



HELLENIC REPUBLIC  
National and Kapodistrian  
University of Athens



## Mastering Conic Sections for a Direct 3D Compton Image Reconstruction

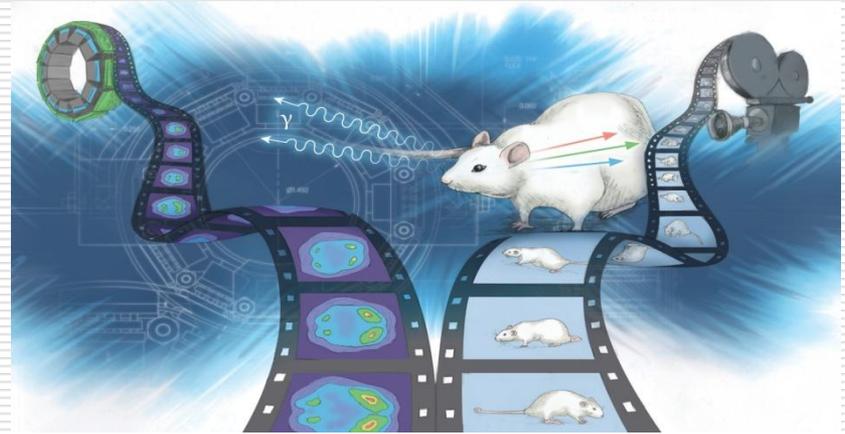


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and E. Stiliaris

# Outline

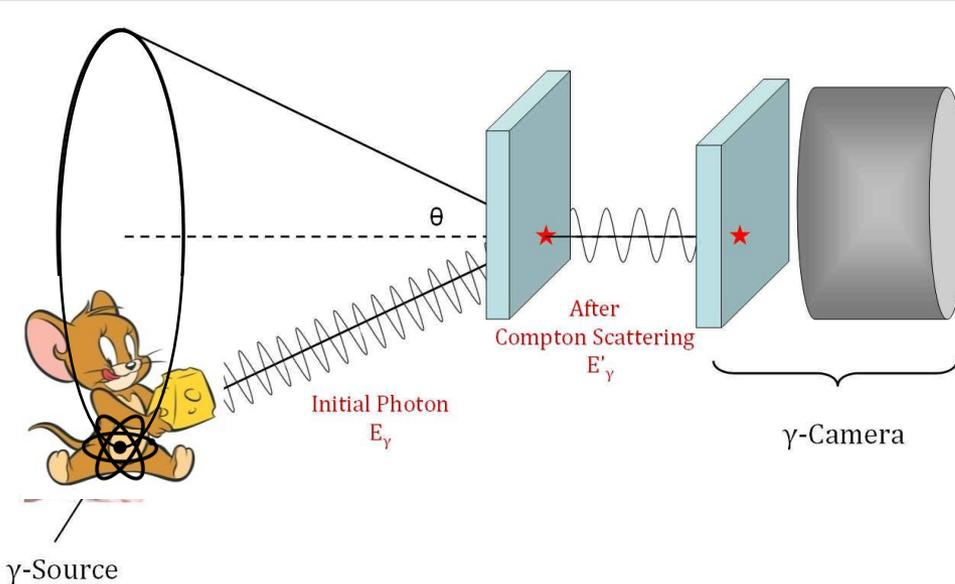
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- Introduction
  - ComptonRec
  - Gate Simulations
    - ▣ Iso Plane Source
    - ▣ One Point Source
    - ▣ Multiple Point Sources
  - Correction Matrix
  - Conclusions
- 



# Compton Camera

Compton scattering effect



## Advantages of the Compton Camera

- ❑ Uses electronic collimation
- ❑ Reconstructs a wide range of energy radiation
- ❑ Provides high sensitivity
- ❑ Improved energy, spatial and angular resolution
- ❑ Lessens the patient dose

$$\theta = \cos^{-1} \left[ 1 + m_0 c^2 \left( \frac{1}{E_\gamma} - \frac{1}{E'_\gamma} \right) \right]$$

# Reconstruction Algorithms

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## IMAGE RECONSTRUCTION

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graph TD; A[IMAGE RECONSTRUCTION] --> B[ANALYTICAL ALGORITHMS]; A --> C[ITERATIVE ALGORITHMS];
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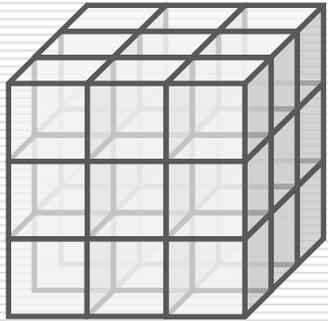
### ANALYTICAL ALGORITHMS

- ❑ Require large number of data and need to solve complex mathematical problems.
- ❑ Unstable
- ❑ Cannot handle complicated factors, present in the Compton Camera, mainly induced by the spatial intensity variation

### ITERATIVE ALGORITHMS

Need the use of spherical harmonics and due to the above mentioned variation are less efficient

# Pixelated Plane



**Volume  
of  
Interest**

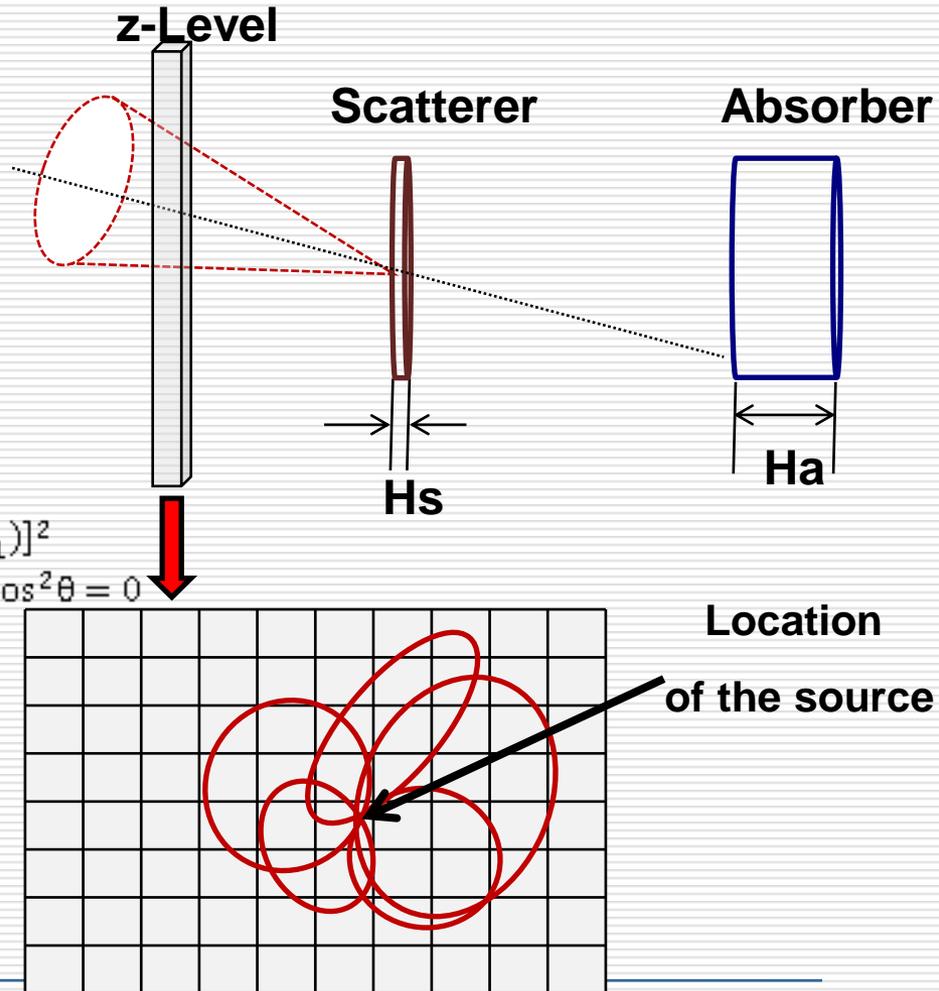
$$(\vec{u} \cdot \vec{d})^2 - (\vec{u} \cdot \vec{u}) \cdot (\vec{d} \cdot \vec{d}) \cdot \cos^2 \theta = 0$$

$$[(x - x_1) \cdot (x_2 - x_1) + (y - y_1) \cdot (y_2 - y_1) + (z - z_1) \cdot (z_2 - z_1)]^2 - [(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2] \cdot \|d\|^2 \cdot \cos^2 \theta = 0$$

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$Cy^2 + (Bx + E)y + (Ax^2 + Dx + F) = 0$$

$$y = \frac{-(Bx + E) \pm \sqrt{(Bx + E)^2 - 4C(Ax^2 + Dx + F)}}{2C}$$



# Image Reconstruction

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## Factors affecting the Detection Efficiency of the System

- 1) The interaction probability for a given tracer energy regarding the systems' characteristics.
  - 2) The physical acceptance of the events recorded by the apparatus.
  - 3) The non uniformity of the angular acceptance.
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# Image Reconstruction

The “Event Selection” component of the program rejects coincidence events that don't fulfill specific kinematical or energetic cuts.

## ❑ Kinematical Cuts

Reject events which a meaningless conical angle  $\theta$

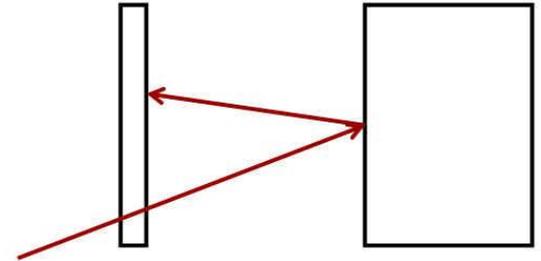
$$(|\cos\theta| > 1)$$

## ❑ Energetic Cuts

The acceptance criterion fulfills the equation

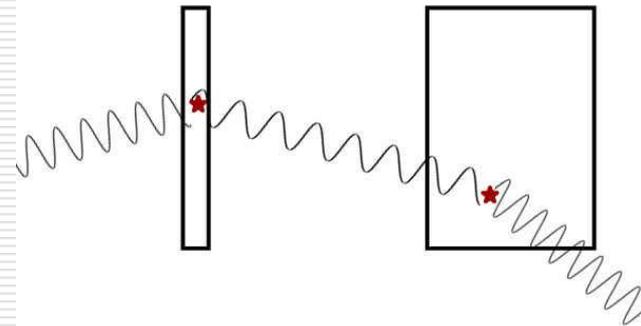
$$E_{\text{abs}} + E_{\text{scat}} = E_{\gamma} \pm \delta E.$$

**Inverse Coincidences**  
First in Absorber and then in Scatterer

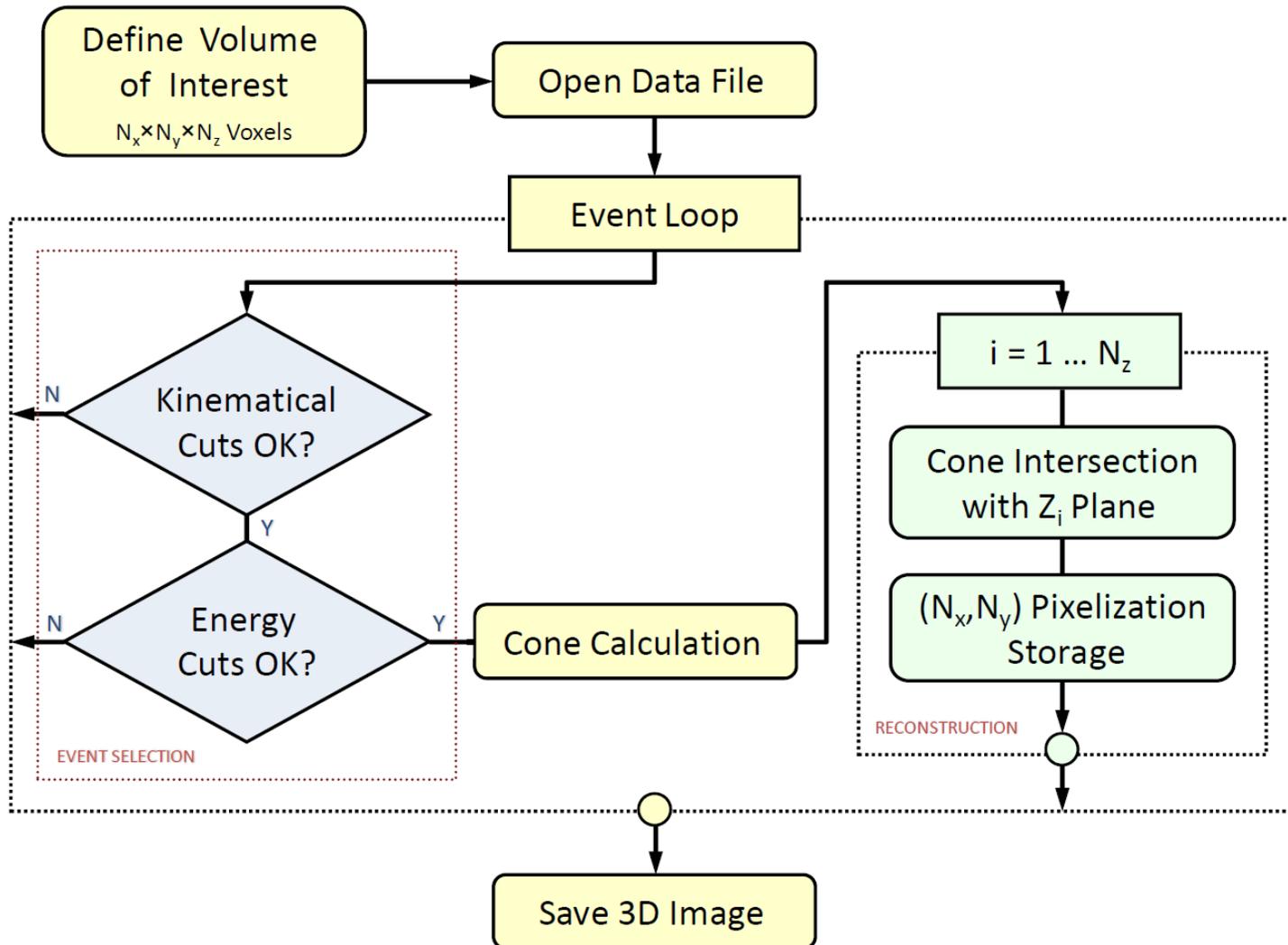


**Missing Energy**  
Photon escapes absorber

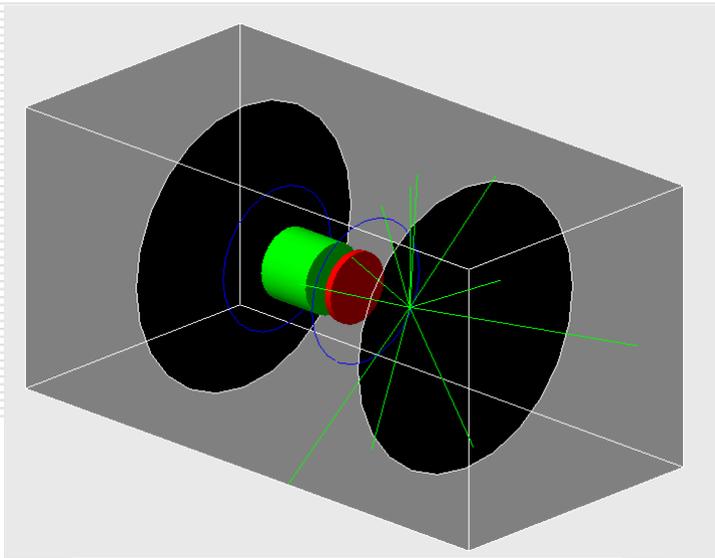
$$E_1 + E_2 < E_{\gamma}$$



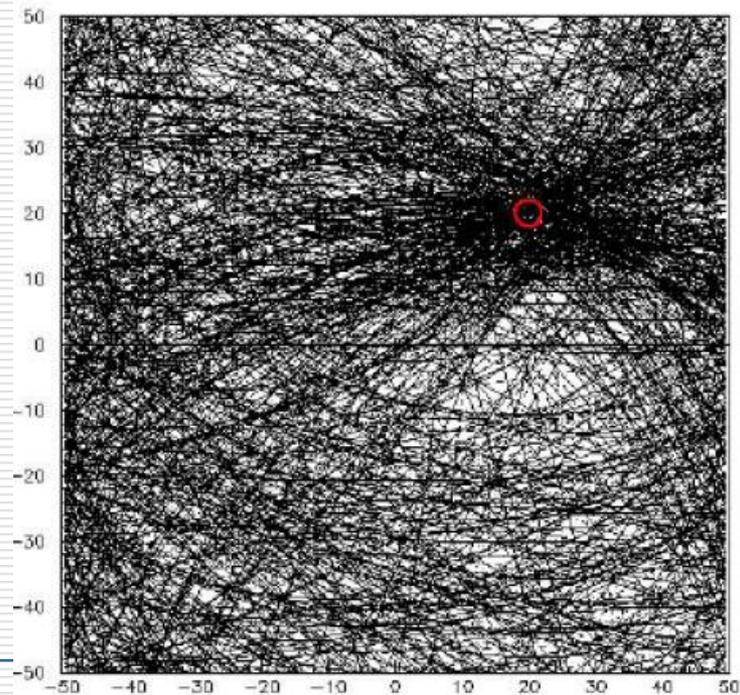
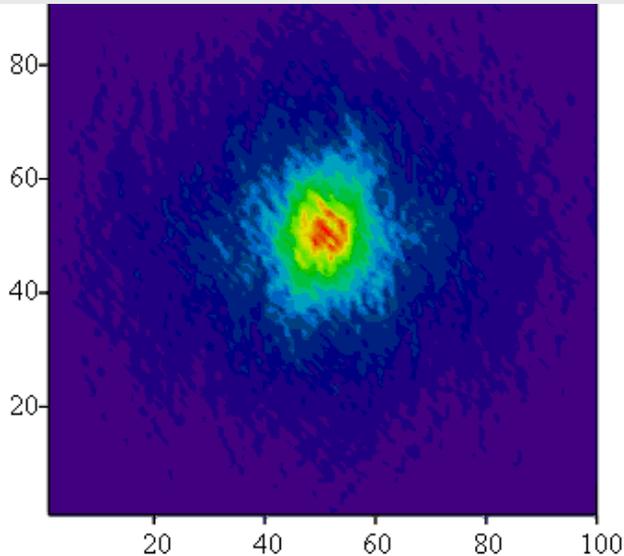
# Image Reconstruction



# Simulating the Compton Camera

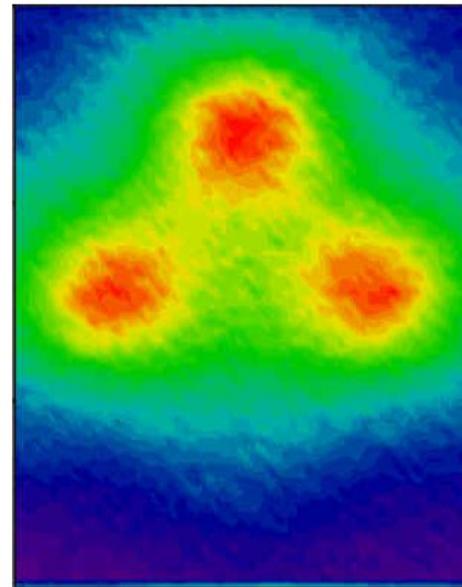
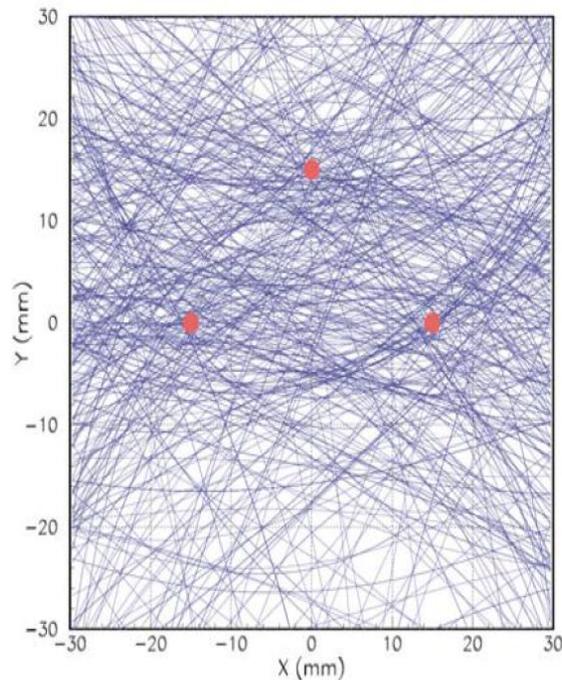


A **2 stage** Compton Camera is modeled using the **GEANT4/Gate** Monte Carlo simulation package, where the scatterer is a 2mm Silicon cylinder and the absorber a 50mm CsI cylinder, both with a 50mm diameter.



# Simulating the Compton Camera

A simple but not symmetric phantom was simulated in the **GEANT4/GATE** environment in order to test the algorithms efficiency, always taking into account the volume effects. The phantom consists of three spherical sources that form a triangle



**Left:** Intersection of the conical surfaces with the reconstruction plane for some of the accumulated events. The location of the three  $\gamma$ -sources is indicated with closed circles.

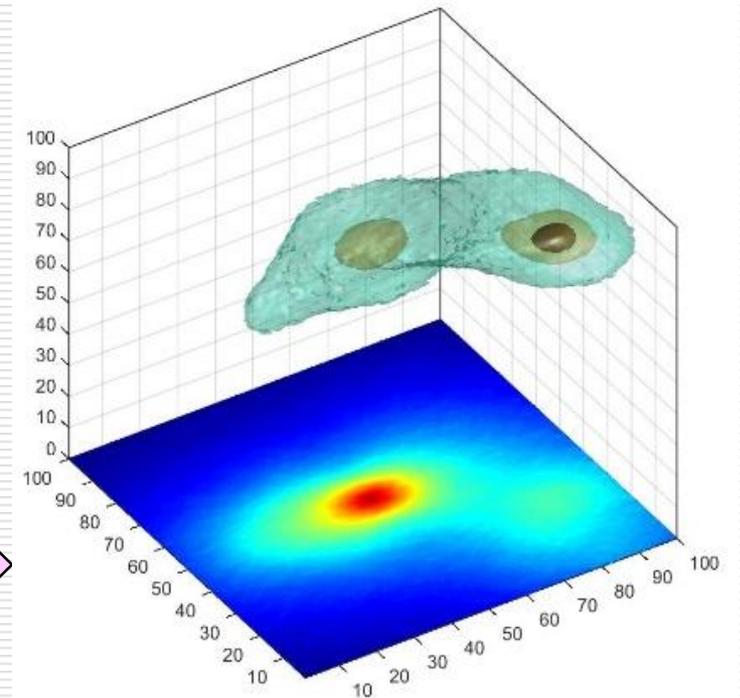
**Right:** Contour of the reconstructed planar image for the same phantom with full statistics

# Simulating the Compton Camera

A second simulation was performed also in the **GEANT4/GATE** environment. The phantom consists of three out of plane spherical sources  $(0,0,-20)$ ,  $(-15,0,-30)$ ,  $(+15,+15,-10)$ .

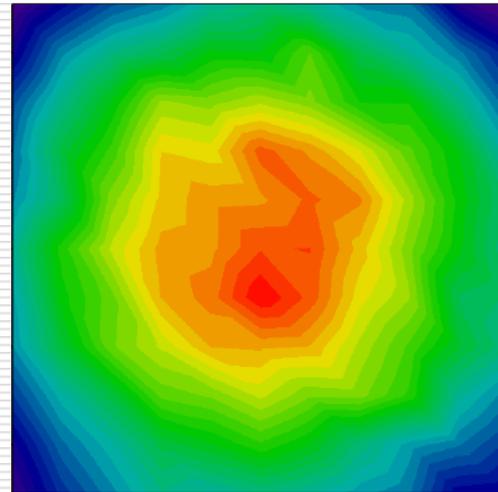
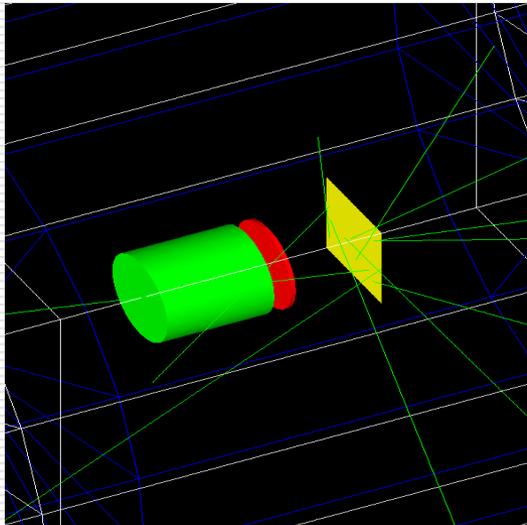


The reconstructed planar image needs density normalization due to the strong dependence on the source distance (volume effects).



# Correcting Image

The parameterization is accomplished by simulating a plane phantom (membrane) emitting isotropically photons of a constant energy  $E_\gamma$ .

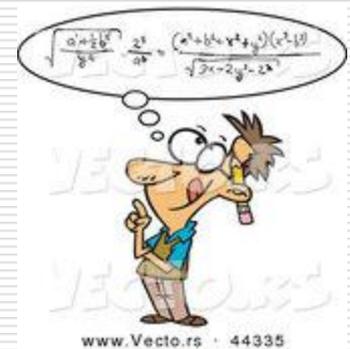
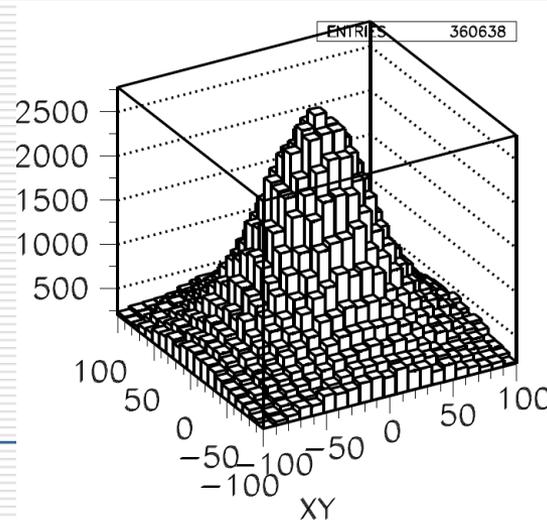
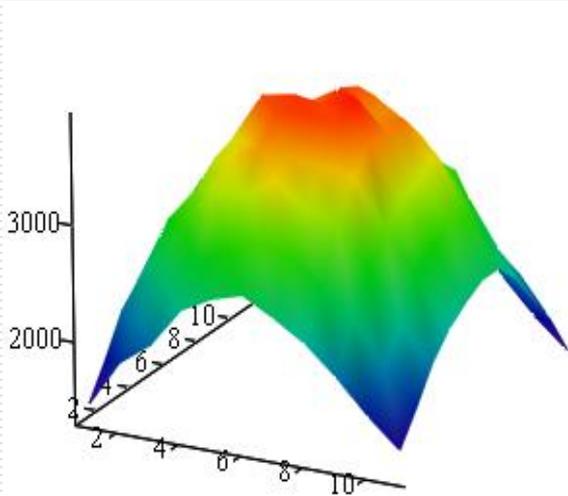


The events detected by the system are filtered so as to discard gammas that have not been fully absorbed or have not followed the desired interaction path. The x - y initial coordinates of the remaining gammas are then stored in a matrix whose density is depicted above.

# Correcting Image

The matrix that contains the amount of the detected gammas by the Compton Camera emitted from the homogeneous source (membrane) is fitted with a two-dimensional Gaussian function. The matrix is obtained by simulating a homogeneous source at a given distance from the camera and selecting all accepted events.

$$I(x,y,D) = A(D) \cdot e^{-\frac{(R-R_0)^2}{2[\sigma(D)]^2}}, \quad R^2 = x^2 + y^2$$

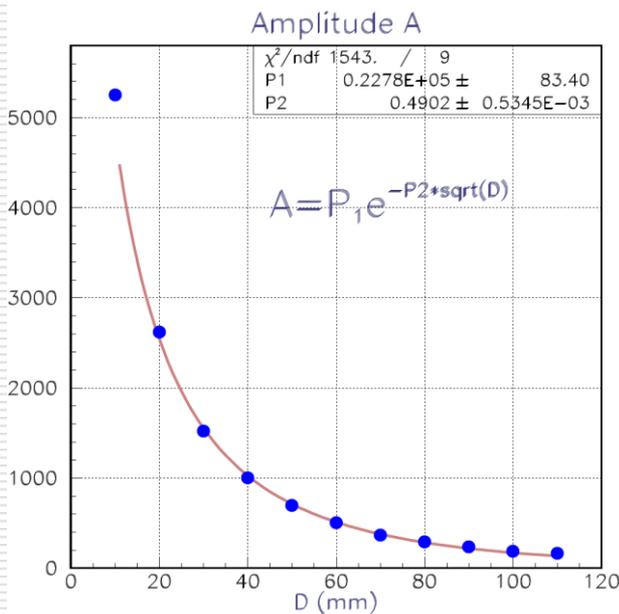


# Correction Matrix

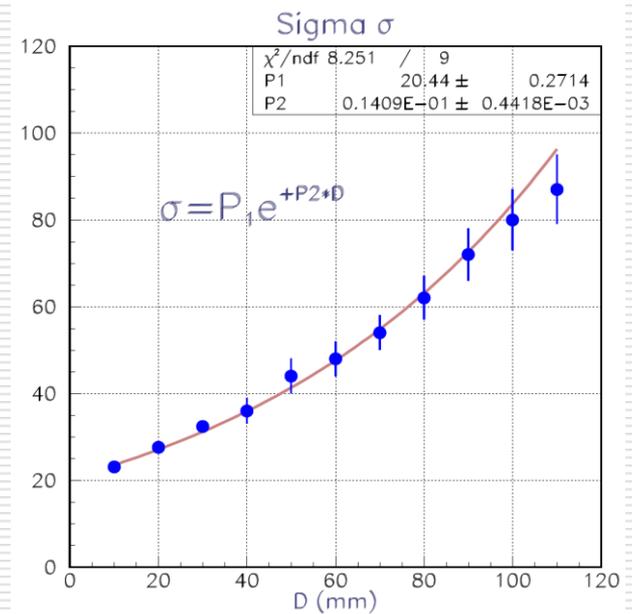
All recorded density distributions  $I(x,y,D)$  for several distances  $D$  are expressed with appropriate functions introducing two parameters :

- ❑  $A(D)$  (amplitude)
- ❑  $\sigma(D)$  (standard deviation)

according to a generalized formula valid for the Compton Camera under investigation.



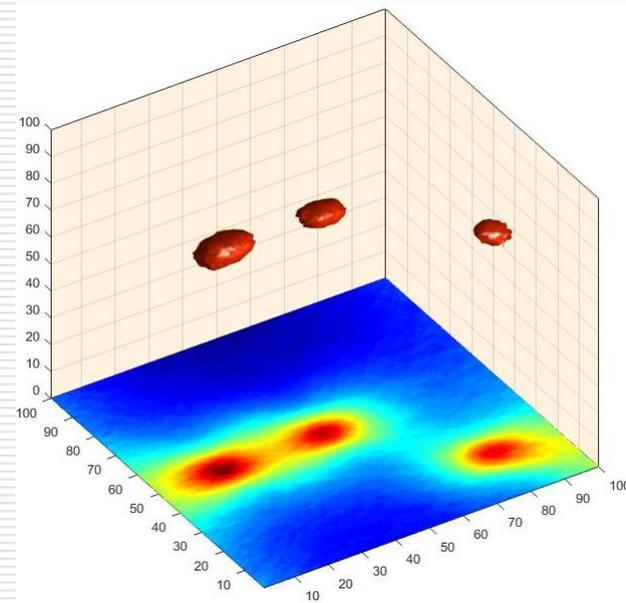
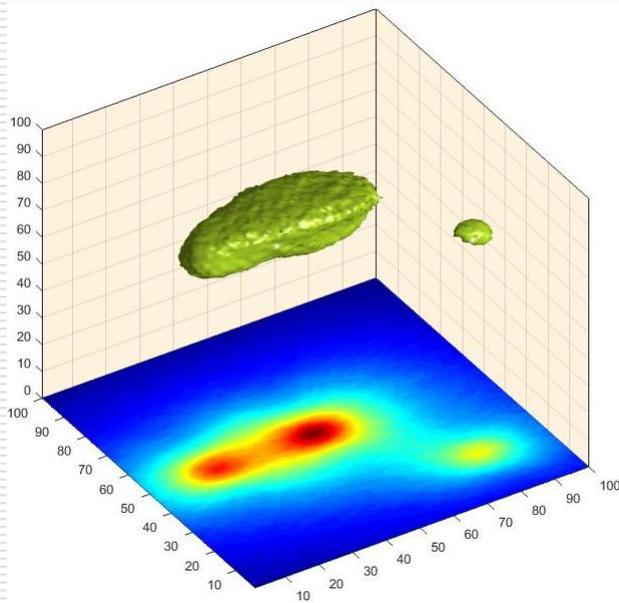
$$A(D) = A_0 e^{-A_1 \sqrt{D}}$$
$$\sigma(D) = \sigma_0 e^{+\sigma_1 D}$$



# Correcting Image

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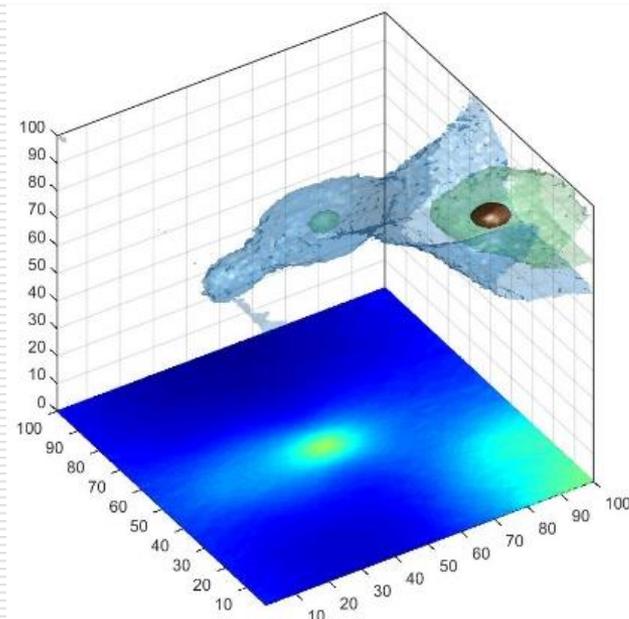
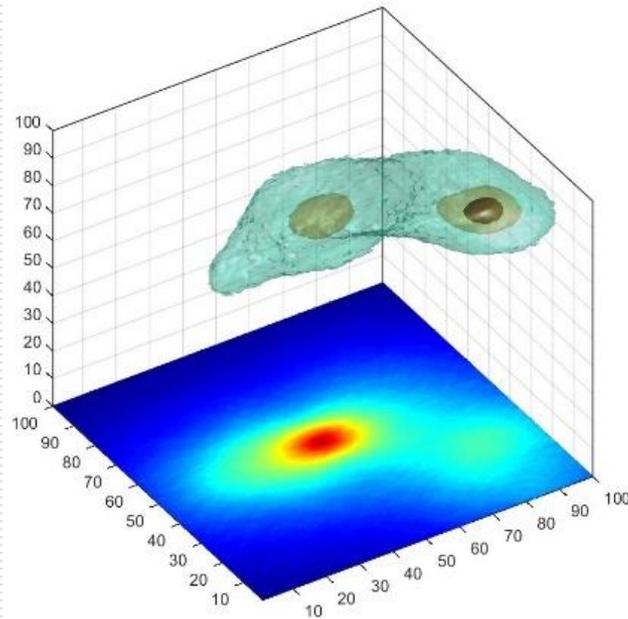
Having defined the general expression for the correction function  $I(x,y,D)$ , a 3D reconstruction can be now performed for several phantoms. The figures below depict the efficiency of the proposed reconstruction method for a phantom, consisting of three identical spherical sources with 2mm in diameter.



# Correcting Image

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Having defined the general expression for the correction function  $I(x,y,D)$ , a 3D reconstruction can be now performed for several phantoms. The figures below depict the efficiency of the proposed reconstruction method for a phantom, consisting of three identical spherical sources with 2mm in diameter.



# Conclusions

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## Correcting Volume Effects and Camera's Acceptance

- The detection efficiency for distributed  $\gamma$ -sources in a Compton Camera can be significantly improved by the presented correction method.
- Homogeneous plane sources are simulated and the collected  $\gamma$ -ray density distribution (correction matrix) is expressed as a function of the distance  $D$ .
- Using a universal two dimensional normal distribution, the two parameters of the correction matrix  $A(D)$  and  $\sigma(D)$  are directly expressed as a function of the distance.
- During the commonly used reconstruction procedure the parameterized correction matrix is utilized to improve the spatial resolution affected by both the geometrical and the physical acceptance of the system.



This technique is going to be applied in various out of plane phantoms with different radiotracer energies.

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**THANK YOU**



**FOR LISTENING**