are needed:

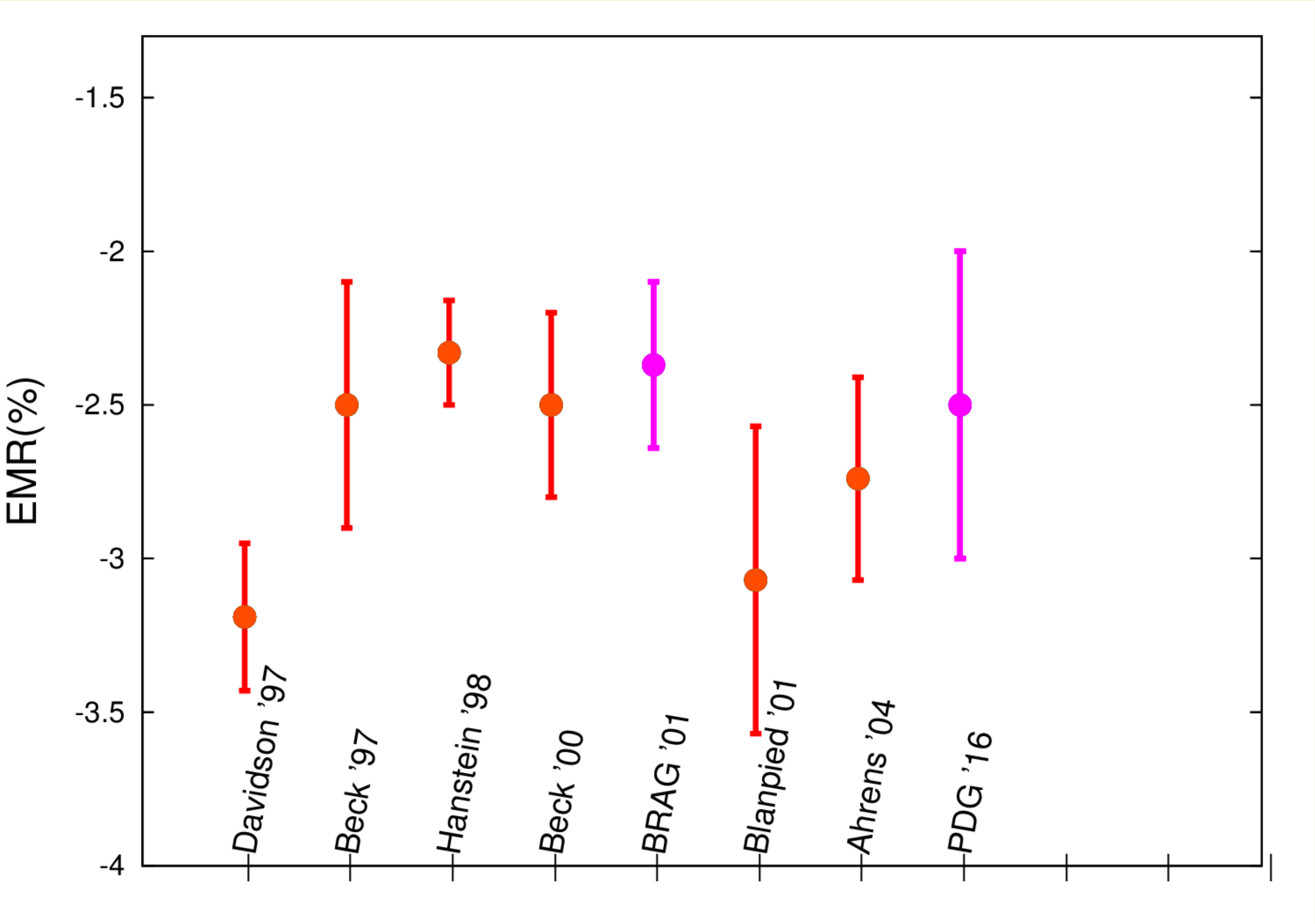
$$A_{\gamma p \to p\pi^0} = A^{1/2} + \frac{2}{3} A^{3/2}$$

$$A_{\gamma p \to n\pi^+} = \sqrt{2}\left(A^{1/2} - \frac{1}{3} A^{3/2} \right)$$

EMR: earlier analyses

Includes statistical, and where available, model and systematic uncertainty

Error estimated by averaging several different analyses



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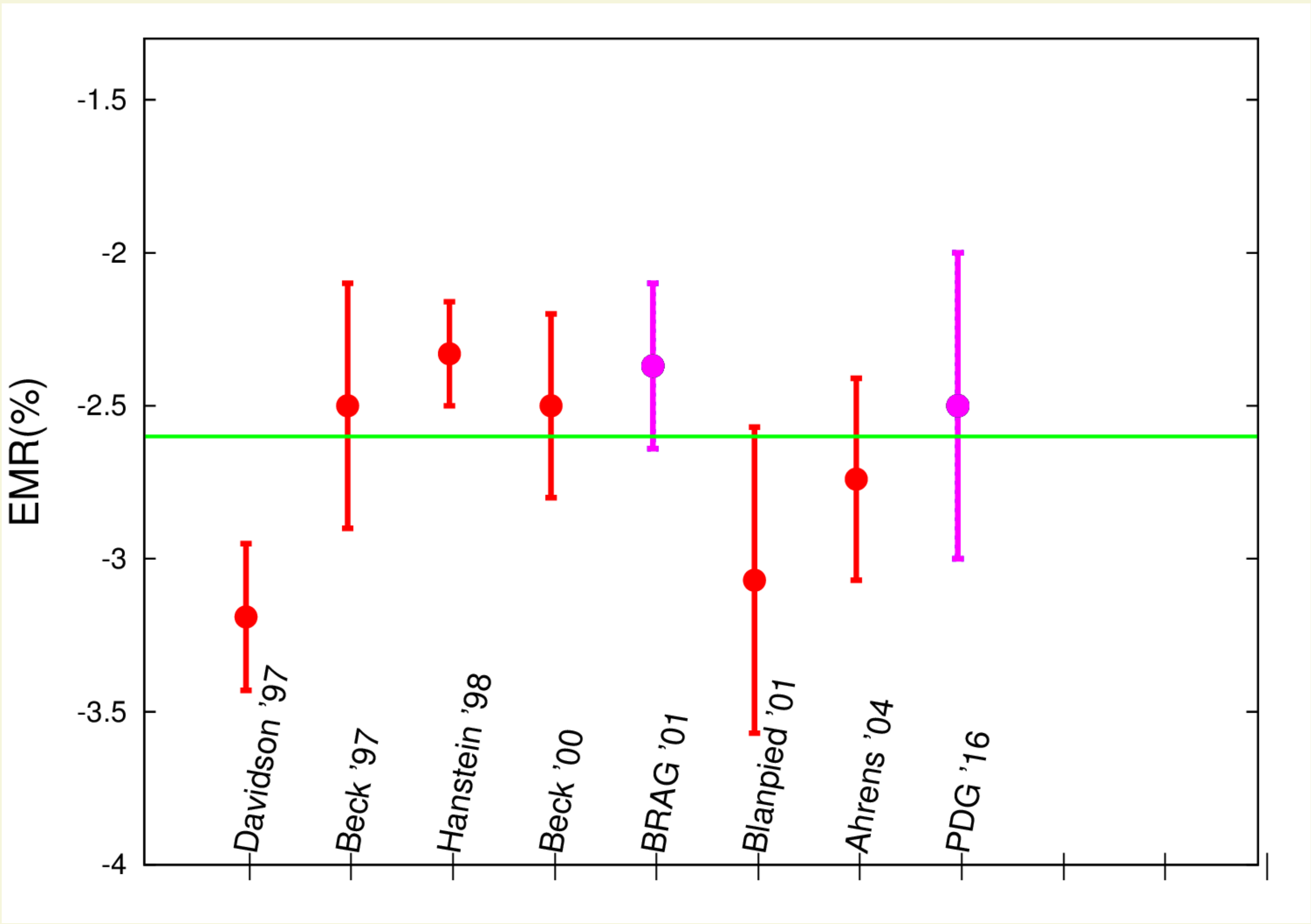
Mean Value



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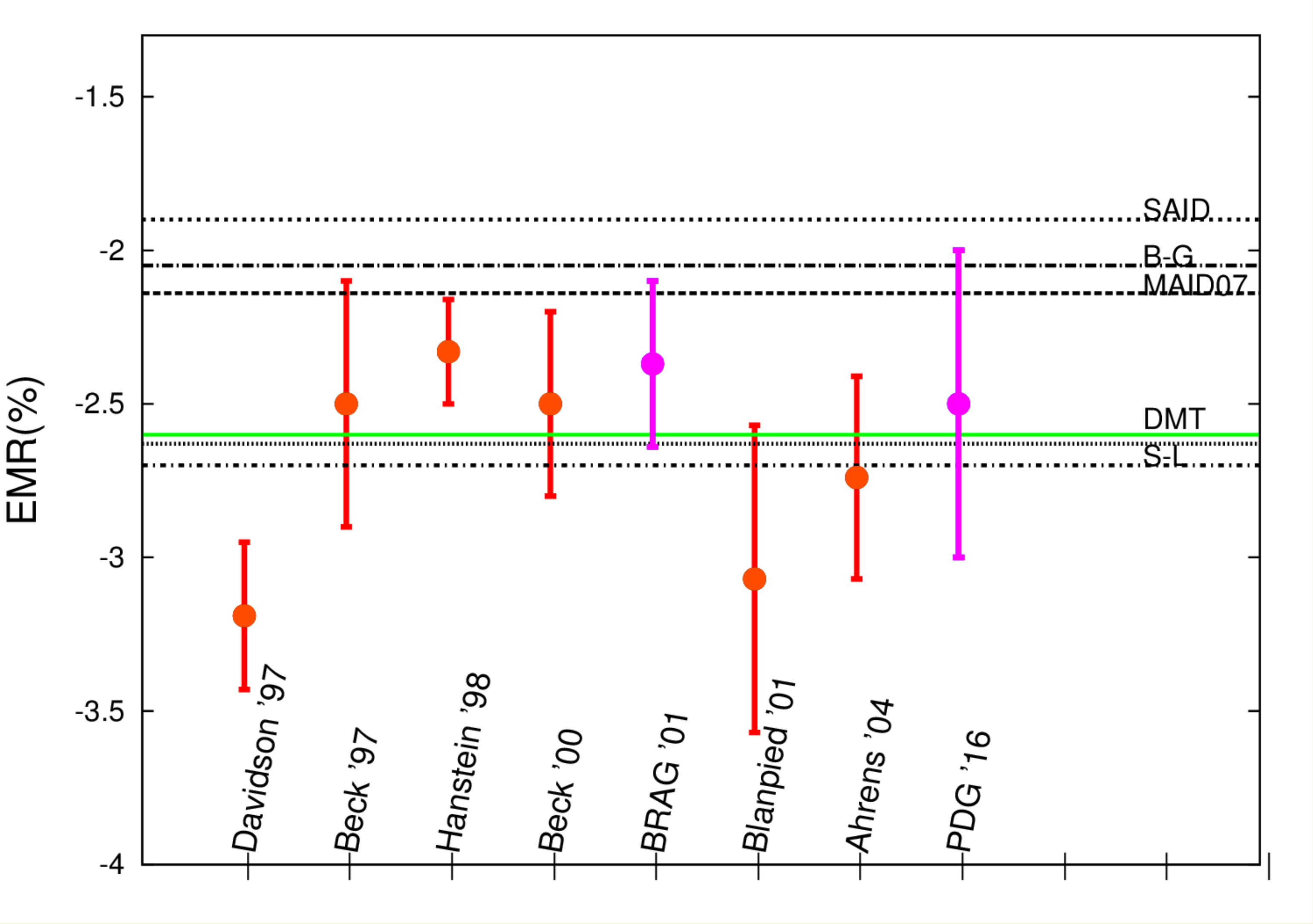
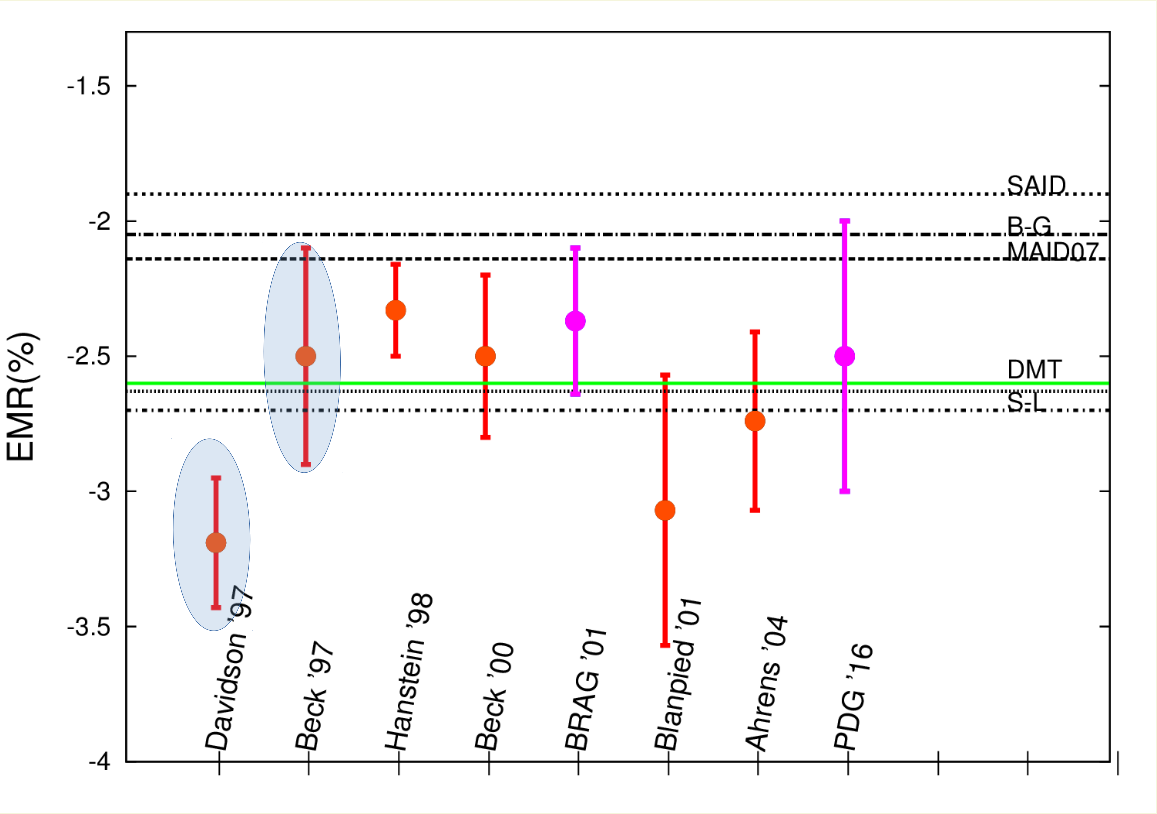
Model prediction　　　　Mean Value



Model dependence in current EMR values



- Determined EMR from the same data and only from the $p\pi^0$ channel. Made different model assumptions.

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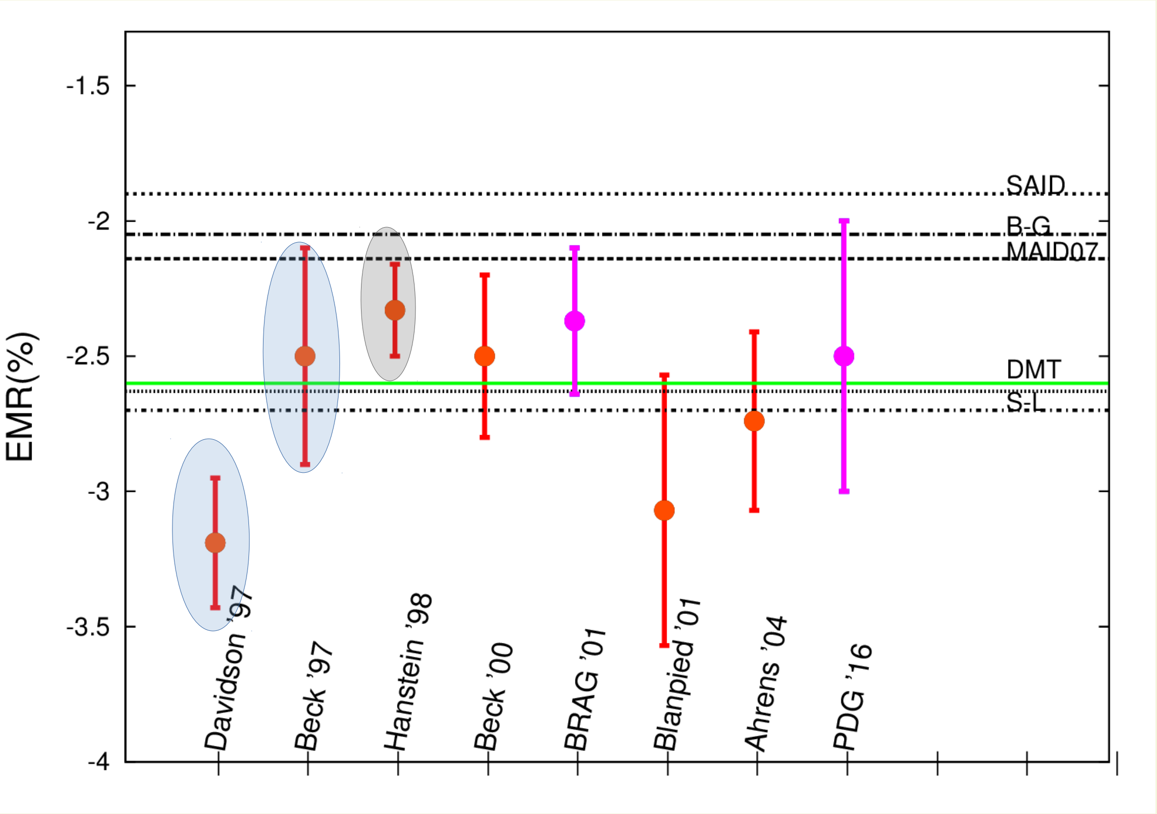

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- Extracted S-P and D_{13} amplitudes.

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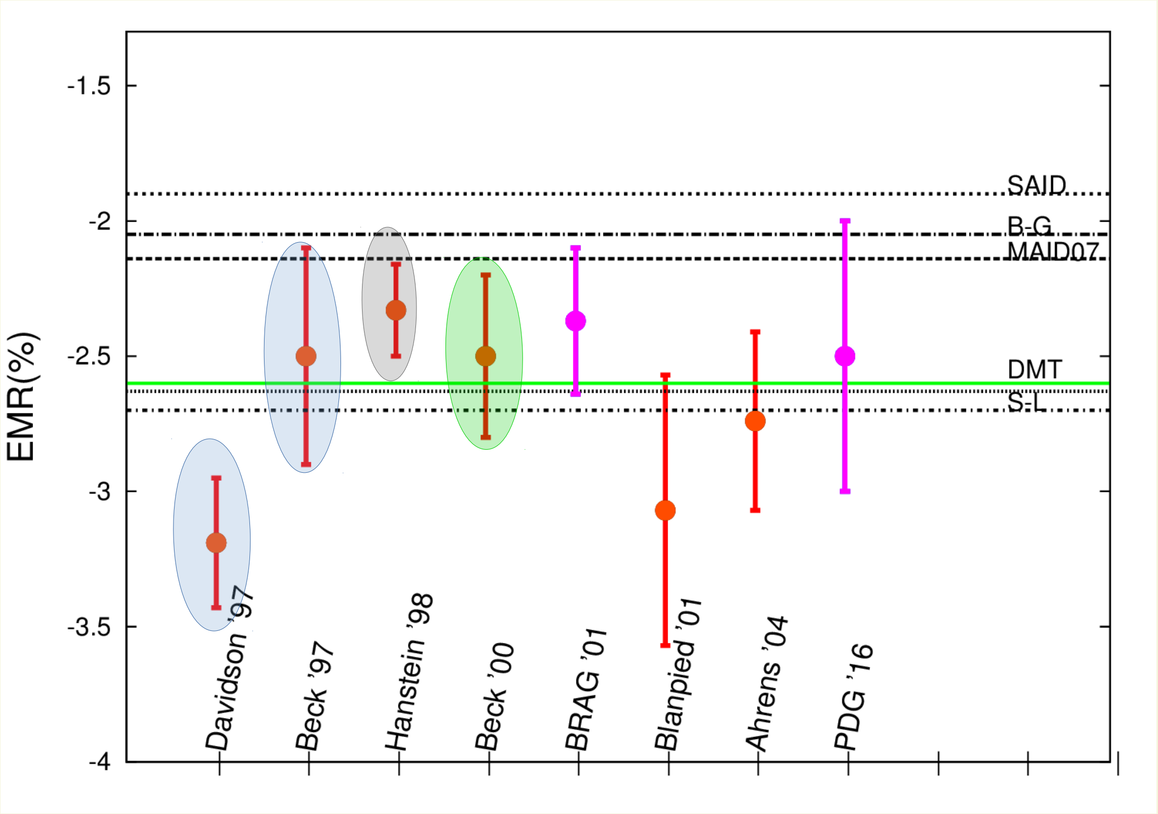

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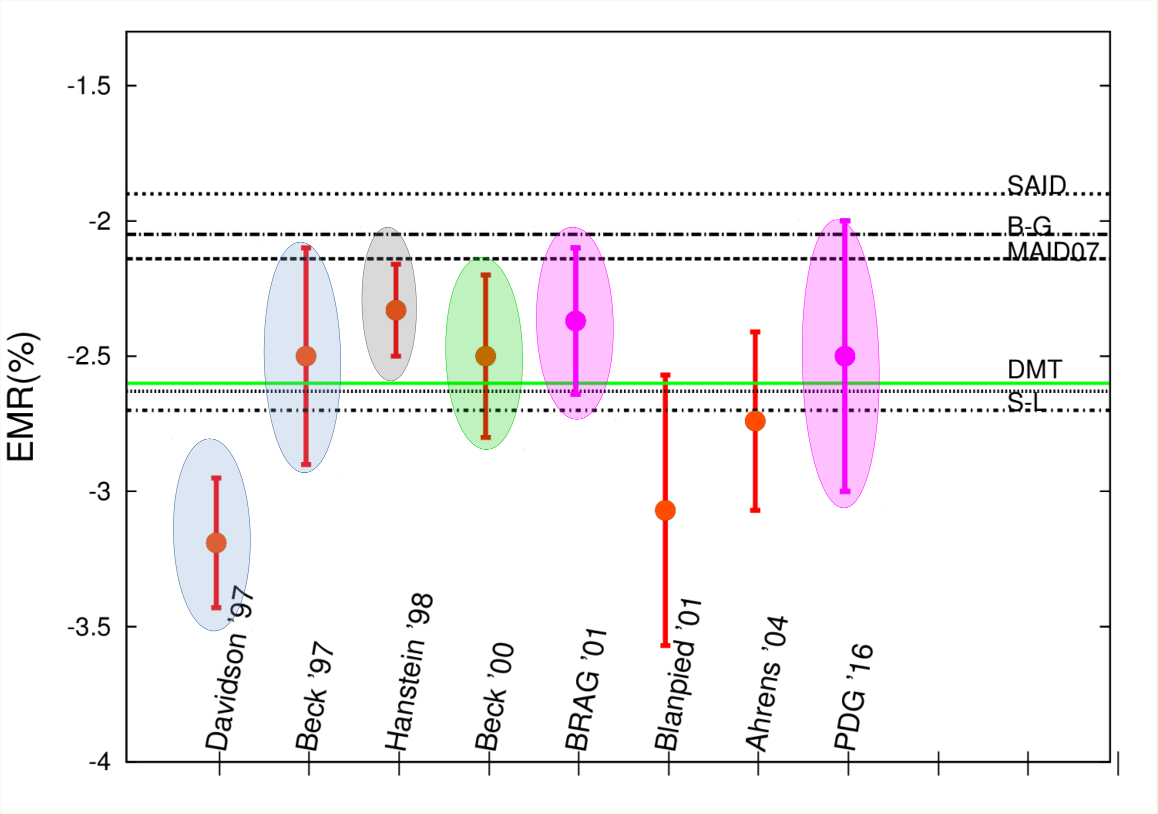

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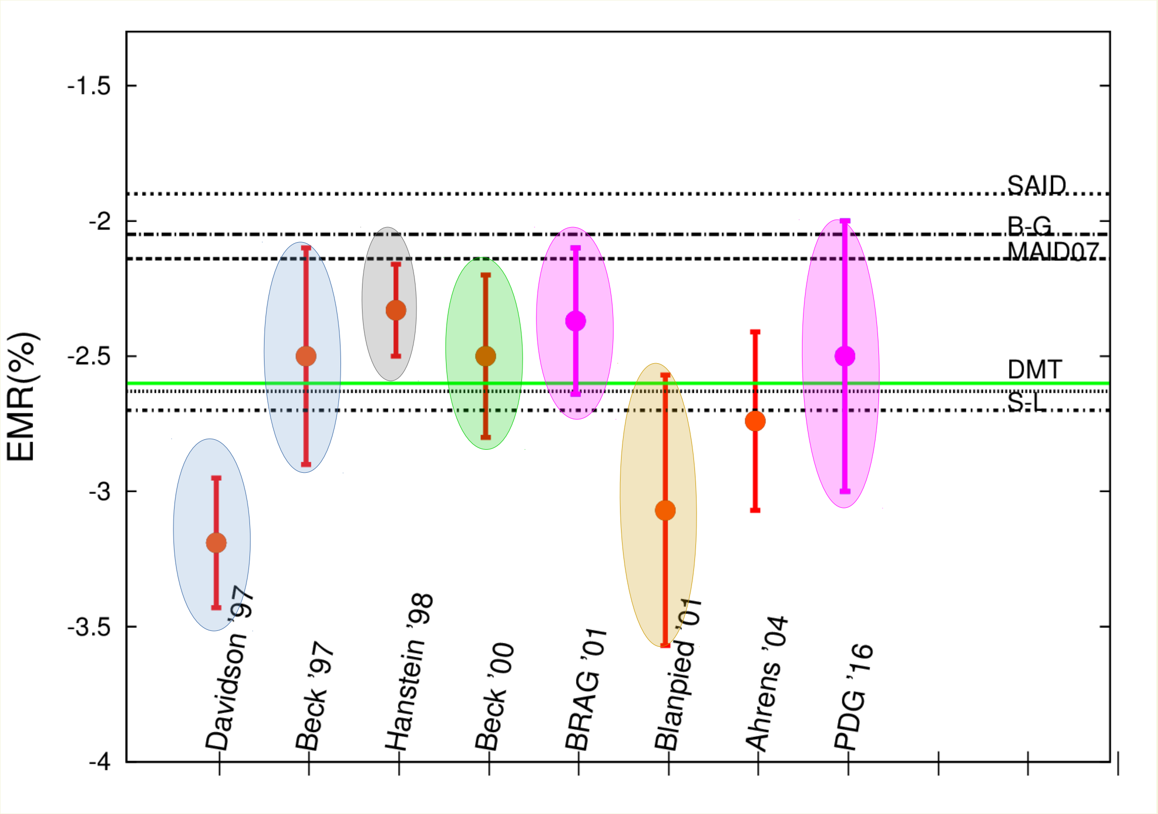

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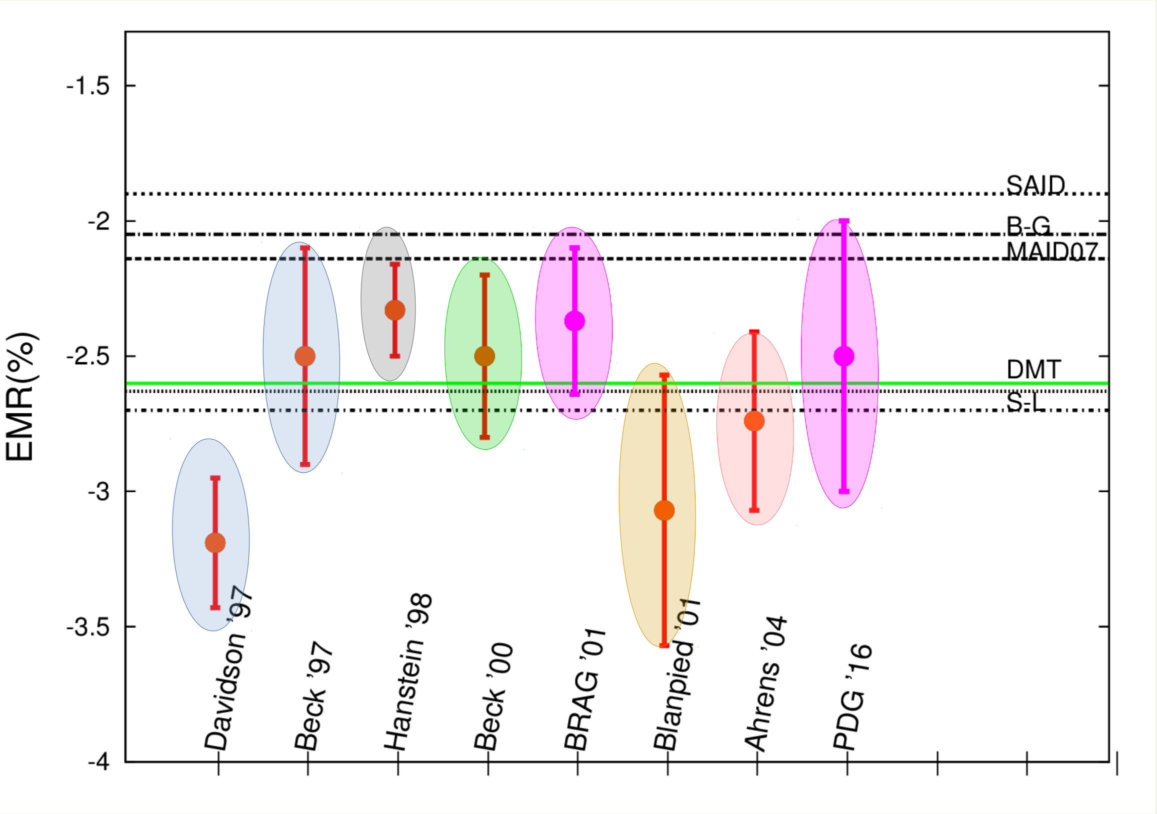
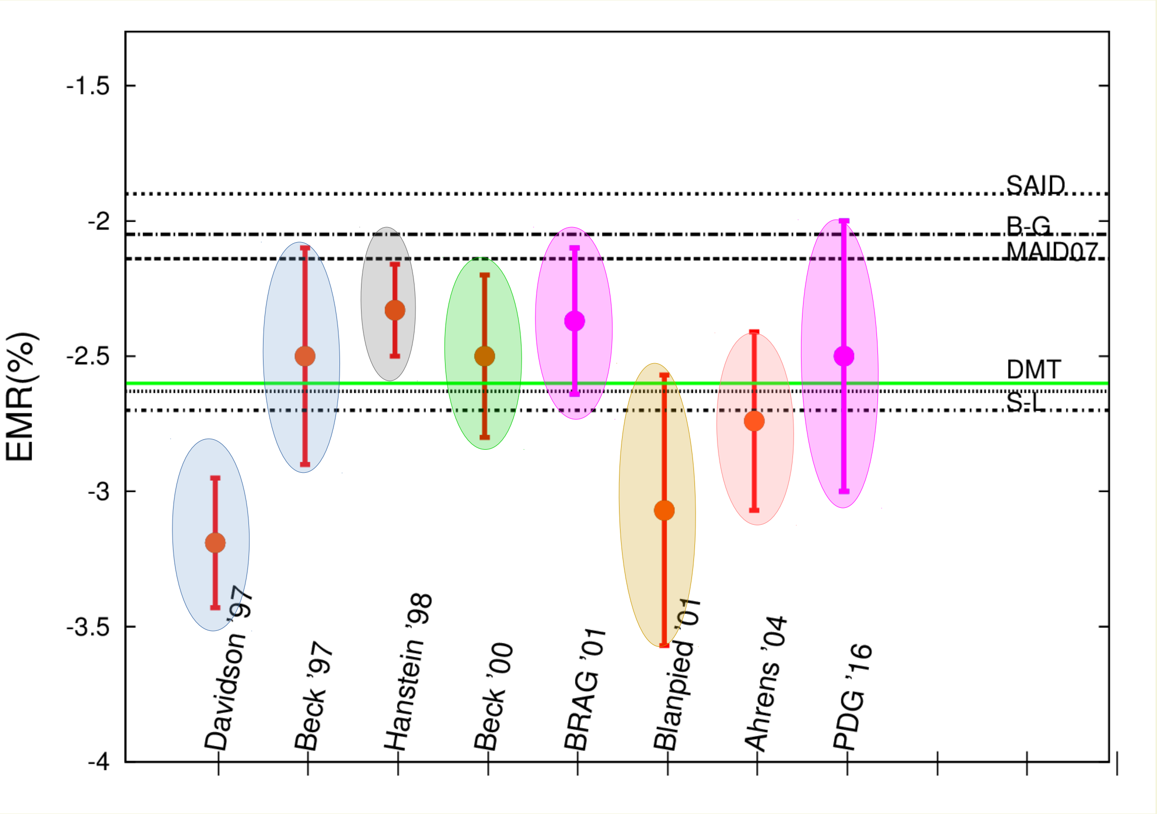
- Data from $p(\gamma,\gamma)$, $p(\gamma,\pi^0)$, $p(\gamma,\pi^+)$. Fitted up to F waves. Different assumptions for different non-resonant amplitudes.

Model dependence in current EMR values



- Determined EMR from the same data and only from the $p\pi^0$ channel. Made different model assumptions.
- Extracted S-P and D_{13} amplitudes.
- Extracted S-P amplitudes. Higher amplitudes calculated in the Born approximation.
- Mean value and uncertainty as a result of averaging different analyses.
- Data from $p(\gamma,\gamma)$, $p(\gamma,\pi^0)$, $p(\gamma,\pi^+)$. Fitted up to F waves. Different assumptions for different non-resonant amplitudes.
- Extracted S-P amplitudes. σ_{EMR} was determined to be $\pm 0.03_{stat}$% and an extra 0.3% was estimated as model error, motivated by the work of the BRAG.

Model dependence in current EMR values



Goal

To extract EMR, at W = 1232.2 MeV, in a model independent fashion for the first time.

Cornerstones of our analysis

- We impose only the constraints dictated by theory and no other. In our case
  - Multipoles with the same quantum numbers l, I, J have the same phase ± nπ (Fermi-Watson theorem)
  - Fix all multipole phases to the phases determined from πN scattering experiments
- Do not assume a L_{cut}. Rather allow all multipole amplitudes to freely vary and let the data decide up to which L_{cut} parameters are relevant
  - e.g. Some analyses extract up to $L_{cut} = 1, 2$ or maybe even 3, but this choice is somewhat arbitrary and preconceived
- Identify possible sources of systematic errors and treat them within the AMIAS framework
  - e.g. Normalization errors, background, uncertainty in pion angle, uncertainty in $E_{\gamma,lab}$

Data used in the analysis

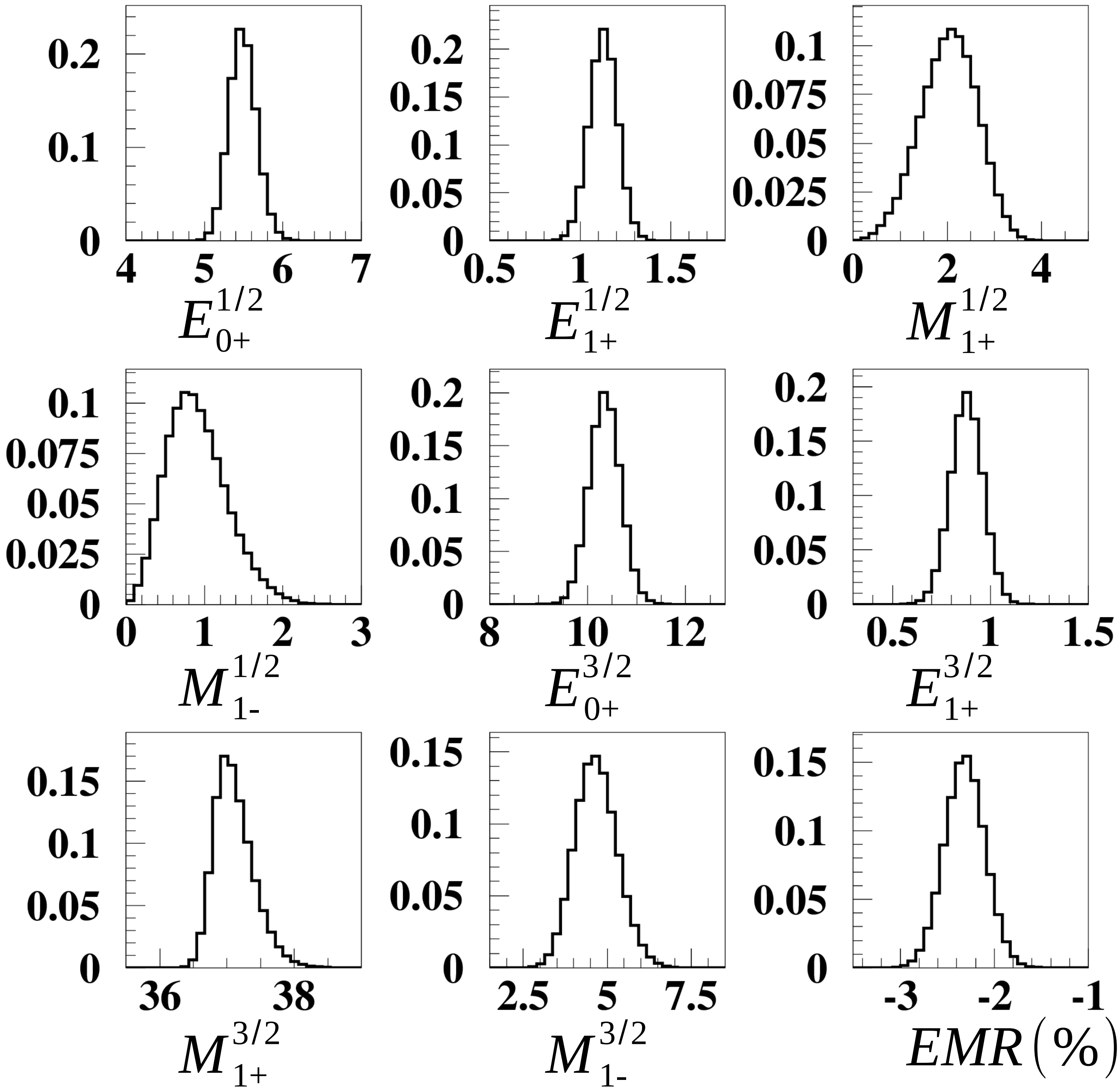
$\gamma p \to p\pi^0$					$\gamma p \to n\pi^+$			
Obs	**E_γ (MeV)**	**Reference**	**#d.p.**		**Obs**	**E_γ (MeV)**	**Reference**	**#d.p.**
$d\sigma_0$	337.6 – 342.0	Adlarson - 2015	30		$d\sigma_0$	335 - 345	Beck - 2000	10
Σ	335 - 345	Leukel – 2001 Beck - 2006	17		Σ	335 - 345	Beck - 2000	10
T	339.0 – 340.06	Schumann - 2015*	18		T	335 - 356	Dutz - 1996	11
F	339.0 – 340.06	Schumann - 2015*	18					
G	326 - 354	Ahrens 2005	3		G	326 - 354	Ahrens 2005	6
P	335 - 365	Belyaev 1983	6		P	330 - 350	Get'man 1981	6

* Unpublished

Results

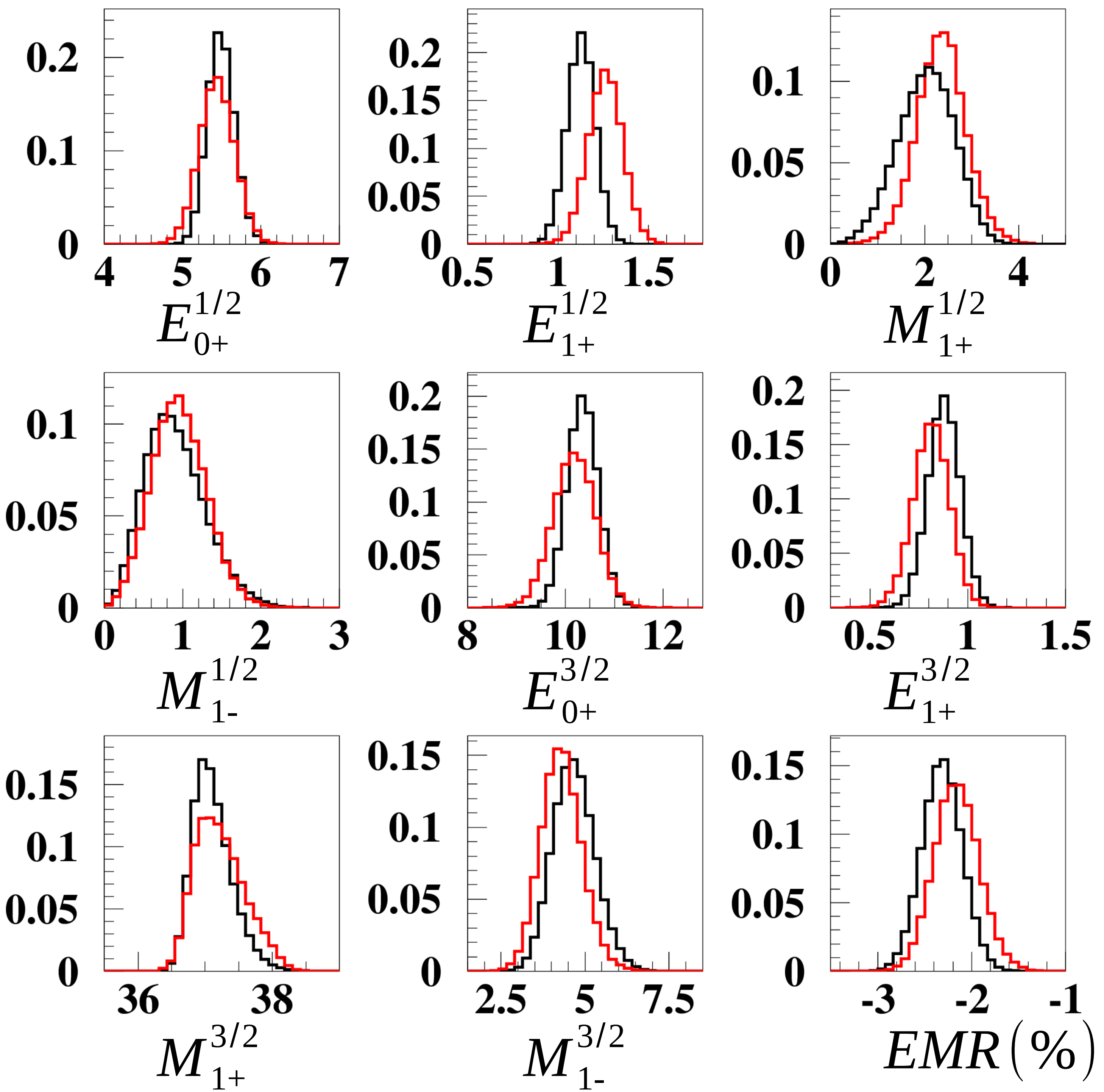
- Following are the Probability Distribution Functions (PDF's) of selected multipole amplitudes.

- Amplitudes up to L = 5 were extracted where convergence was reached.
  - *Convergence:* χ^2_{min} *reached and extracted amplitudes remain unchanged when going from* L_{cut} *to* $L_{cut} + 1$



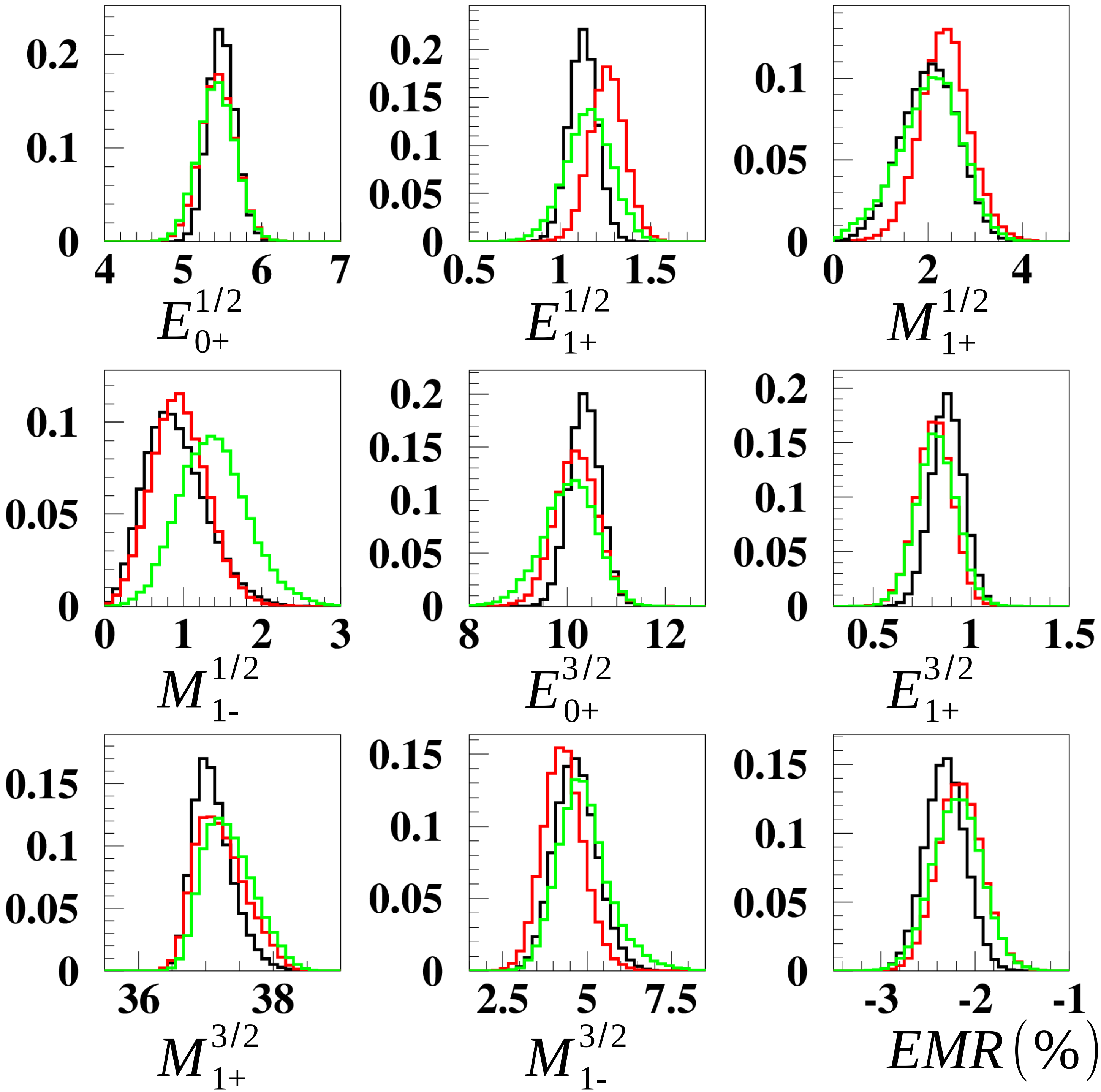
Only L = 1 Amplitudes Shown here

- **L ≤ 1**



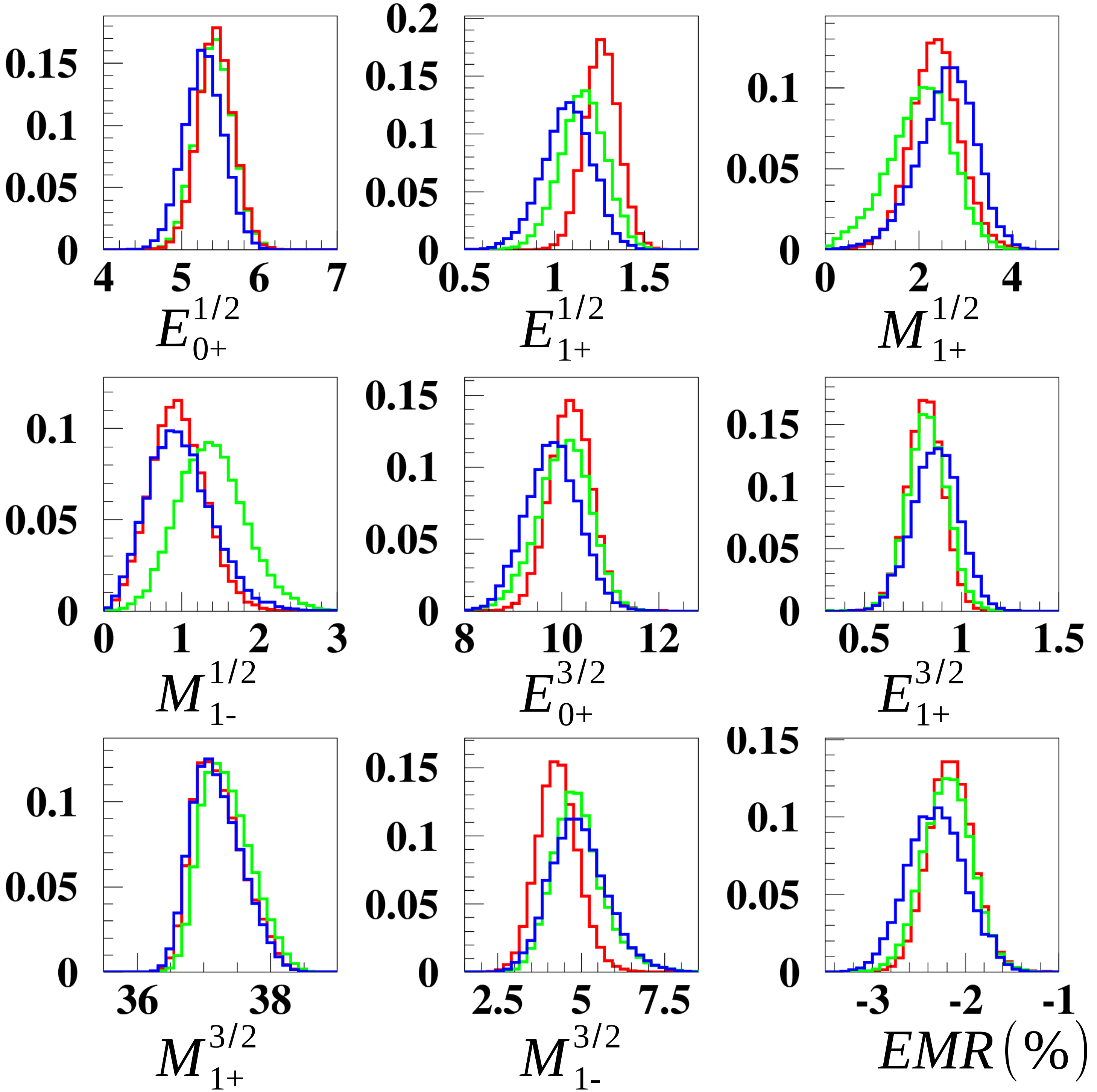
Only L = 1 Amplitudes Shown here

- **L ≤ 1**
- **L ≤ 2**



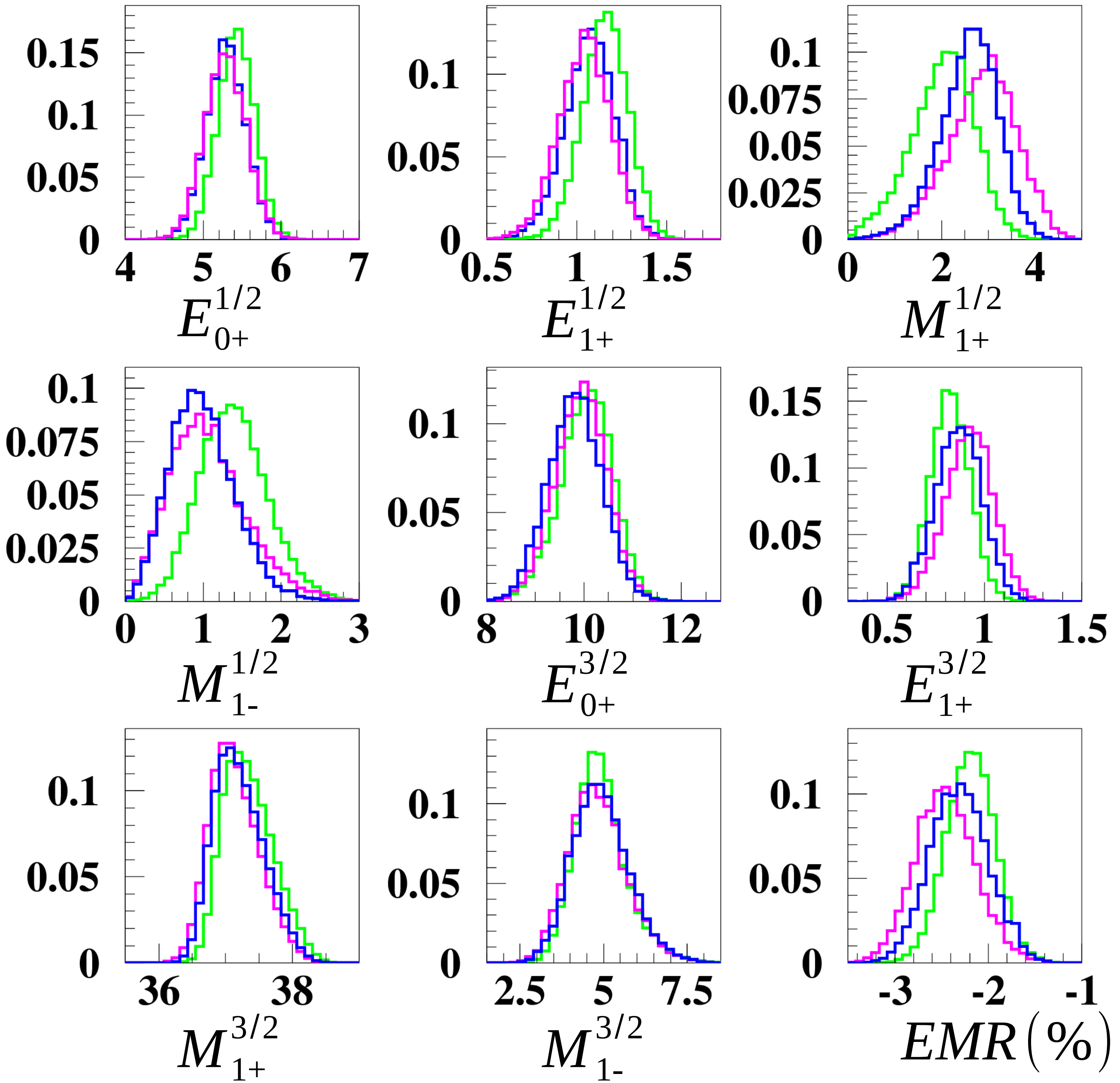
Only L = 1 Amplitudes Shown here

- **L ≤ 1**
- **L ≤ 2**
- **L ≤ 3**



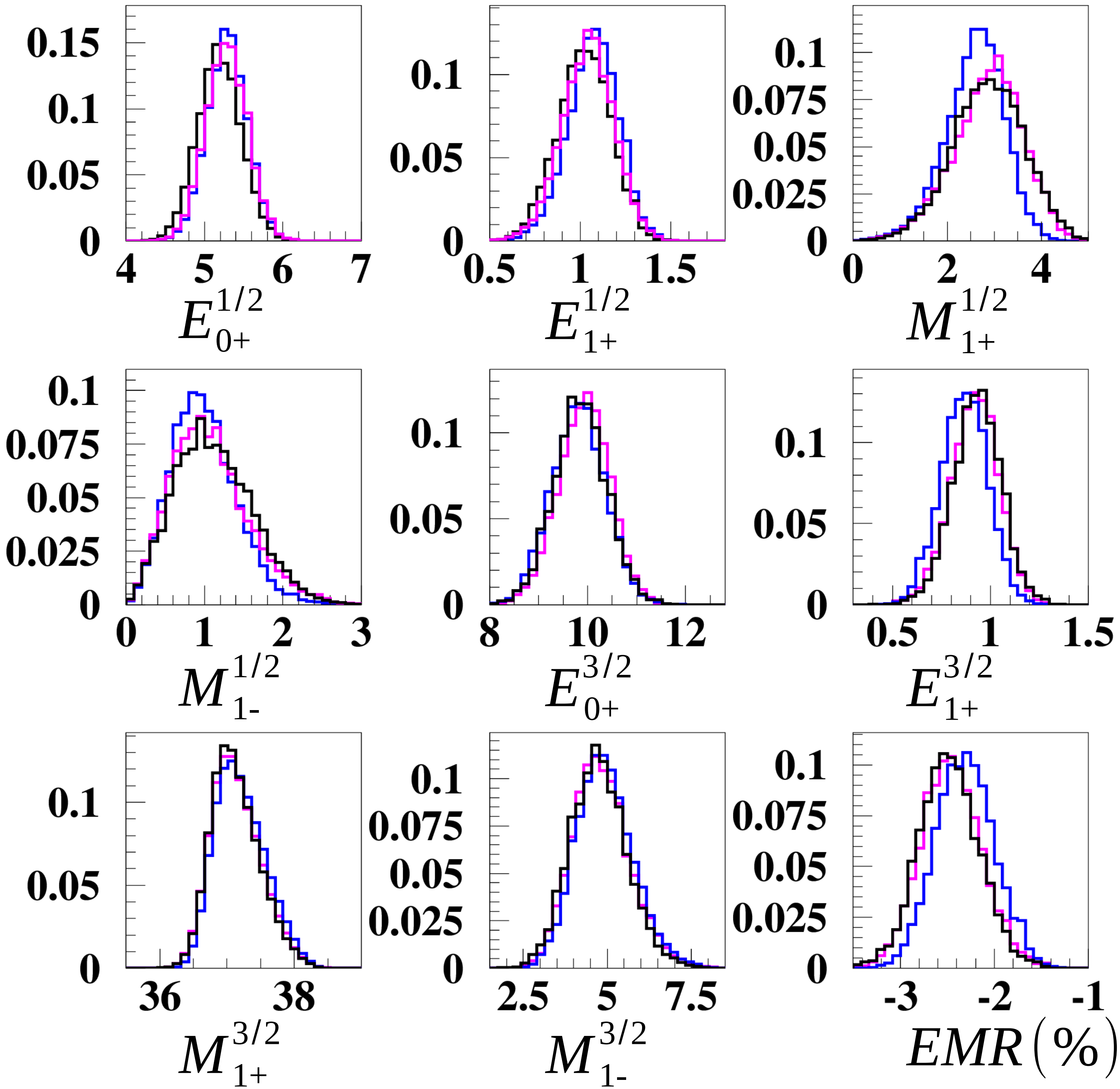
Only L = 1 Amplitudes Shown here

- $L \leq 2$
- $L \leq 3$
- $L \leq 4$



Only L = 1
Amplitudes
Shown here

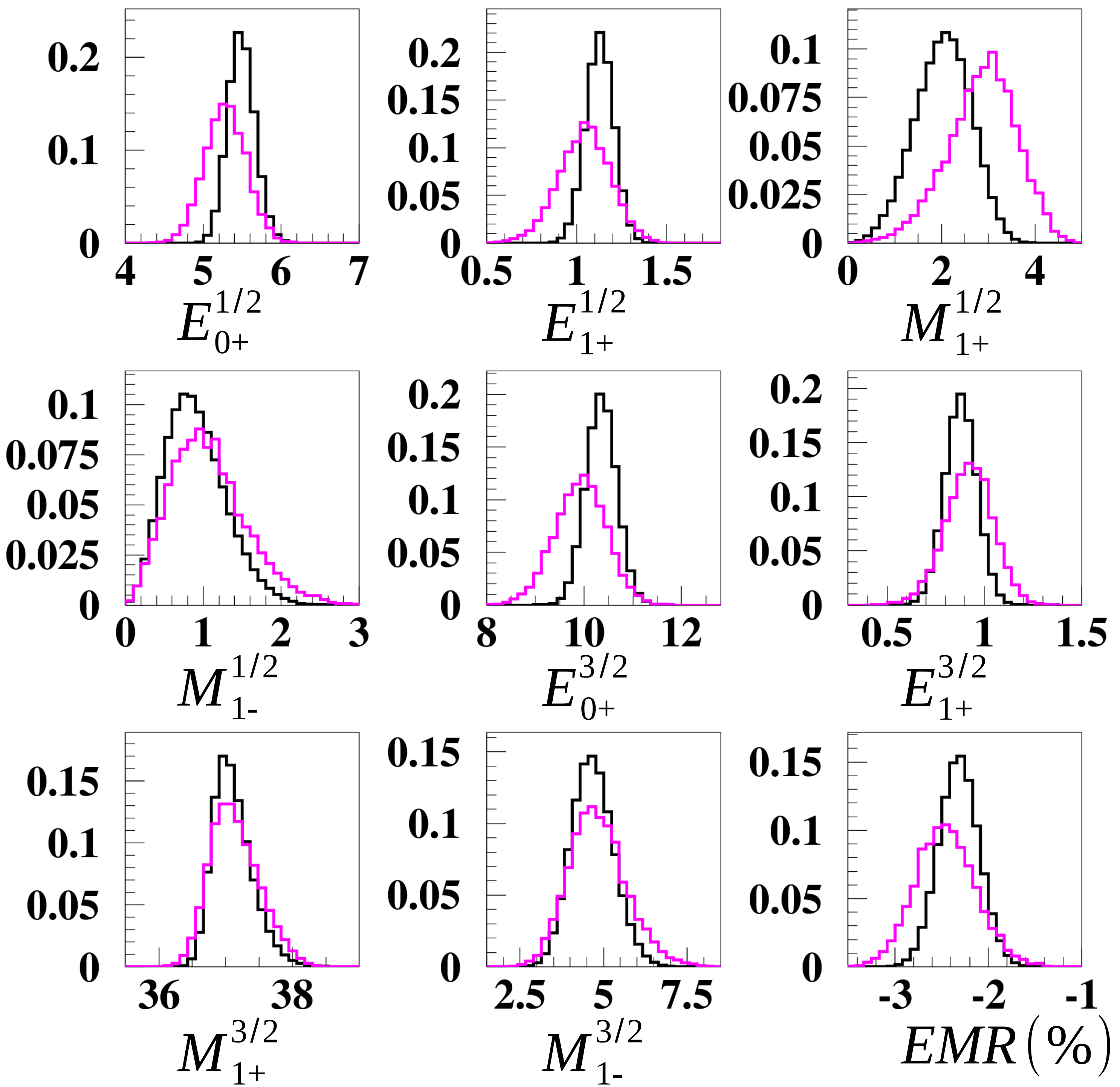
- L ≤ 3
- L ≤ 4
- L ≤ 5



Only L = 1
Amplitudes
Shown here

- **L ≤ 4**
- **L ≤ 5**
- **L ≤ 6**

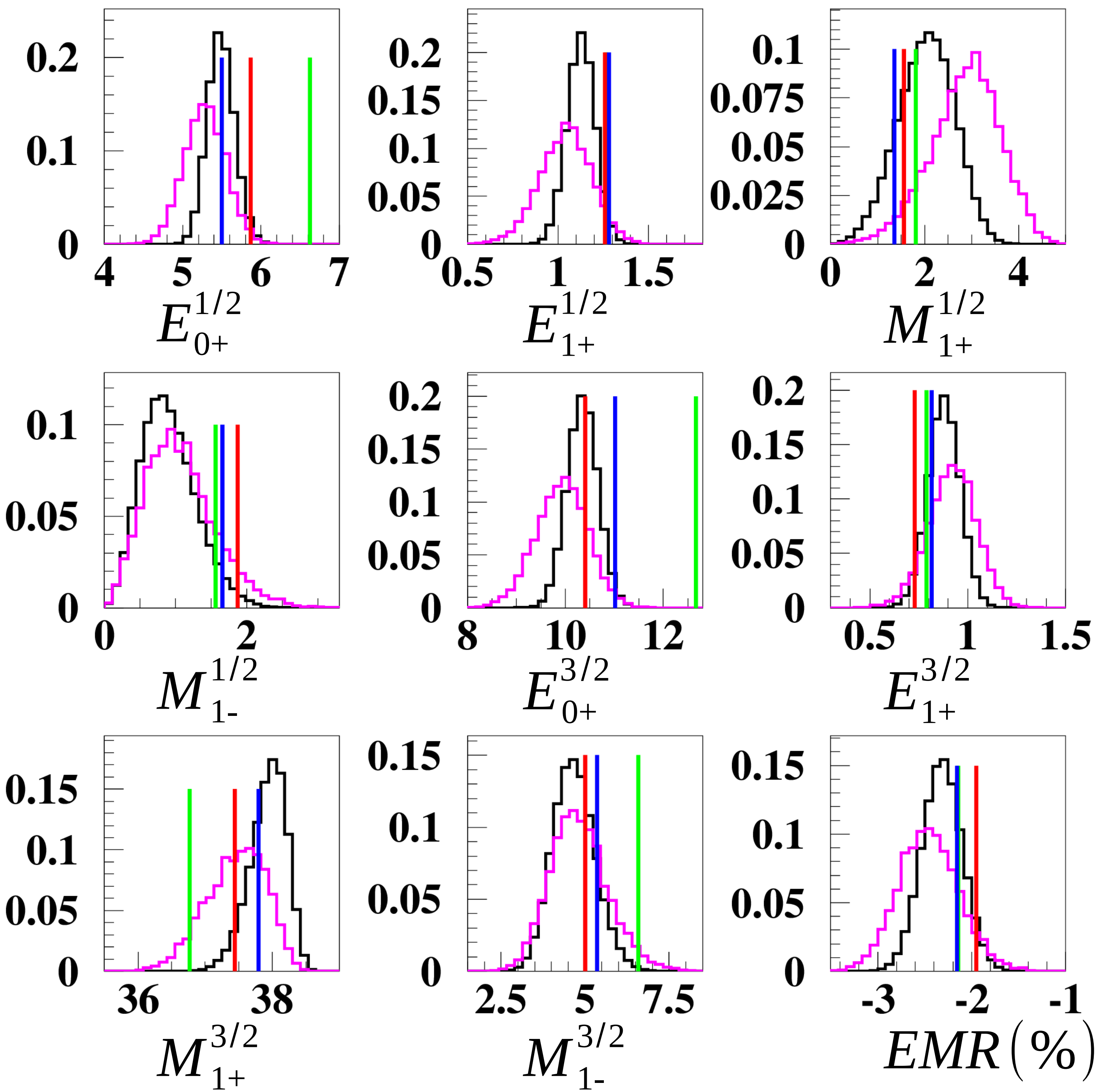
Convergence



L = 1 amplitudes

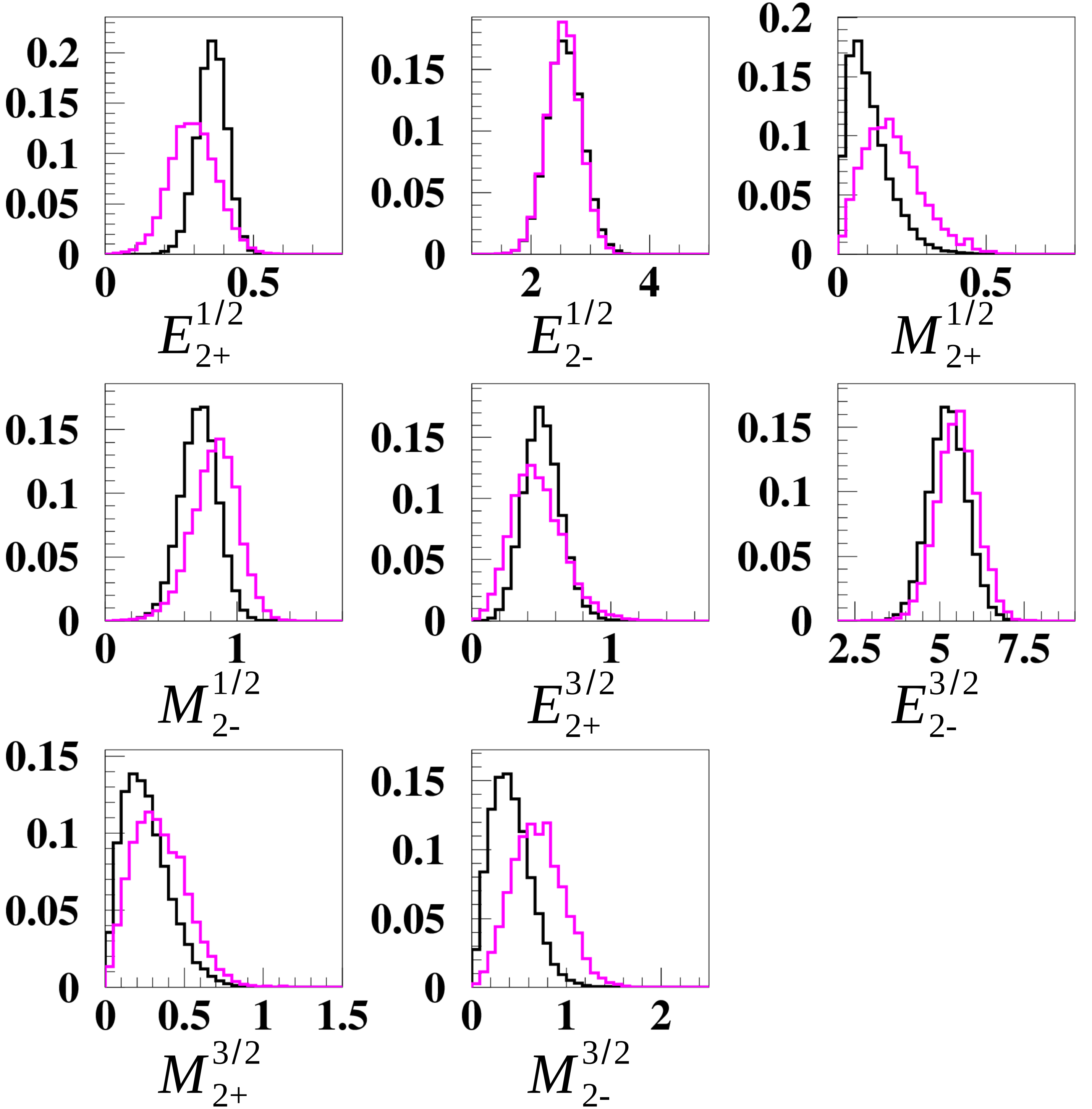
- **L ≤ 1**
- **L ≤ 5**

L = 1 amplitudes



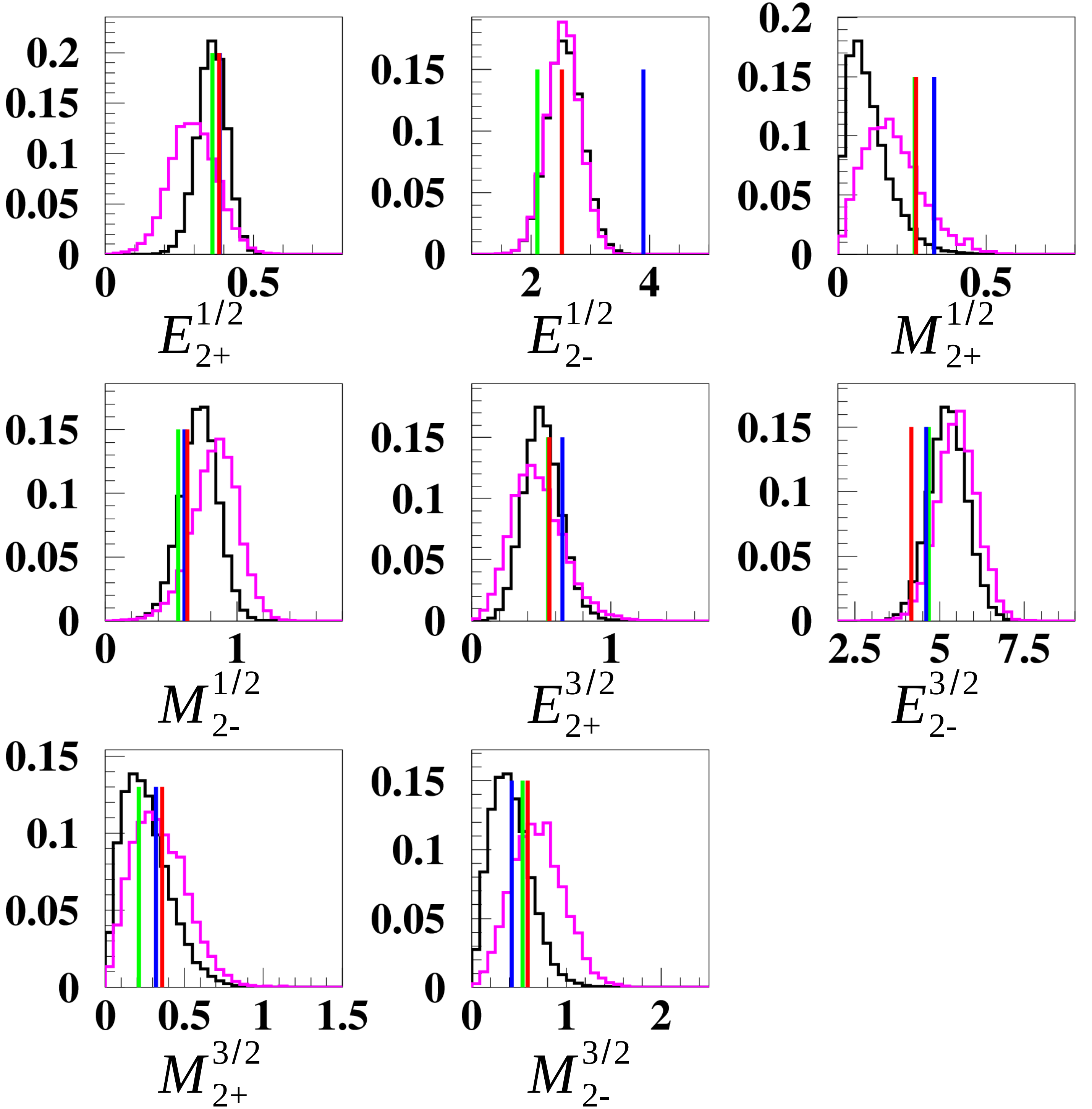
- L ≤ 1
- L ≤ 5

- MAID07
- SAID (PR15)
- B-G(2014-02)



L = 2 amplitudes

- **L ≤ 2**
- **L ≤ 5**



L = 2 amplitudes

- L ≤ 2
- L ≤ 5

- MAID07
- SAID (PR15)
- B-G(2014-02)

- The AMIAS analyses where all **$L \leq 5$** and all **$L \leq 6$** amplitudes are allowed to vary show identical results, therefore, convergence is reached.

- Maximum information is extracted from the data.

- No model assumptions made (e.g. where to place L_{cut}, e.t.c.).

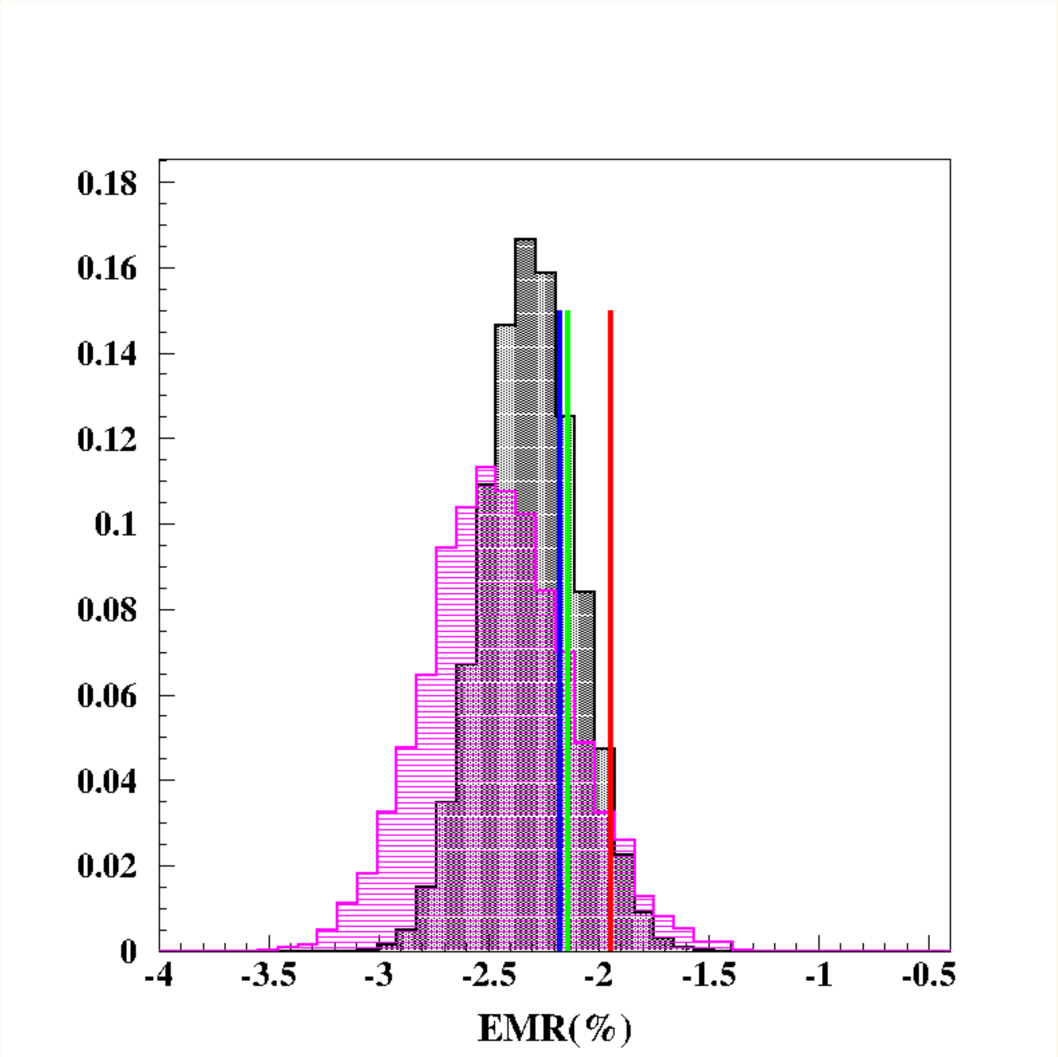
$L_{cut} \leq$	**χ^2_{min}**	EMR(%)
1	120	-2.3 ± 0.2
2	109	$-2.18^{+0.26}_{-0.23}$
3	88	-2.2 ± 0.3
4	83	-2.3 ± 0.3
5	80	-2.5 ± 0.3
6	80	-2.5 ± 0.3

- Model Dependent analysis

$$EMR(\%) = -(2.3 \pm 0.2)$$

- Model Independent analysis

$$EMR(\%) = -\left(2.5\,^{+0.3}_{-0.4}\right)$$

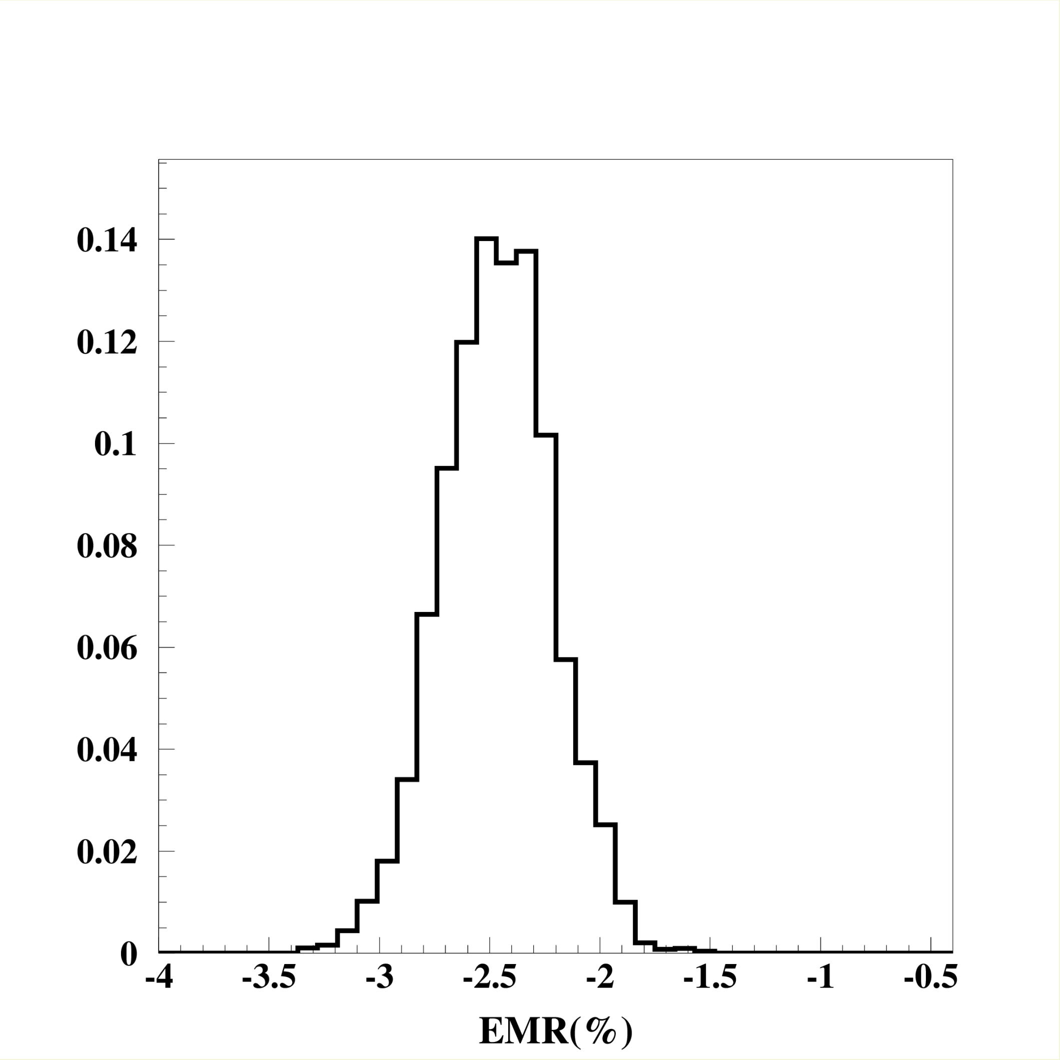


Model Predictions:

- **SAID (PR15): -2.2**
- **SAID (CM12): -1.9**
- **MAID07: -2.1**

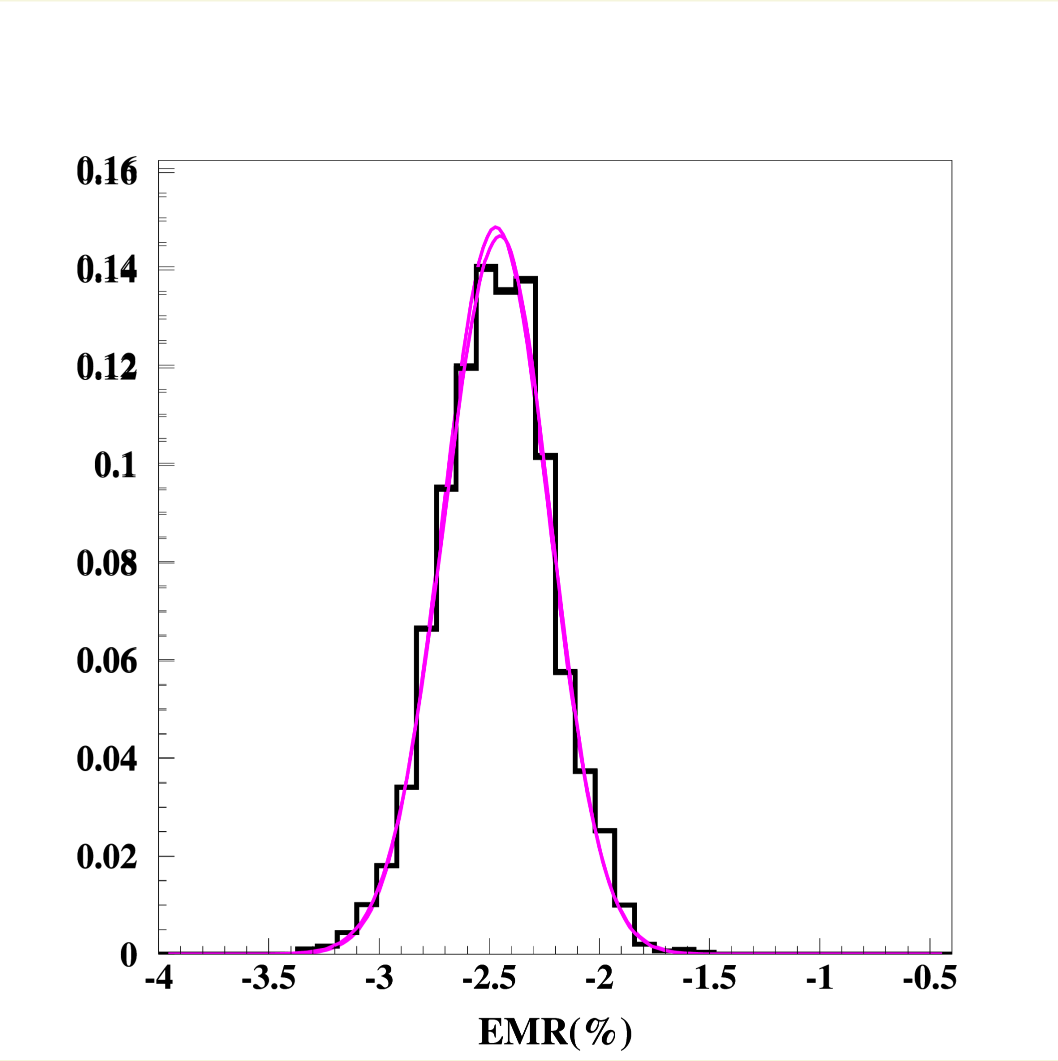
- Model Dependent analysis underestimates the derived errors
- Model predictions naturally closer to the Model Dependent analysis

- Full dataset: statistical errors

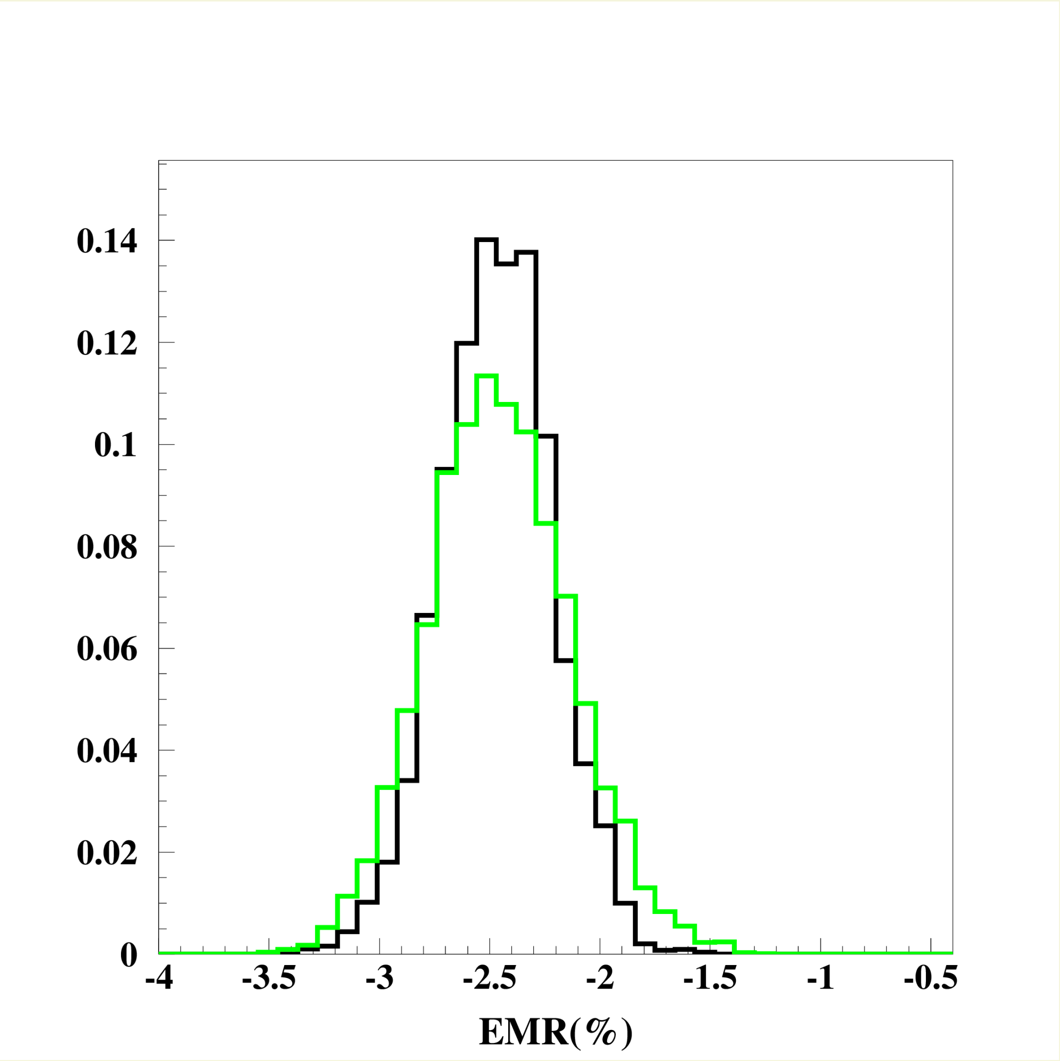


- Full dataset: statistical errors

$$EMR(\%) = -\left(2.47\ {}^{+0.24}_{-0.24}\right)$$



- Full dataset: statistical errors $EMR(\%) = -\left(2.47\,^{+0.24}_{-0.24}\right)$
- Full dataset: statistical & systematic errors $EMR(\%) = -\left(2.53\,^{+0.34}_{-0.35}\right)$

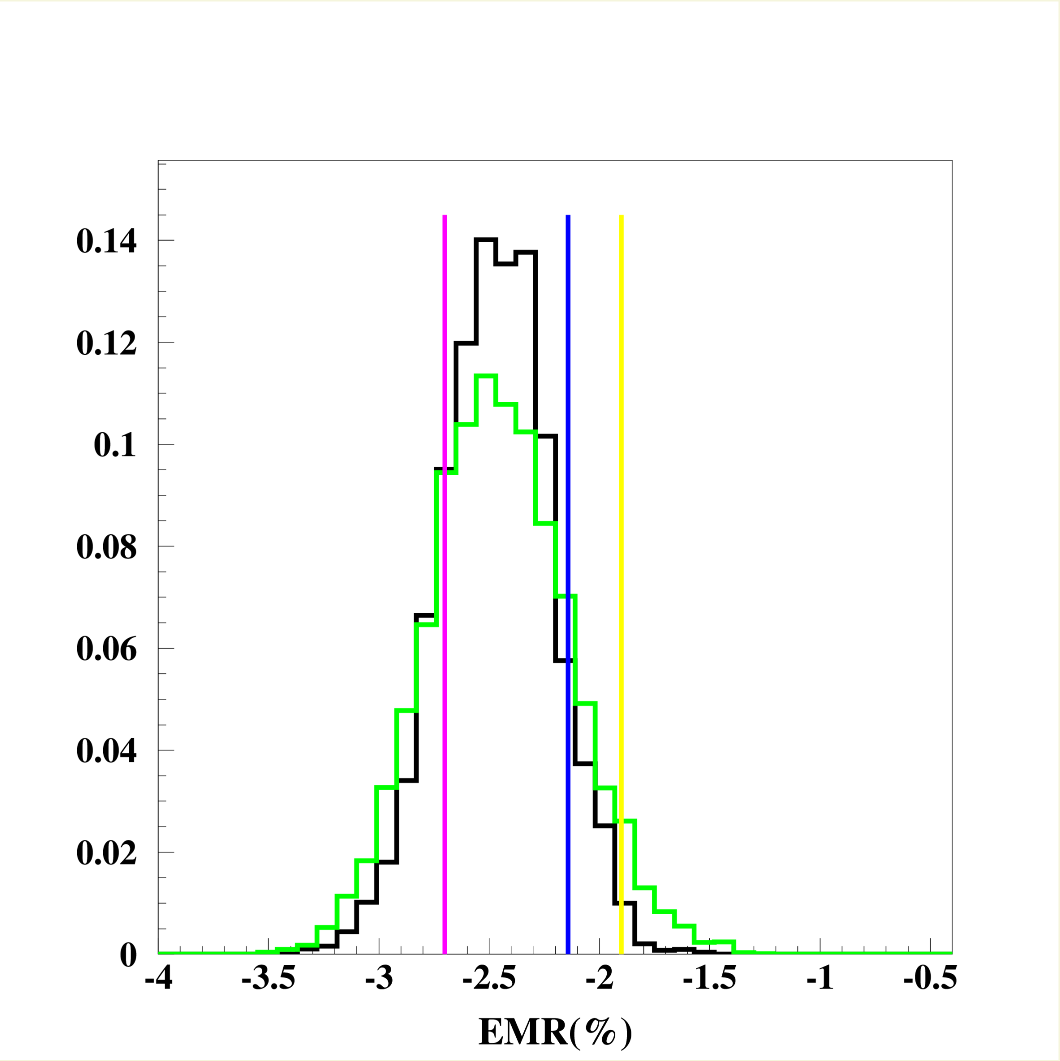


- Full dataset: statistical errors

$$EMR(\%) = -\left(2.47 \begin{smallmatrix} +0.24 \\ -0.24 \end{smallmatrix}\right)$$

- Full dataset: statistical & systematic errors

$$EMR(\%) = -\left(2.53 \begin{smallmatrix} +0.34 \\ -0.35 \end{smallmatrix}\right)$$



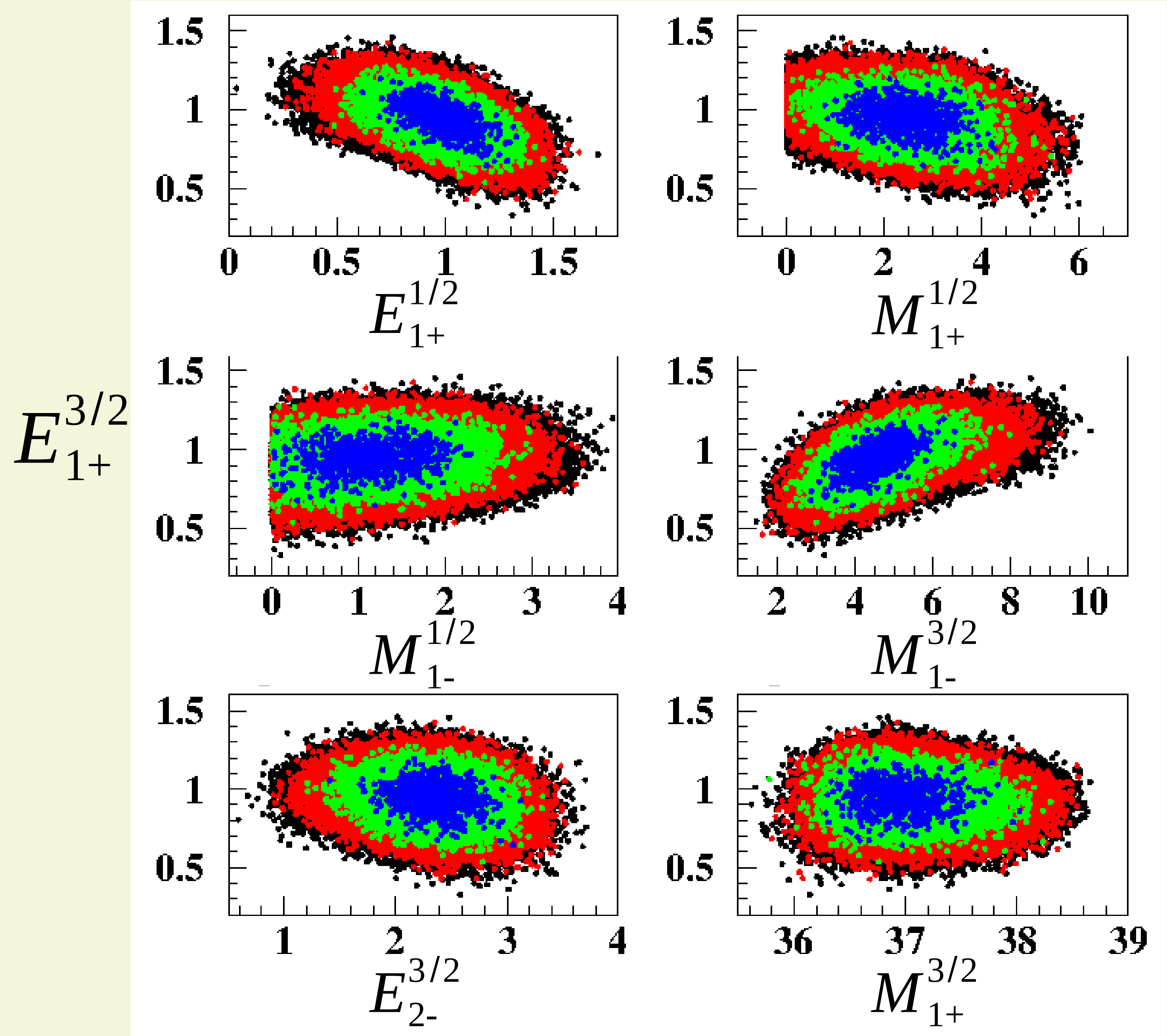
Model Predictions:

- **SAID (CM12): -1.9**
- **MAID07: -2.1**
- **Sato - Lee: -2.7**

Correlation Plots:

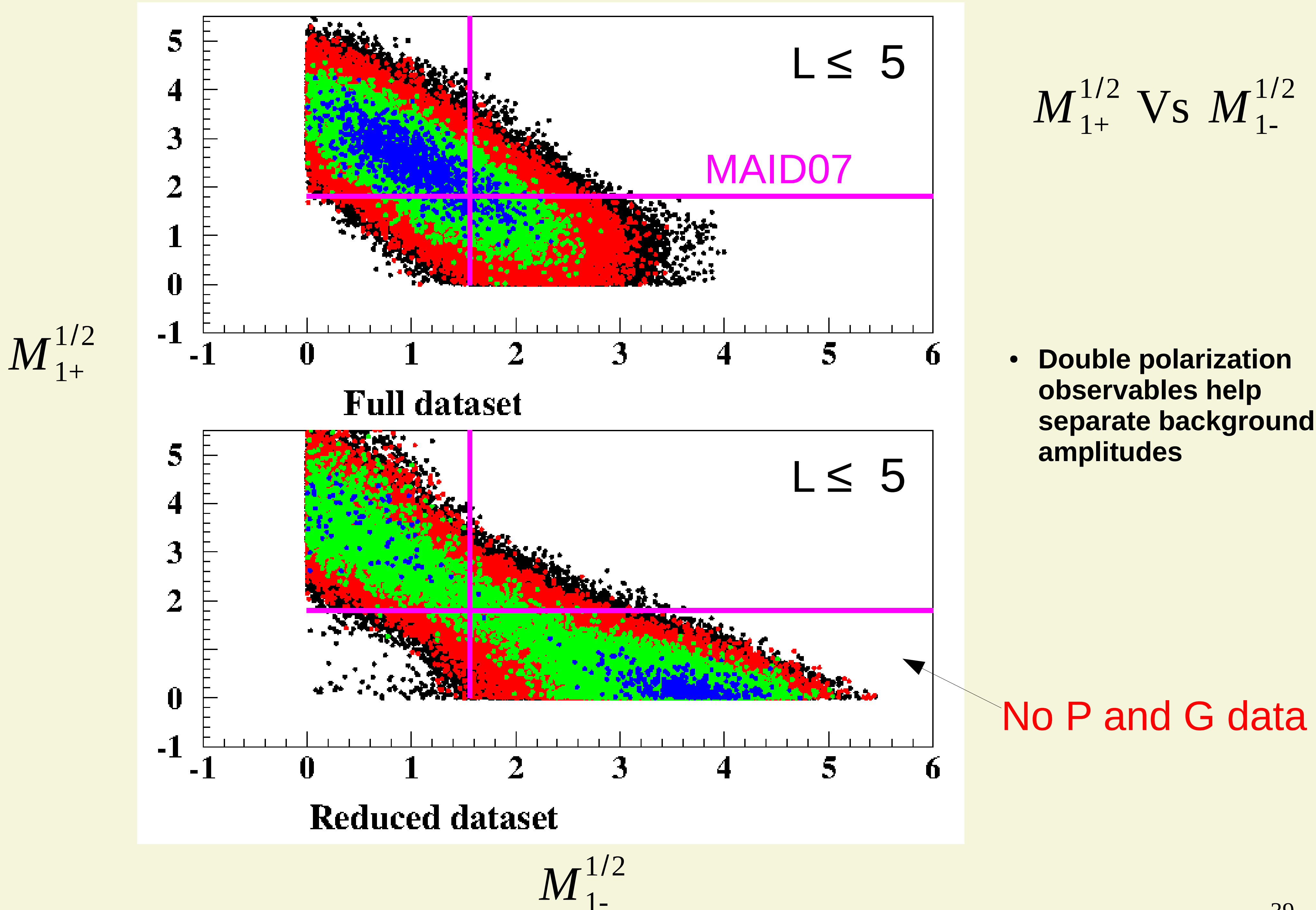
- 2-dimensional scatter plots of parameters
- Color coded according to the χ^2 value of each event (adds another dimension)

Correlations of $E_{1+}^{3/2}$



- **Mildly correlated with background amplitudes**
- **No correlation between the two resonant amplitudes**

$M_{1+}^{1/2}$ Vs $M_{1-}^{1/2}$



- **Double polarization observables help separate background amplitudes**

- Full dataset: statistical errors

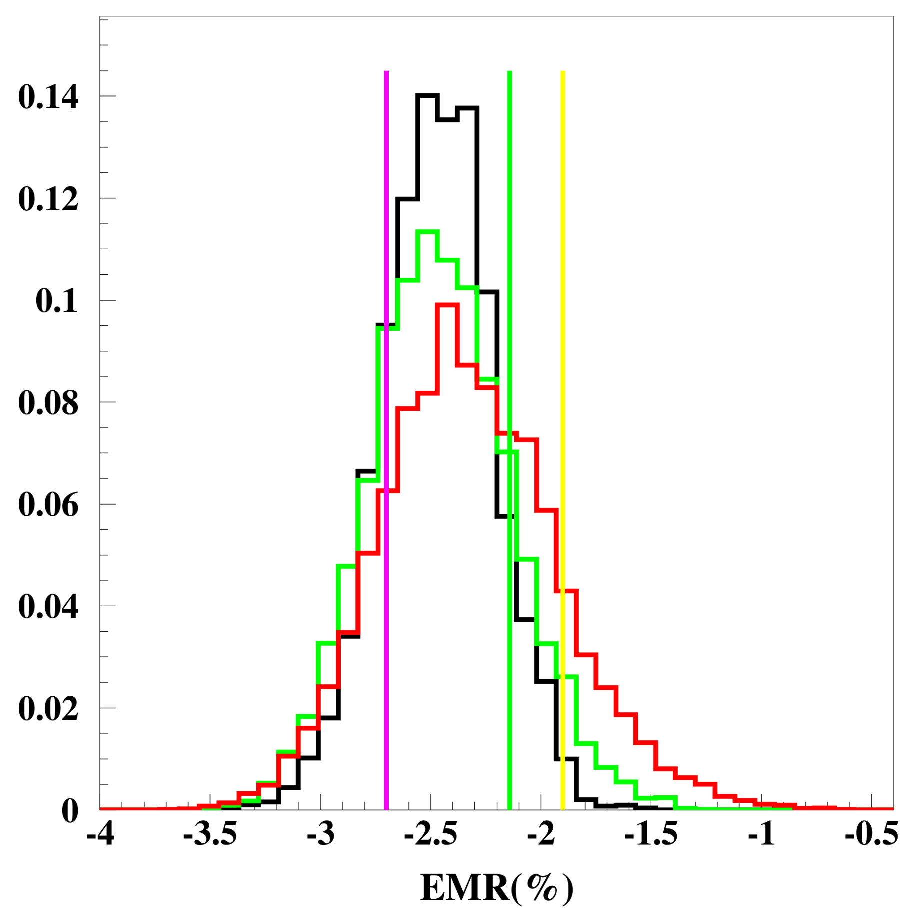
$$EMR(\%) = -\left(2.47\,^{+0.24}_{-0.24}\right)$$

- Full dataset: statistical & systematic errors

$$EMR(\%) = -\left(2.53\,^{+0.34}_{-0.35}\right)$$

- Reduced dataset (No P, No G)

$$EMR(\%) = -\left(2.45\,^{+0.47}_{-0.31}\right)$$



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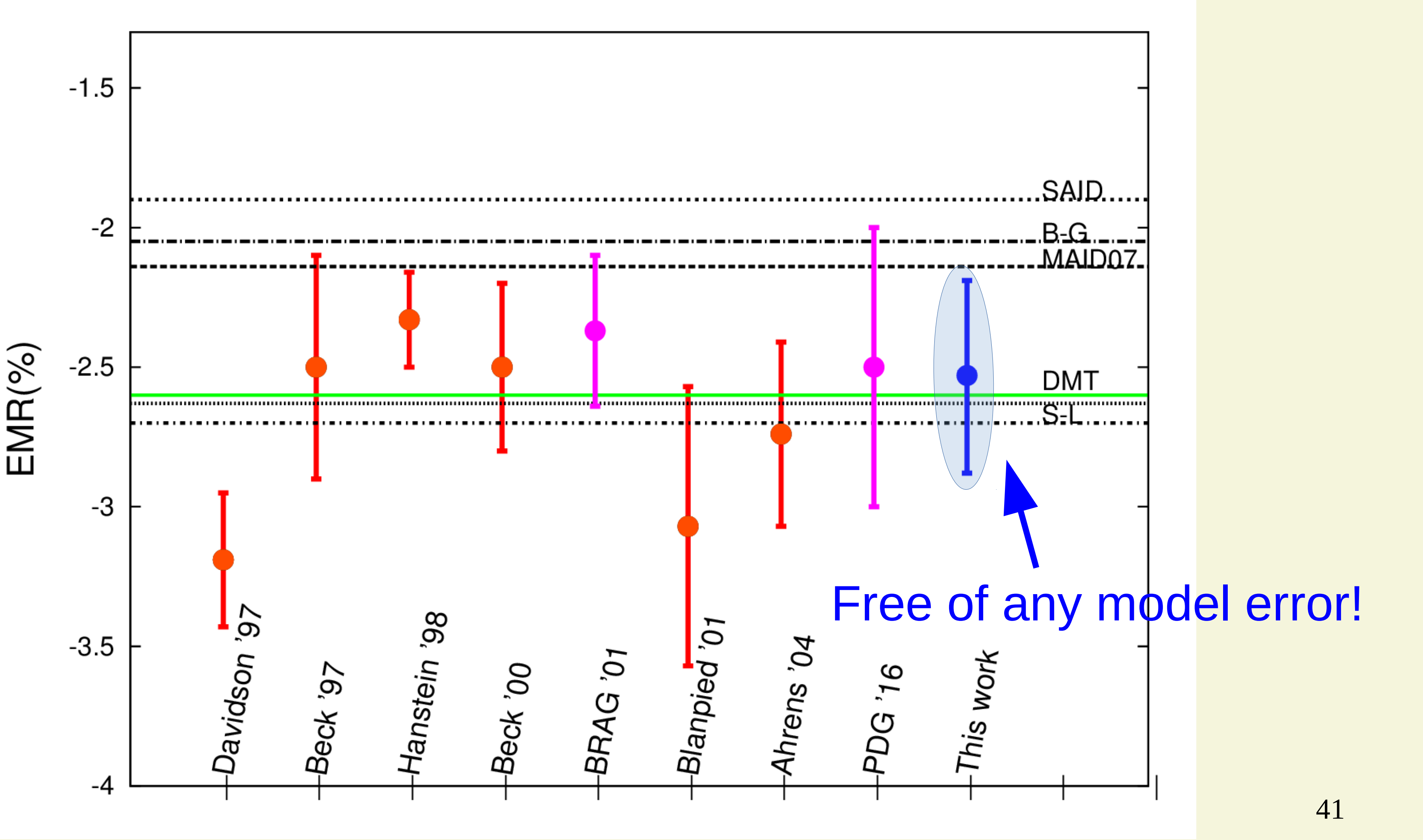
Model prediction   Mean Value



Summary / Conclusions

Using AMIAS, a model independent amplitude extraction from the most recent photoproduction data at the Δ(1232) was performed. It was found:

- Multipole amplitudes up to L = 5 were required to reach convergence.
- Some extracted amplitudes were found to be highly correlated between them.
- The background amplitudes were found to be more correlated than the resonant amplitudes.
- Through correlations, background amplitudes affect the extracted value of the resonant amplitudes and EMR.

Summary / Conclusions

- A $EMR(\%) = -2.5 \pm 0.3_{(\mathrm{stat+syst})}$ for the first time free of model error was determined.

- Good compatibility with phenomenological models and earlier analyses confirms the validity of the model assumptions behind the analysis methods used up to now.

Acknowledgements

I would like to thank

Reinhard Beck
Michael Ostrick
Sergey Prakhov
Lothar Tiator
Yannick Wunderlich

Thank you

www.cyi.ac.cy

The Cyprus Institute

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