

Theory of Heavy Ion Charge Exchange Reactions as Probes for Beta-Decay



H. Lenske

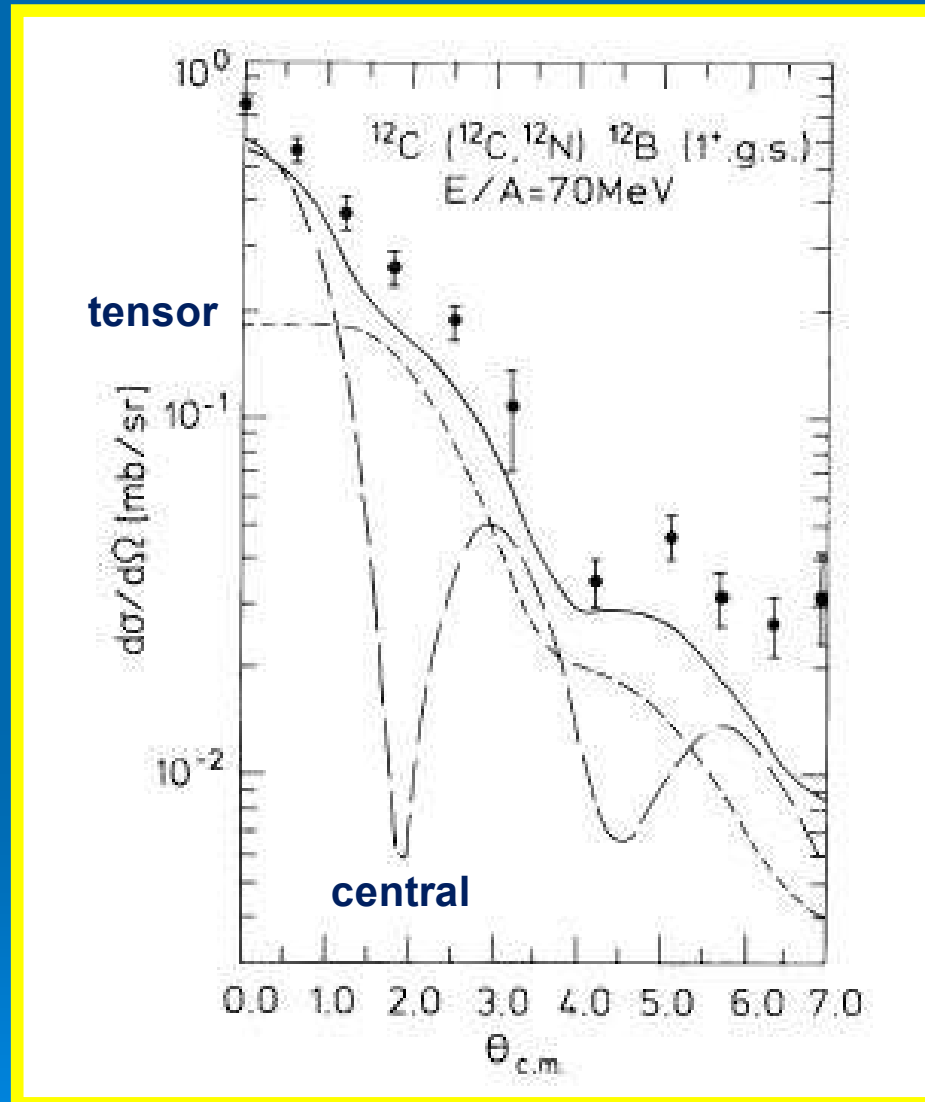
Institut für Theoretische Physik, JLU Giessen

Agenda:

- Heavy ion Single Charge Exchange (SCE) reactions
- SCE reactions and $1\nu 1\beta$ decay
- 2-step Double Single Charge Exchange reactions (DSCE)
- 1-step DCE Mechanism: „Majorana“ DCE reactions (MDCE)
- Connection to NME of $2\nu 2\beta$ and $0\nu 2\beta$ double beta decay
- First DCE results
- Summary and Outlook

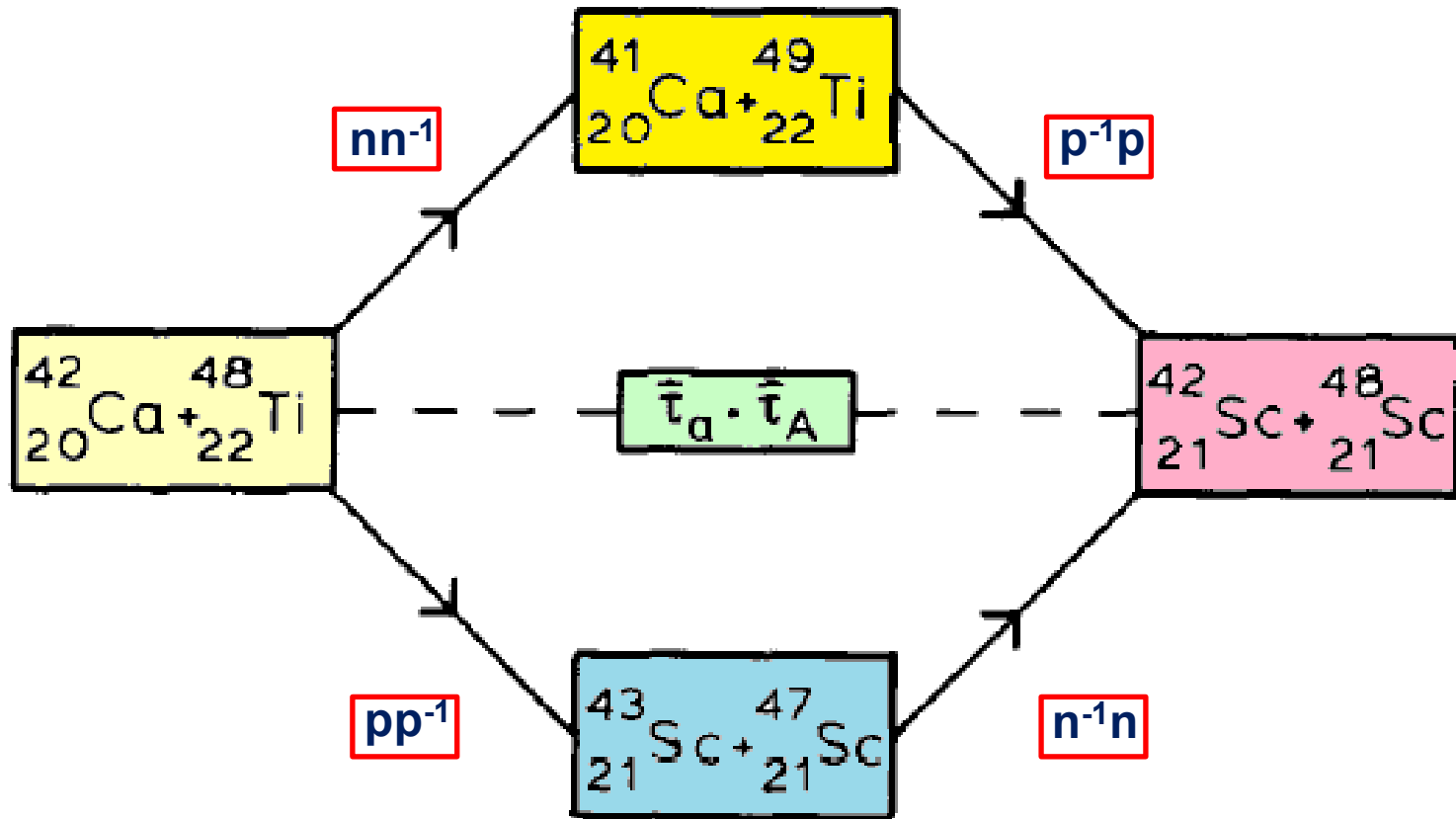
Single Charge Exchange Reactions

Heavy Ion SCE Reactions: Rank-1 Central and Rank-2 Tensor Interaction



H. Lenske et al.,
Phys. Rev. Lett.
62, 1457 (1989)

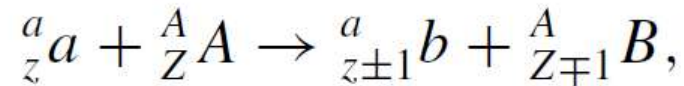
Heavy Ion Single Charge Exchange Dynamics: 1-Step Direct and 2-Step Transfer Sequential Charge Exchange



C. Brendel, H.L. *et al.*, Nuclear Physics A477 (1988) 162

H. Lenske, HINPW 5, 2019

Heavy Ion Single Charge Exchange Reactions



Peripheral, coherent reaction \rightarrow Direct Reaction Theory

$$M_{\alpha\beta}(\mathbf{k}_\beta, \mathbf{k}_\alpha) = \langle \chi_\beta^{(-)}, bB | T_{NN} | aA, \chi_\alpha^{(+)} \rangle$$

$$T_{NN}(\mathbf{p}) = \sum_{S=0,1, T=0,1} \{ V_{ST}^{(C)}(p^2) [\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_B]^S + \delta_{S1} V_T^{(Tn)}(p^2) S_{12}(\mathbf{p}) \} [\boldsymbol{\tau}_a \cdot \boldsymbol{\tau}_A]^T$$

Ion-Ion ISI and FSI Interactions \rightarrow Optical Potential

$$U_{\text{opt}} = \langle aA | T_{NN} | aA \rangle \approx \int \frac{d^3 p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} V_{00}^{(c)}(\mathbf{p}) \rho_a(\mathbf{p}) \rho_A(\mathbf{p}) + \dots$$

H. Lenske, J. Bellone, M. Colonna, J.-A. Lay, PRC 98:044620 (2018)

Separation of Reaction and Nuclear Dynamics

$$M_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) = \int \frac{d^3p}{(2\pi)^3} \langle \chi_\beta^{(-)} | e^{-i\mathbf{p}\cdot\mathbf{r}} \mathcal{U}_{\alpha\beta}(\mathbf{p}) | \chi_\alpha^{(+)} \rangle$$

$$\mathcal{U}_{\alpha\beta} \sim \langle J_b M_b J_B M_B | T_{NN}^{(C)} + T_{NN}^{(Tn)} \dots | J_a M_a J_A M_A \rangle$$

$$N_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta, \mathbf{p}) = \frac{1}{(2\pi)^3} \langle \chi_\beta^{(-)} | e^{-i\mathbf{p}\cdot\mathbf{r}} | \chi_\alpha^{(+)} \rangle,$$

$M_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta, \mathbf{p})$

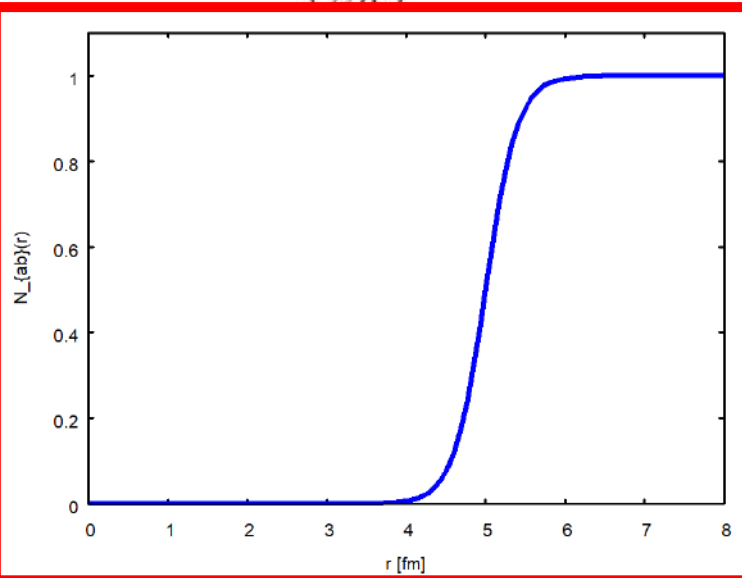
$\mathbf{k}_\beta, \mathbf{p}$

Dis

ix:

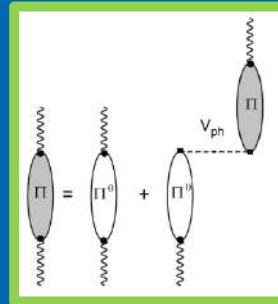
$$N_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta, \mathbf{p})_{p=}$$

$$\delta(k_\alpha - k'_\alpha) \frac{1}{4\pi} S_\alpha$$



PHYSICAL REVIEW C **98**, 044620 (2018)

Nuclear Response to Charge Changing Interactions



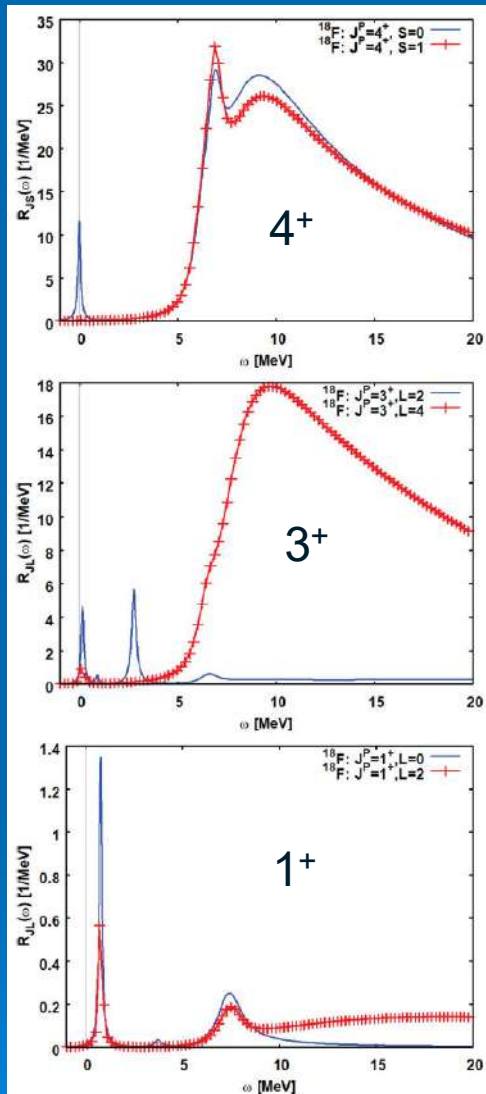
- $A \leq 12$: Transition densities from Shell Model calculations
- $A \geq 12$: Transition densities by RPA methods - „ccQRPA“:

$$\Omega_c^\dagger = \sum_{pn} \left(x_{pn}^{(c)} \left[\alpha_p^\dagger \alpha_n^\dagger \right]_c - y_{pn}^{(c)*} \left[\alpha_n \alpha_p \right]_c \right)$$

Response Functions and Polarization Tensor

$$R_\lambda(\omega) = \sum_c \left| \langle c | T_\lambda | 0 \rangle \right|^2 \delta(E_c - \omega) = -\frac{1}{\pi} \text{Im} \left(\langle 0 | T_\lambda^\dagger G(\omega) T_\lambda | 0 \rangle \right)$$

$^{18}\text{O} \rightarrow ^{18}\text{F}$



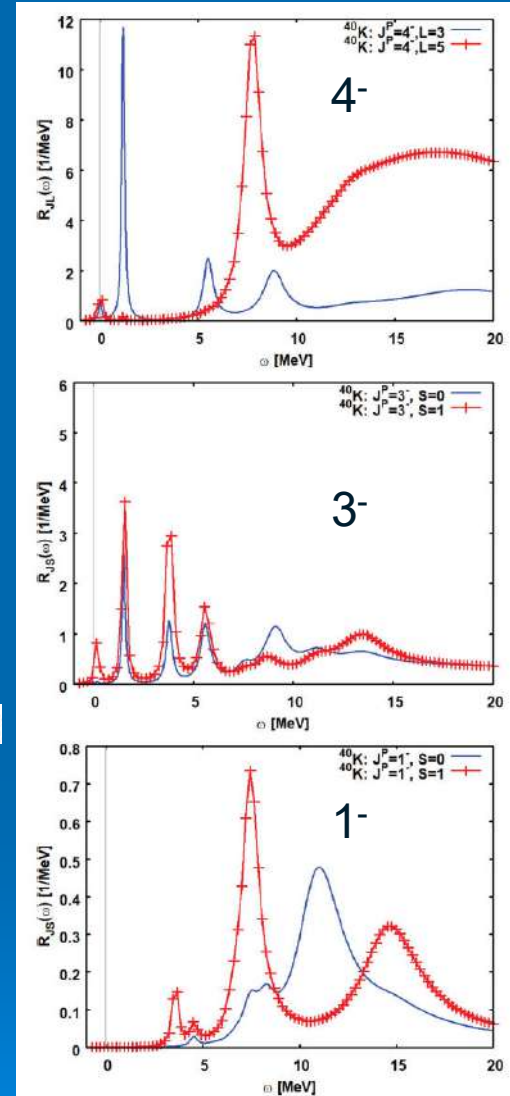
QRPA Response Functions

Transition Operator:

$$T_{LSJM} = \left(\frac{r}{R_d} \right)^L [\sigma^S \otimes Y_L]_{JM} \tau_{\pm}$$

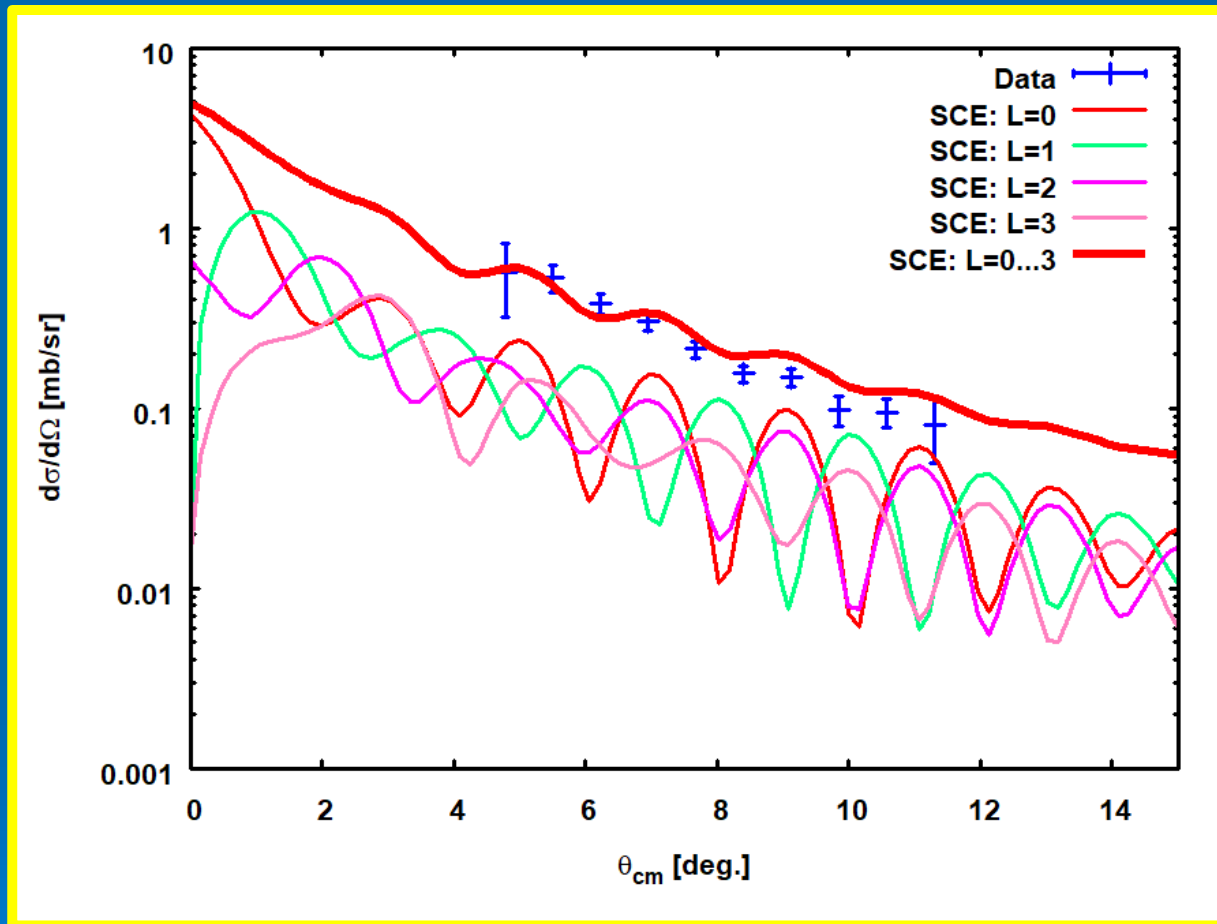
PHYSICAL REVIEW C **98**, 044620 (2018)

$^{40}\text{Ca} \rightarrow ^{40}\text{K}$



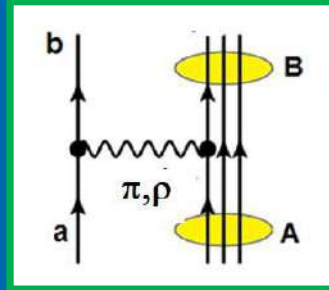


$$T_{\text{Lab}} = 15 \text{ A MeV}$$



Data: NUMEN collaboration, to be published

Nuclear Interactions and beta-Decay



Strong Interaction

Weak Interaction

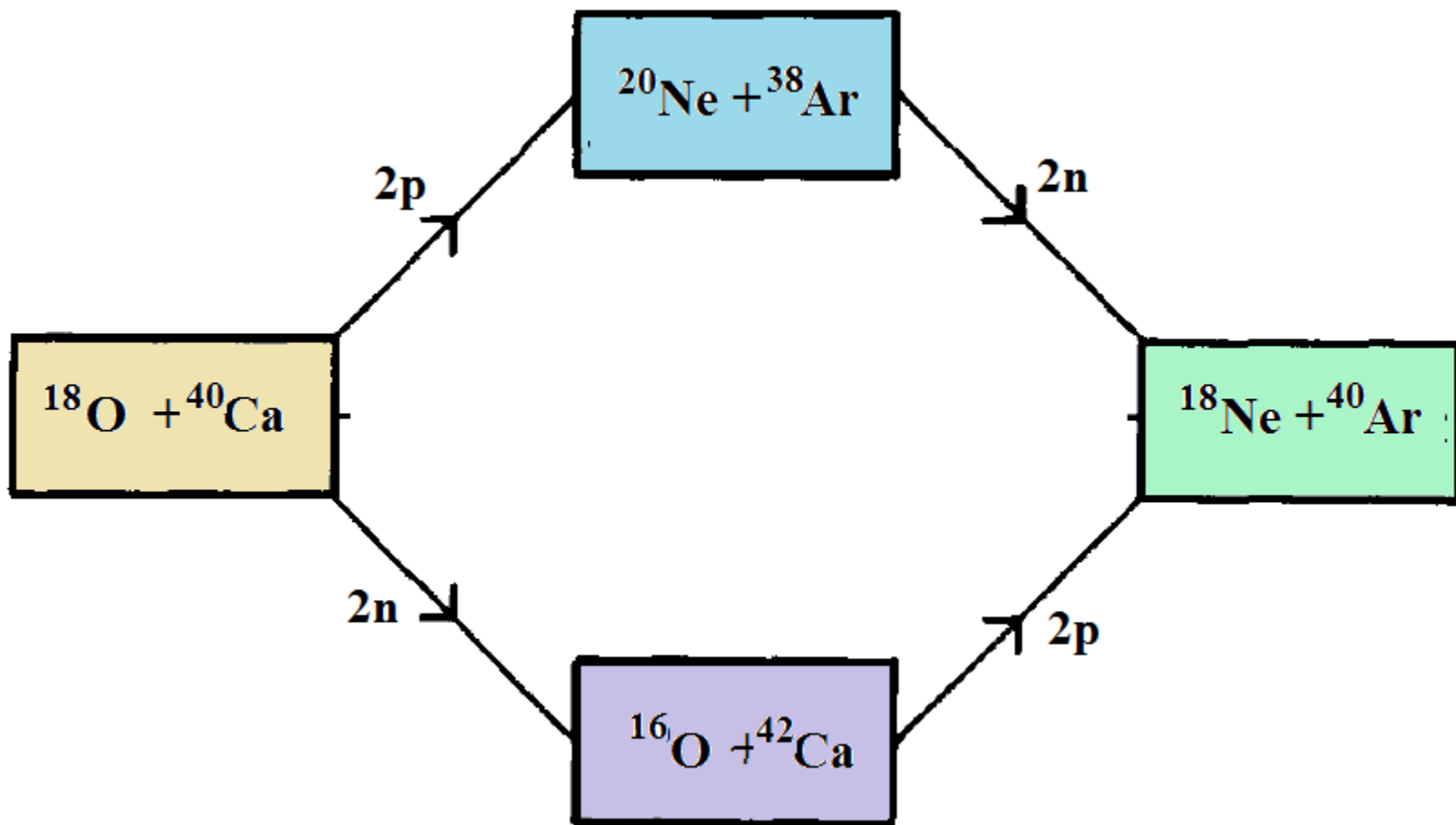
$V_{NN} \sim V_{01}(q^2) \tau_{\pm} \tau$	\leftrightarrow	$g_F(q^2) \tau_{\pm}$	"Fermi"
$+ V_{11}(q^2) \sigma_1 \cdot \sigma_2 \tau_{\pm} \tau$	\leftrightarrow	$g_A(q^2) \sigma \tau_{\pm}$	"Gamow-Teller"
$+ V_{T1}(q^2) S_{12} \tau_{\pm} \tau$	\leftrightarrow	$g_M(q^2) \sigma \times q \tau_{\pm}$	"weak magnetic"
$+ \dots$			

Rank-2 tensor operator: $S_{12} = \frac{1}{q^2} [3\sigma_1 \cdot q \sigma_2 \cdot q - \sigma_1 \cdot \sigma_2 q^2]$

Nuclear Double Charge Exchange Processes

Scheme of a Double Charge Exchange Reaction

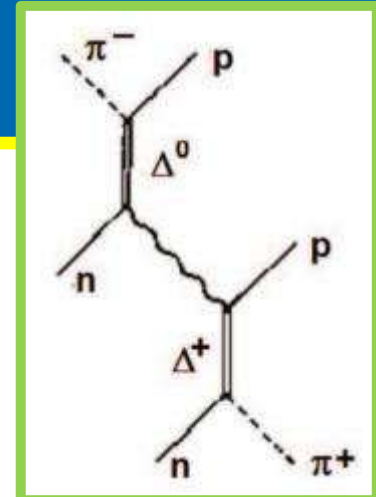
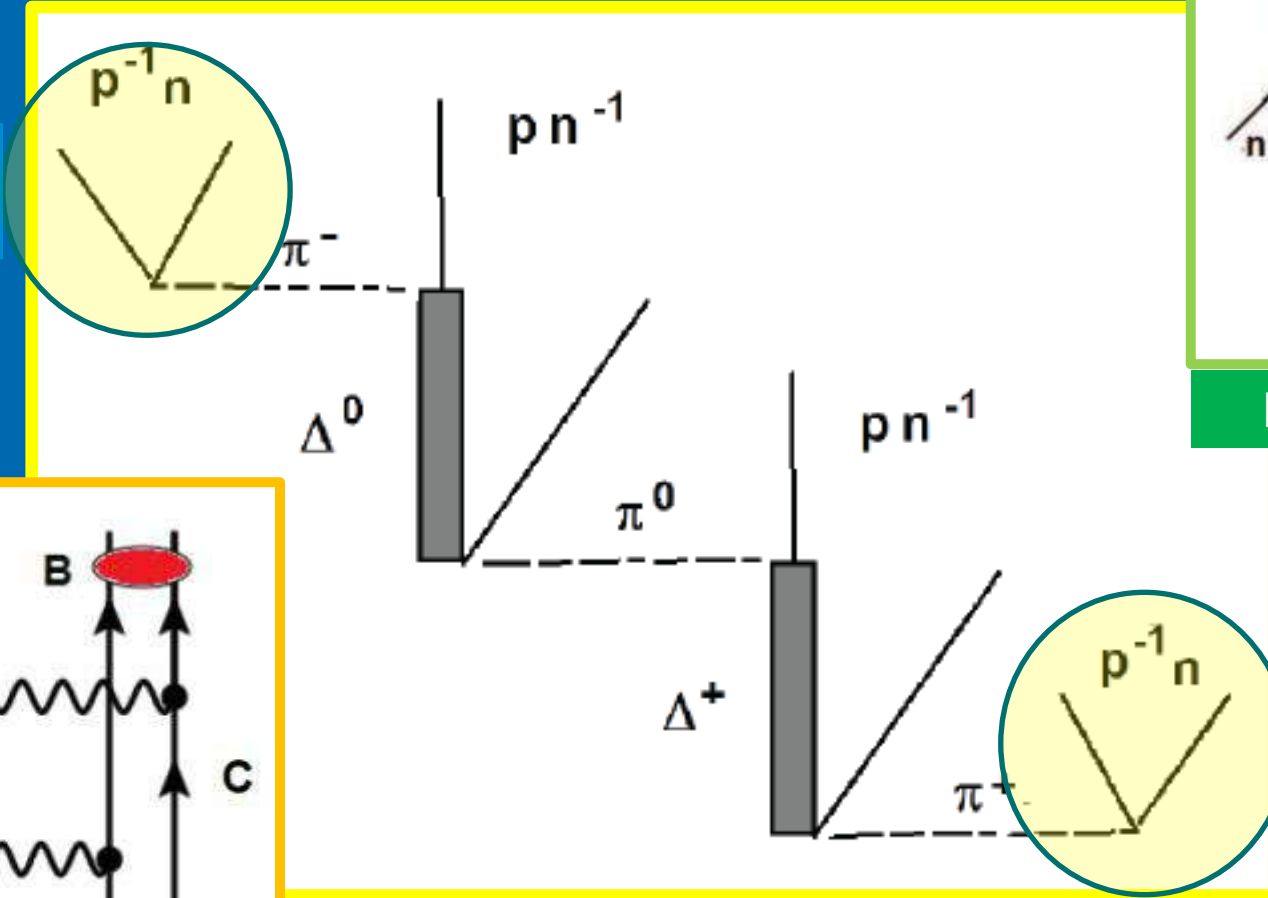
The „traditional“ 2N transfer picture ~ 1975...2015



Heavy-Ion DCE by the Two-Nucleon Mechanism

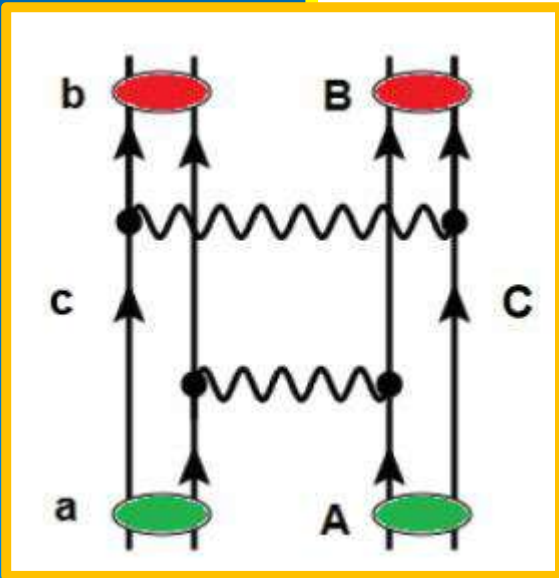
...and double SCE

ph-Source
Projectile

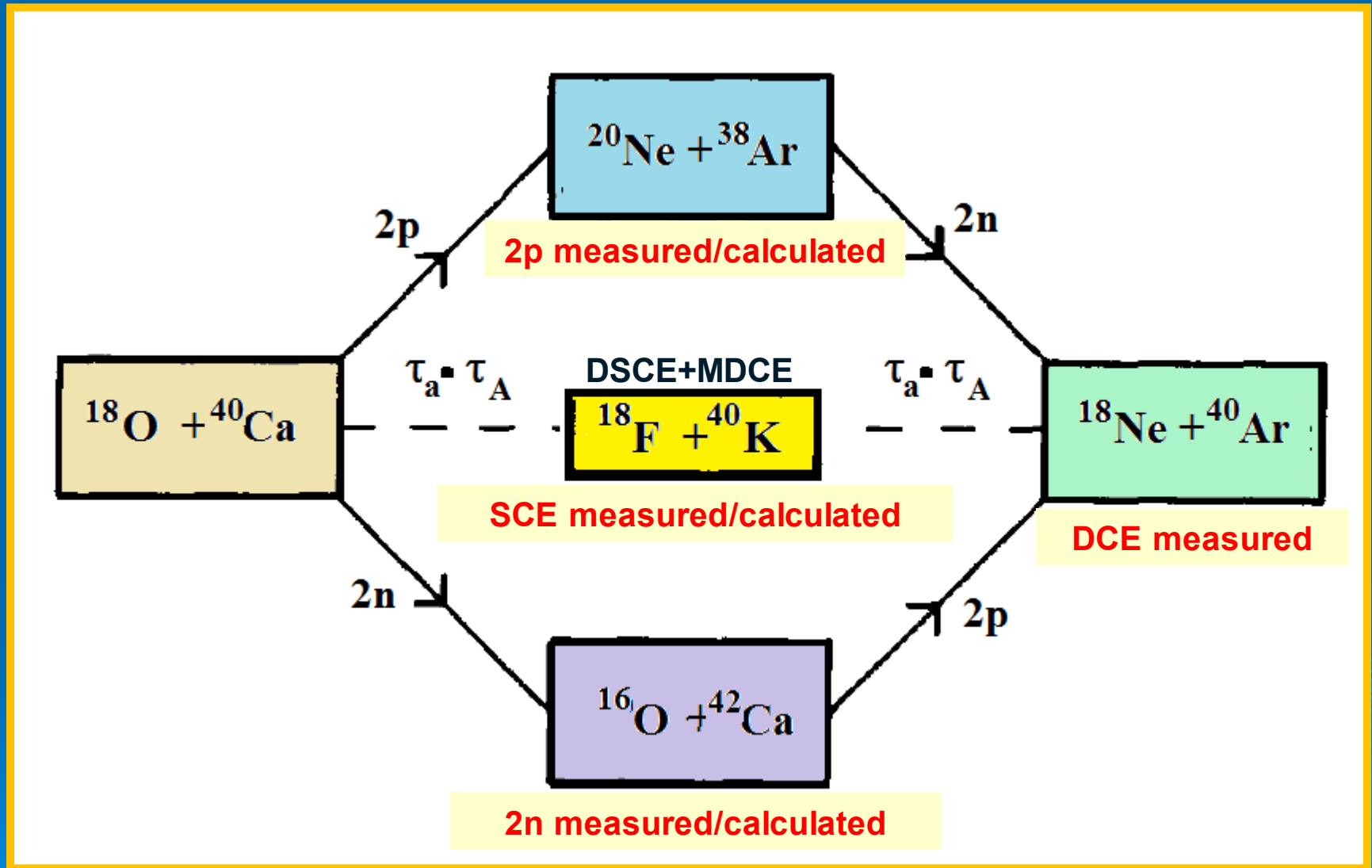


Pion-DCE

ph-Source
Projectile



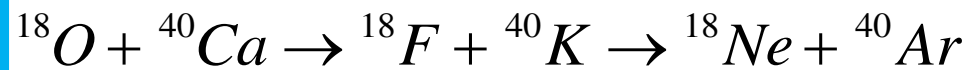
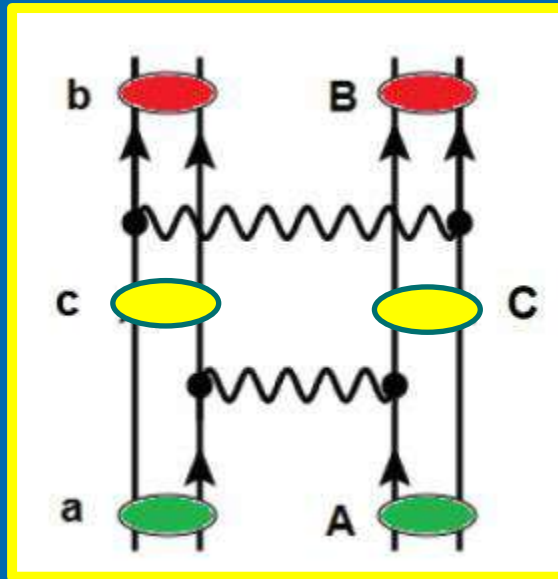
The NUMEN Scheme of a Double Charge Exchange Reaction: Double-SCE (DSCE) plus Majorana-DCE (MDCE)



Double-Single Charge Exchange (DSCE) Reactions

DSCE:

Double Charge Exchange by Sequential Single Charge Exchange



Reaction Amplitude

$$M_{\alpha\beta}^{(DSCE)}(\mathbf{k}_{bB}, \mathbf{k}_\alpha) = \langle \chi_\beta^{(-)}, bB | T_{NN} \mathcal{G}^{(+)}(\omega) T_{NN} | aA, \chi_{aA}^{(+)} \rangle.$$

Evaluation of the DSCE Amplitude

$$M_{\alpha\beta}^{(DSCE)}(\mathbf{k}_\beta, \mathbf{k}_\alpha) = \sum_{c,C} \int \frac{d^3 k_\gamma}{(2\pi)^3} M_{bB,cC}^{(SCE)}(\mathbf{k}_\beta, \mathbf{k}_\gamma) G_{cC}(\omega_\gamma, \omega_\alpha) \tilde{M}_{cC,aA}^{(SCE)}(\mathbf{k}_\gamma, \mathbf{k}_\alpha),$$

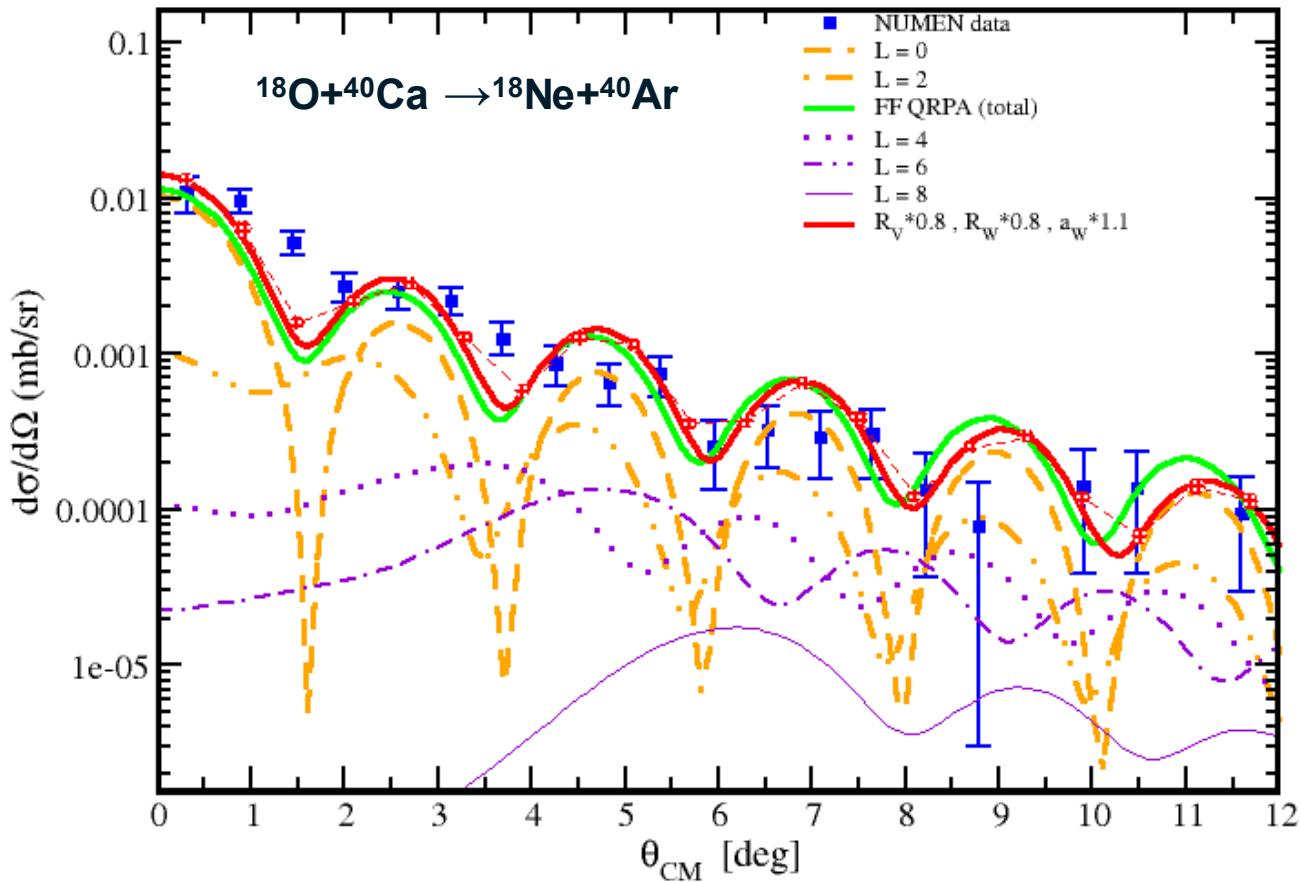
ISI/FSI → Distortion Coefficient:

$$N_{\beta\gamma}(\mathbf{p}) = \frac{1}{(2\pi)^3} \langle \chi_\beta^{(-)} | e^{-i\mathbf{p}\cdot\mathbf{r}} | \chi_\gamma^{(+)} \rangle$$

...and making use of the analytic properties of RPA Green's functions:

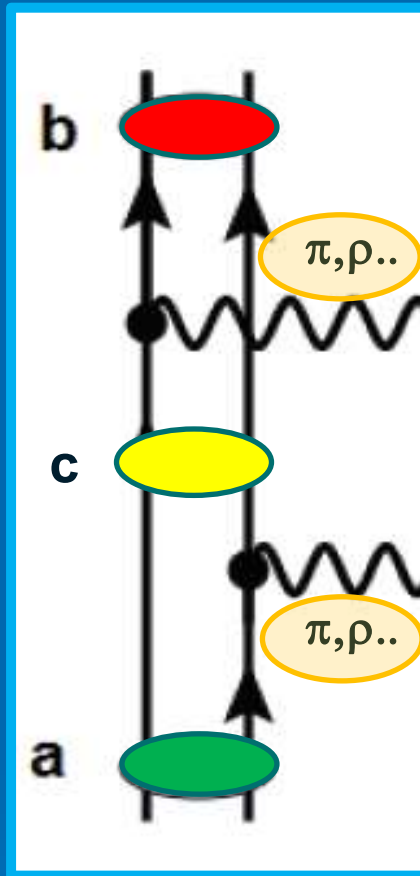
$$M_{\beta\alpha}^{(DSCE)}(\mathbf{k}_\beta, \mathbf{k}_\alpha) = \sum_{S_1, S_2, T=1} \int \frac{d^3 k_\gamma}{(2\pi)^3} \int d^3 p_1 d^3 p_2 N_{\beta\gamma}(\mathbf{p}_2) \tilde{N}_{\gamma\alpha}(\mathbf{p}_1) t_{S_2 T}(p_2^2) t_{S_1 T}(p_1^2) \\ \times \oint \frac{d\zeta}{2i\pi} \Pi_{S_2 S_1}^{(ba)\dagger} \left(\frac{1}{2} \tilde{\kappa} - \zeta - i\eta, \mathbf{p}_2, \mathbf{p}_1 \right) \cdot \Pi_{S_2 S_1}^{(BA)} \left(\frac{1}{2} \tilde{\kappa} + \zeta + i\eta', \mathbf{p}_2, \mathbf{p}_1 \right)$$

DSCE Results in Pole Approximation

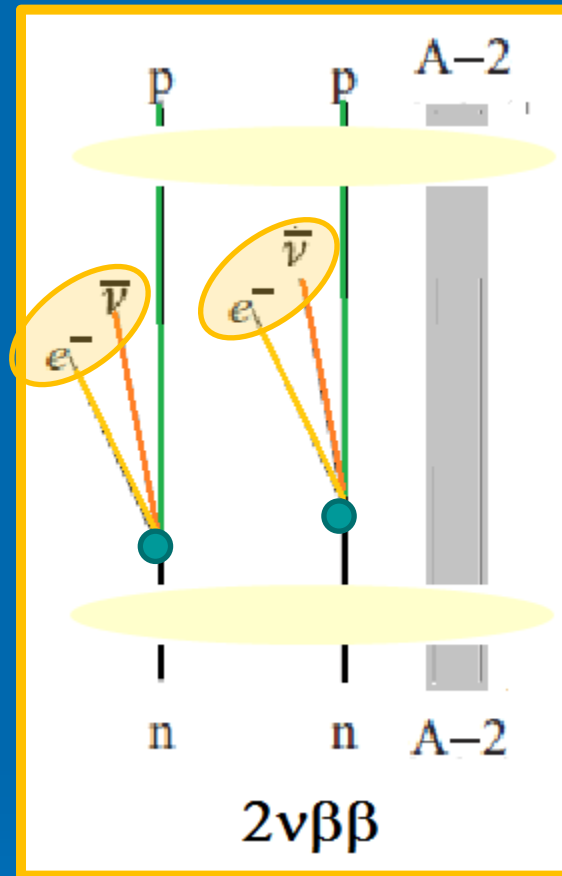


Theory: J. Bellone, M. Collona, H.L. (2019);
Data: F. Cappuzzello et al., EPJ A51 (2015)

DSCE and $2\nu 2\beta$ Beta Decay



2nd order
Strong Interaction



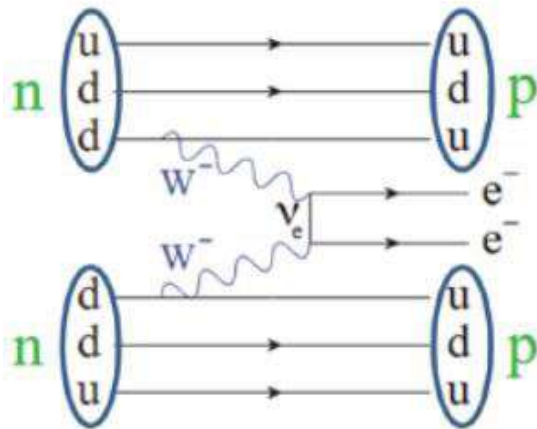
2nd order
Weak Interaction

Double Charge Exchange through 2-Body Interactions:

„Majorana“ DCE

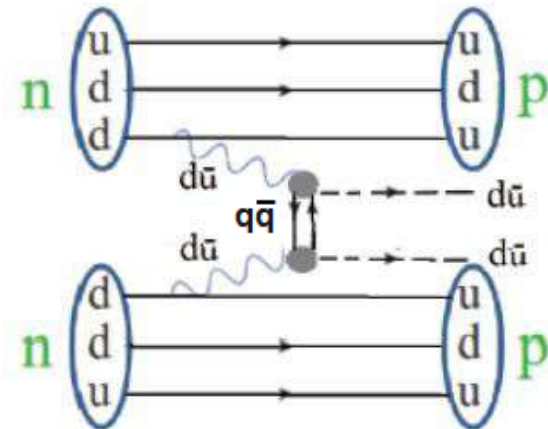
Weak Interaction $0\nu 2\beta$ decay and Strong Interaction Analogue

$0\nu 2\beta$ Decay



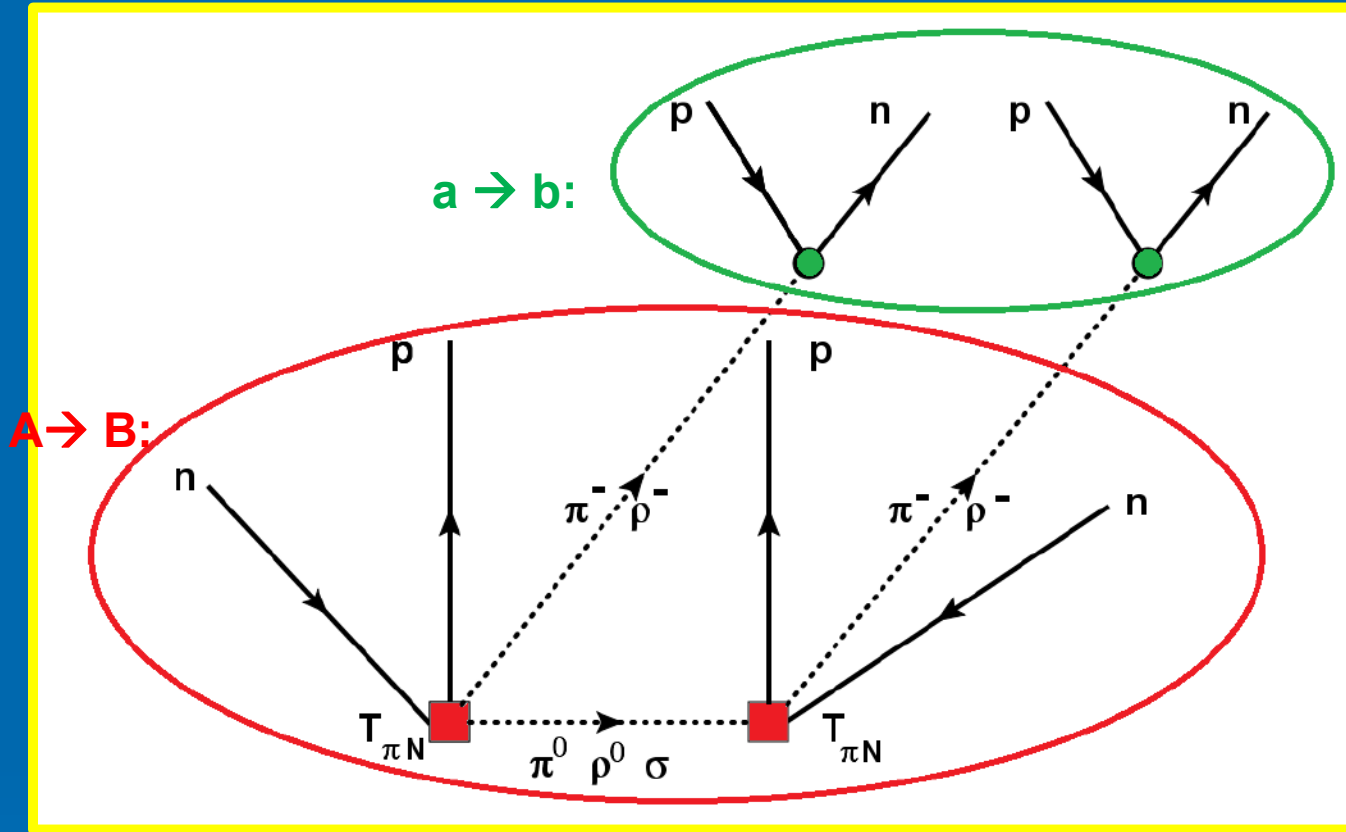
- simultaneous $d \rightarrow u$ $\Delta q = +1$ transitions by emission of a virtual weak gauge boson W^-
- $W^- \rightarrow e^- + \bar{\nu}_e / \nu_e$: decay into electron and Majorana neutrino
- Correlation of the two events by exchange of the virtual $\nu_e \bar{\nu}_e$ pair
- Emission of two electrons ON their mass-shell: $p_e^2 = m_e^2$
- Direct observation (in principle)

Hadronic Analogue

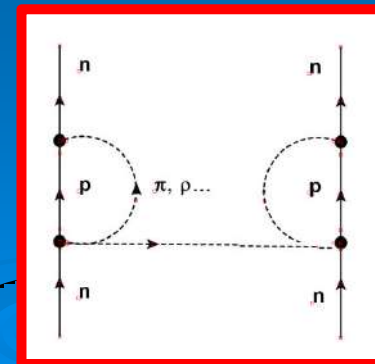


- simultaneous $d \rightarrow u$ $\Delta q = +1$ transitions by emission of a virtual $d\bar{u}$ vector pair $\leftrightarrow \rho^-$ meson
- $\rho^- \rightarrow \pi^- + \pi^0$: decay into a pair of pions
- Heavy vector mesons ρ^{*-}
- Correlation of the two events by exchange of the virtual $q\bar{q}$ pair as contained in $\pi^0 \cong (d\bar{d} + u\bar{u})/\sqrt{2}$
- Emission of two π^- OFF their mass-shell: $p_{\pi}^2 \neq m_{\pi}^2$
- No direct observation

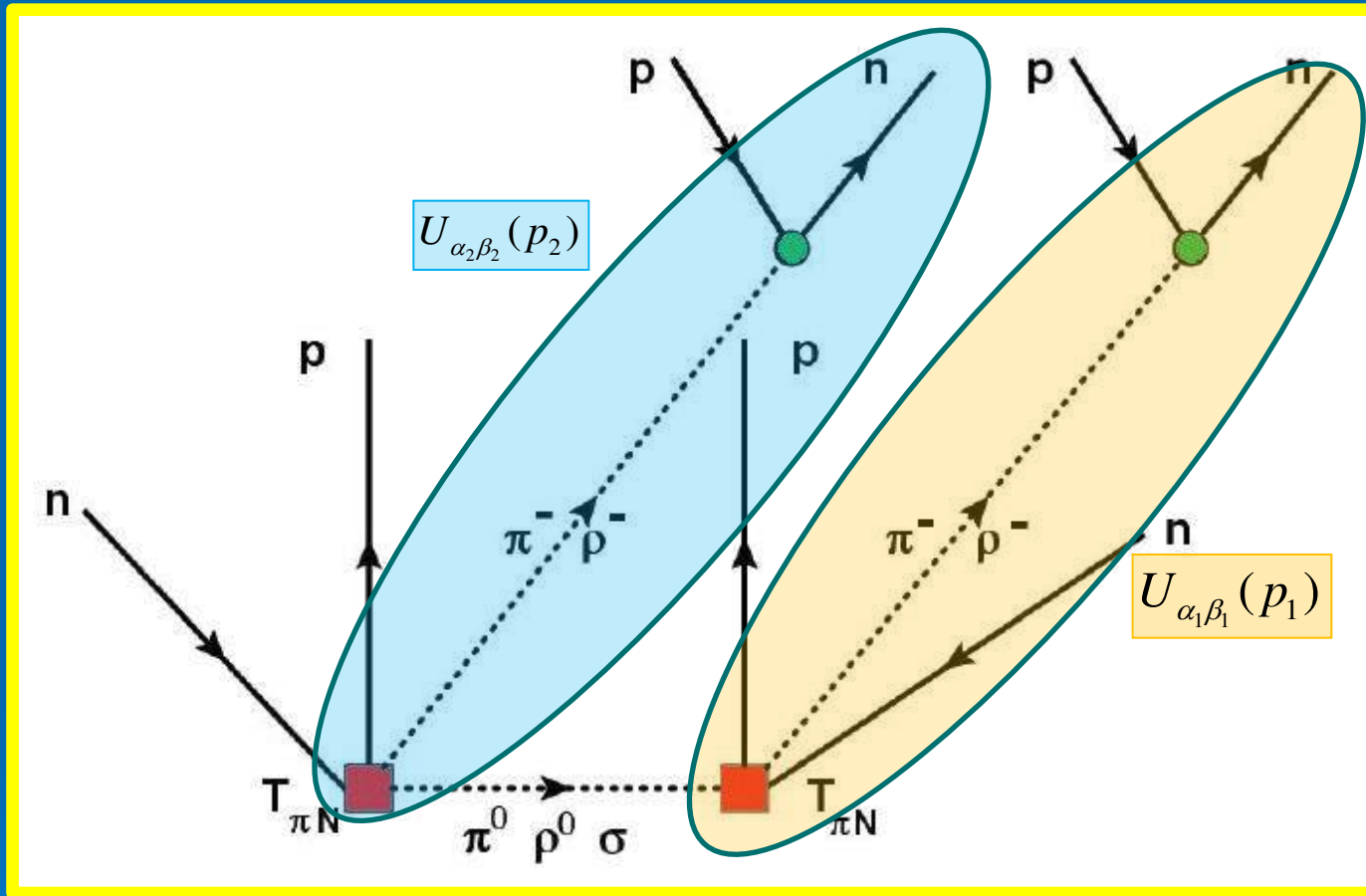
Majorana DCE Transitions



...two-nucleon DCE mechanism ↔ SRC:

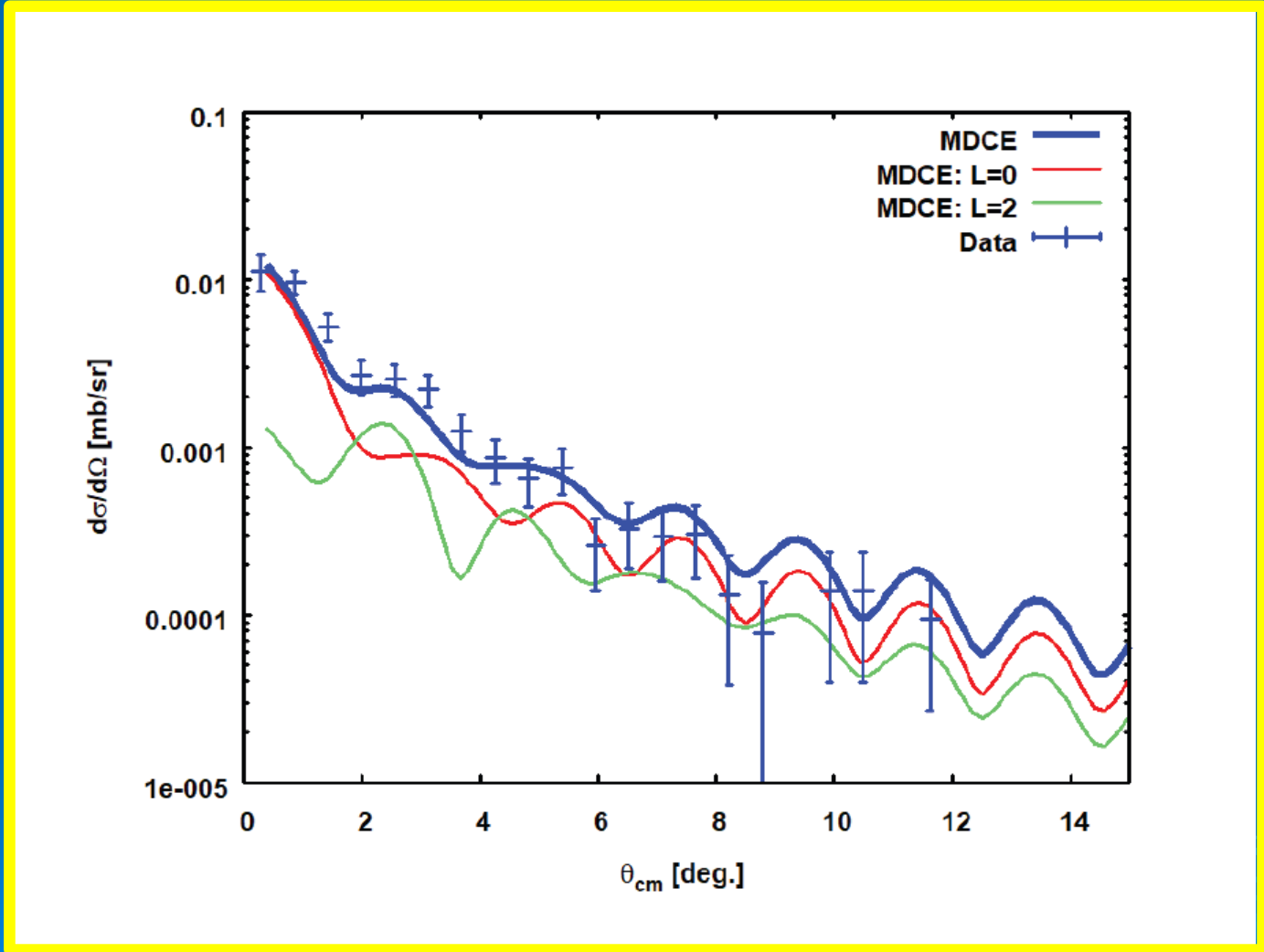


The Majorana DCE Transition Form factor



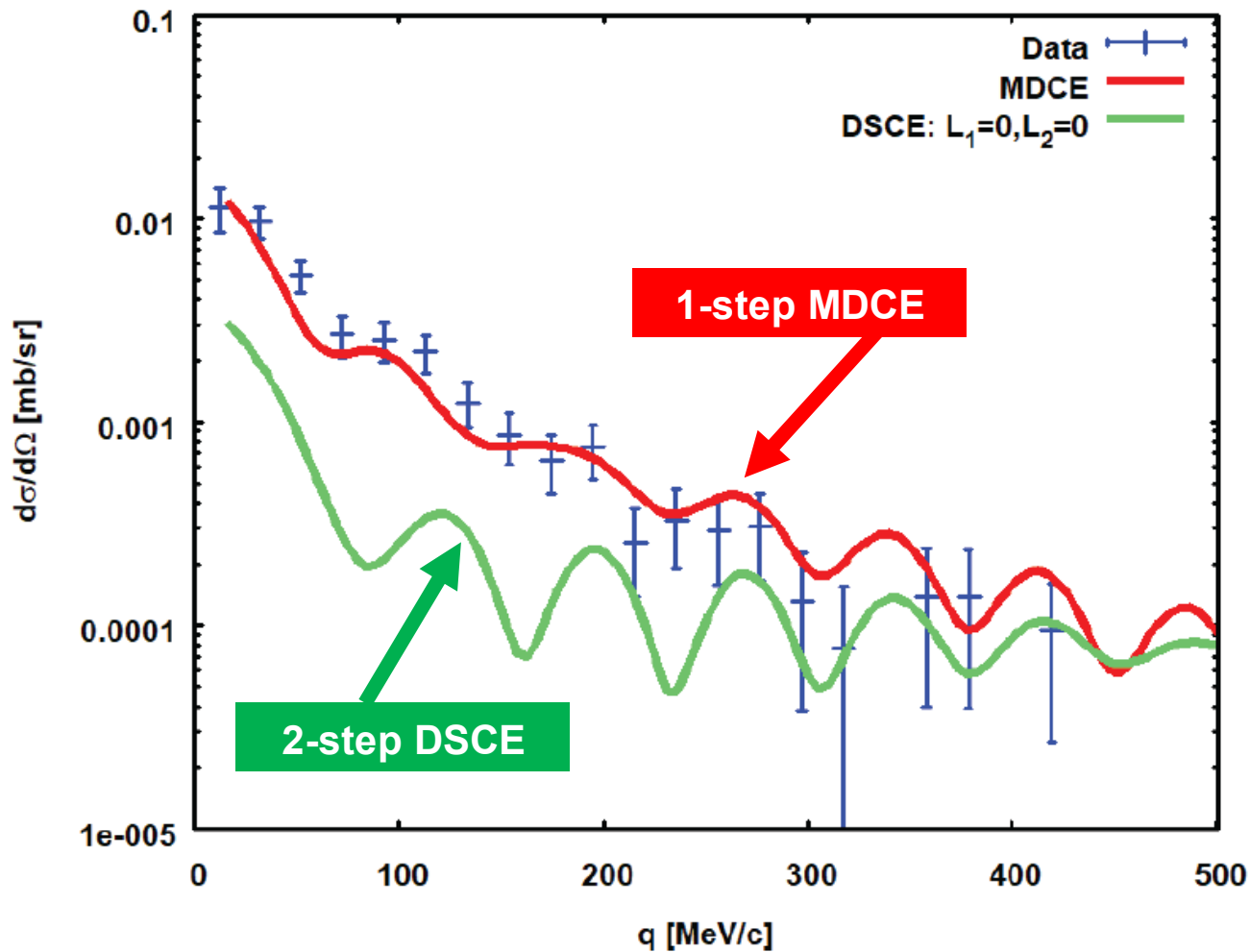
$$F_{\alpha\beta}(\vec{q}) \approx g_{\pi N}^2 \int d^3 p_1 \int d^3 p_2 U_{\alpha_2 \beta_2}(\vec{p}_2) D_{\pi^0}(\vec{p}_1 - \vec{p}_2) U_{\alpha_1 \beta_1}(\vec{p}_1) \delta(\vec{p}_1 + \vec{p}_2 - \vec{q}) + \dots$$

MDCE Cross Section



Data: F. Cappuzzello et al., EPJ A51 (2015)

MDCE and DSCE Cross Sections



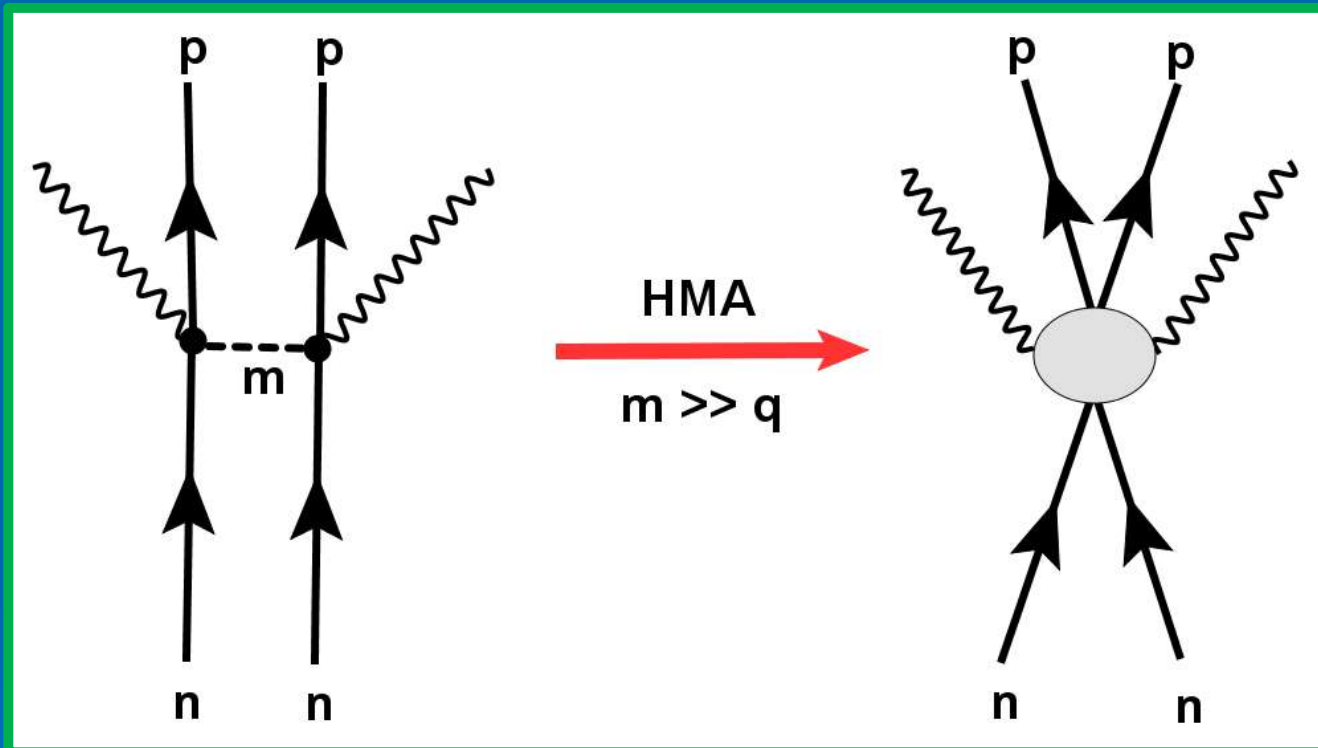
Data: F. Cappuzzello et al., EPJ A51 (2015)

Summary and Outlook

- Theory of heavy ion SCE reactions: direct 1-step, transfer 2-step
- 2-step Double-SCE reaction mechanism: awaiting exploration
- 1-step Majorana-DCE : a new reaction mechanism
- Effective rank-2 **IsoTensor** ion-ion interaction
- Investigations of rare processes :
 - Probing nuclear 2-body CC currents and short range correlations
 - Probing NME of $2\nu 2\beta$ & $0\nu 2\beta$ -type in a hadronic surrogate process
- **Gateway to precision physics with heavy ions**

...together with J. Bellone, M. Colonna (Catania), J.-A. Lay (Sevilla),
E. Santopinto (Genova) and the NUMEN@LNS collaboration

The MDCE Vertex Heavy Meson Approximation



...being measured on-shell at HADES@GSI via
 $pp \rightarrow pp\pi^+\pi^-$

Effective DCE IsoTensor Interaction

DCE Reaction Amplitude

$$M_{\alpha\beta} \sim \langle \chi_{\beta}^{(-)\dagger}, bB | V^{(MDCE)} + V^{(DSCE)} | aA, \chi_{\alpha}^{(+)} \rangle = M_{\alpha\beta}^{(MDCE)} + M_{\alpha\beta}^{(DSCE)}$$

DSCE Interaction

$$V^{(DSCE)}(\mathbf{13}, \mathbf{24}) \sim \sum_{cC} T_{NN}(\mathbf{3}, \mathbf{4}) \mathcal{G}_{cC}(\mathbf{2} - \mathbf{4}, \mathbf{1} - \mathbf{3}) T_{NN}(\mathbf{2}, \mathbf{1})$$

MDCE Interaction

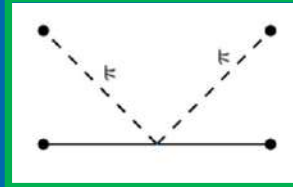
$$V^{(MDCE)}(\mathbf{13}, \mathbf{24}) \sim T_{\pi^{-}p, \pi^0n}(\mathbf{1}, \mathbf{3}) D_{\pi^0}(\mathbf{1} - \mathbf{2}) T_{\pi^0n, \pi^{-}p}(\mathbf{2}, \mathbf{4})$$

→ Rank-2 iso-tensor interactions
with spin operators of rank $S=0,1,2$

Work in Progress:

EFT Approach to MDCE in Time-Ordered Perturbation Theory

(with Genova Group: E. Santopinto et al.)



Axial Vector

$$\mathcal{L}_{AV} = -\frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \boldsymbol{\tau} \Psi \cdot \partial_\mu \boldsymbol{\pi}.$$

covariant

→

$$\hat{\mathcal{L}}_{AV} = -\frac{g_A}{2f_\pi} \bar{N} \boldsymbol{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla}) \boldsymbol{\pi} N$$

Non-relativistic

Weinberg-Tomozawa

$$\mathcal{L}_{WT} = -\frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi}) \Psi$$

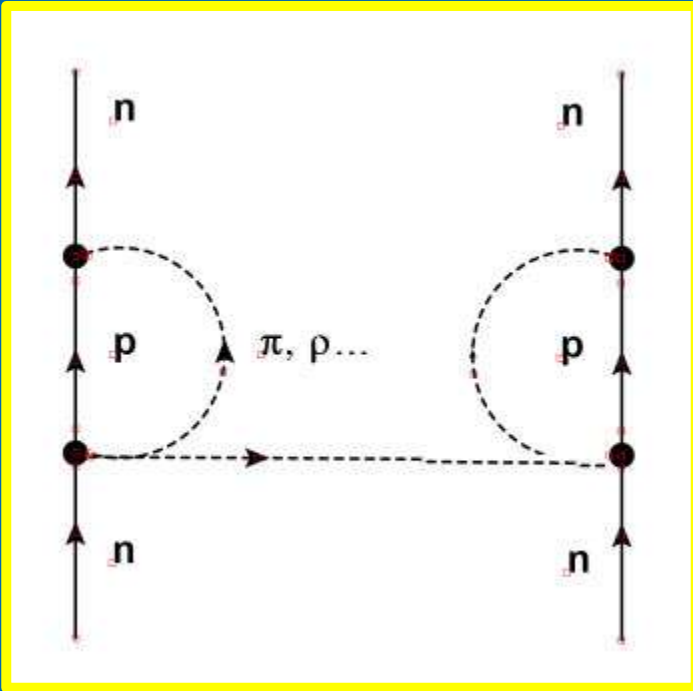
covariant

→

$$\hat{\mathcal{L}}_{WT} = -\frac{1}{4f_\pi^2} \bar{N} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) N$$

Non-relativistic

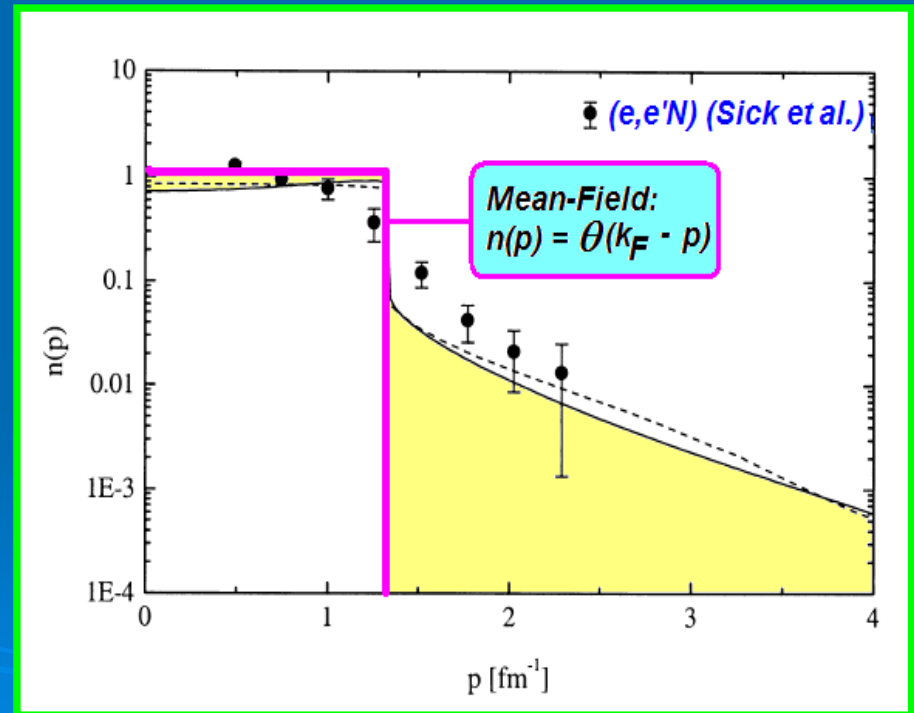
Backups



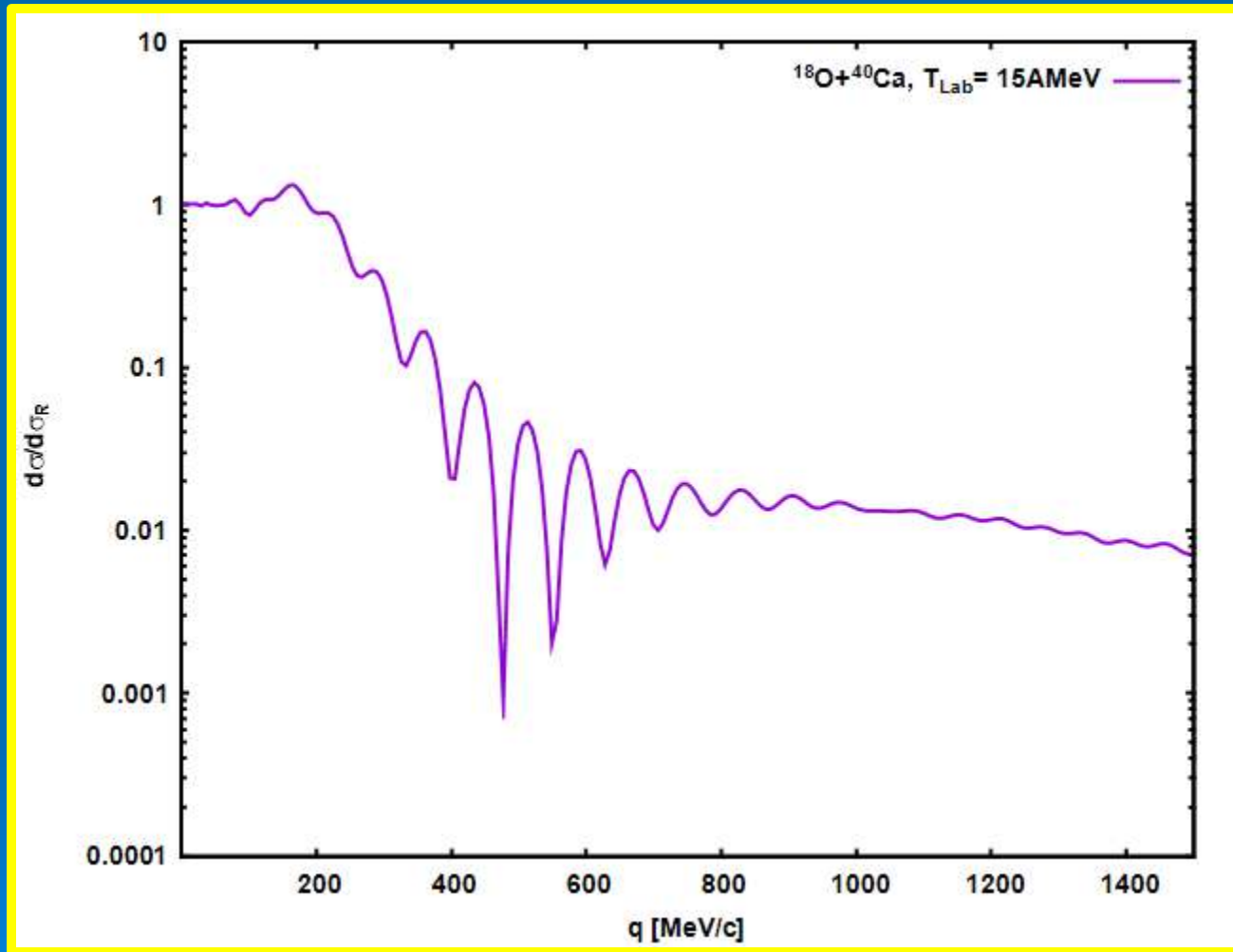
...diagrams known from nuclear short range correlations!

(Vertex and wave function renormalization)

...~ 10...20% contribution to nuclear ground states (P. Konrad, H.L. NPA 756 2005).



$^{18}\text{O} + ^{40}\text{Ca}$ Elastic Scattering: Nearside-Farside Interference



The $0\nu 2\beta 0^+ \rightarrow 0^+$ Nuclear Matrix Element

$$M^{0\nu} = \frac{4\pi R}{g_A^2(0)} \sum_L \int d^3x_1 \int d^3x_2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{q(q+E_d)} \langle 0_F^+ | \mathcal{J}_{L,\mu}^\dagger(\mathbf{x}_1) \mathcal{J}_L^{\mu\dagger}(\mathbf{x}_2) | 0_I^+ \rangle$$

Nuclear charge-changing (CC) Currents \mathcal{J}_L :

- Vector
- Pseudo-vector
- Axial-vector
- Magnetic

Hadronic CC Currents and Transition Amplitude

$$\mathcal{J}_V^\mu = \bar{\Psi}_N \gamma^\mu \boldsymbol{\tau} \Psi_N$$

$$\mathcal{J}_A^\mu = \bar{\Psi}_N \gamma^\mu \gamma_5 \boldsymbol{\tau} \Psi_N$$

$$\mathcal{J}_S = \bar{\Psi}_N \gamma_5 \boldsymbol{\tau} \Psi_N.$$

...requires πN
T-matrix!

$$m_\pi \mathcal{T}_{\pi N}^{(CC)} = T_V(s, t) \mathcal{J}_V^\mu \cdot \partial_\mu (\phi_\pi \times \phi_\pi)$$

$$+ T_A(s, t) \mathcal{J}_A^\mu \cdot (\phi_{\mu, \rho} \times \phi_\pi)$$

$$+ T_P(s, t) \mathcal{J}_A^\mu \cdot \partial_\mu (\phi_\sigma \phi_\pi)$$

$$+ T_S(s, t) \mathcal{J}_S (\phi_\sigma \phi_\pi).$$

Nucleon Iso-spinor Fields:

$$\Psi_N \equiv (\psi_p, \psi_n)^T$$

Meson Iso-vector Fields:

$$\phi_\pi = (\phi_{\pi^-}, \phi_{\pi^0}, \phi_{\pi^+})^T, \quad \phi_\rho^\mu = (\phi_{\rho^-}^\mu, \phi_{\rho^0}^\mu, \phi_{\rho^+}^\mu)^T$$

Meson Iso-scalar Field:

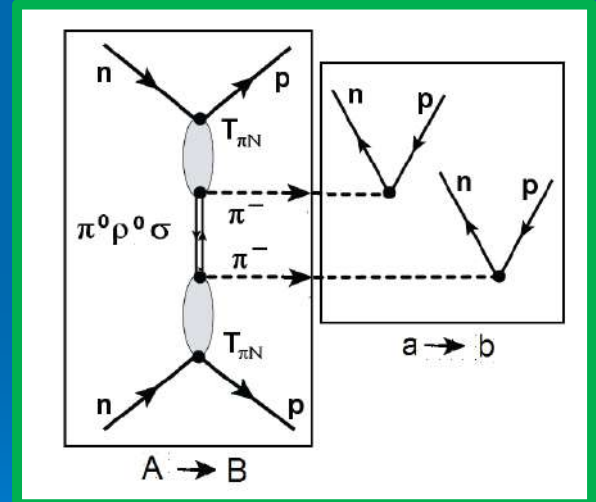
$$\phi_\sigma$$

The MDCE Reaction Amplitude

$$M_{\alpha\beta}^{(MDCE)} = \left\langle \chi_{\beta}^{(-)} \left| U_{\alpha\beta}(\mathbf{r}_{\alpha}, \mathbf{r}_{\beta}) \right| \chi_{\alpha}^{(+)} \right\rangle$$

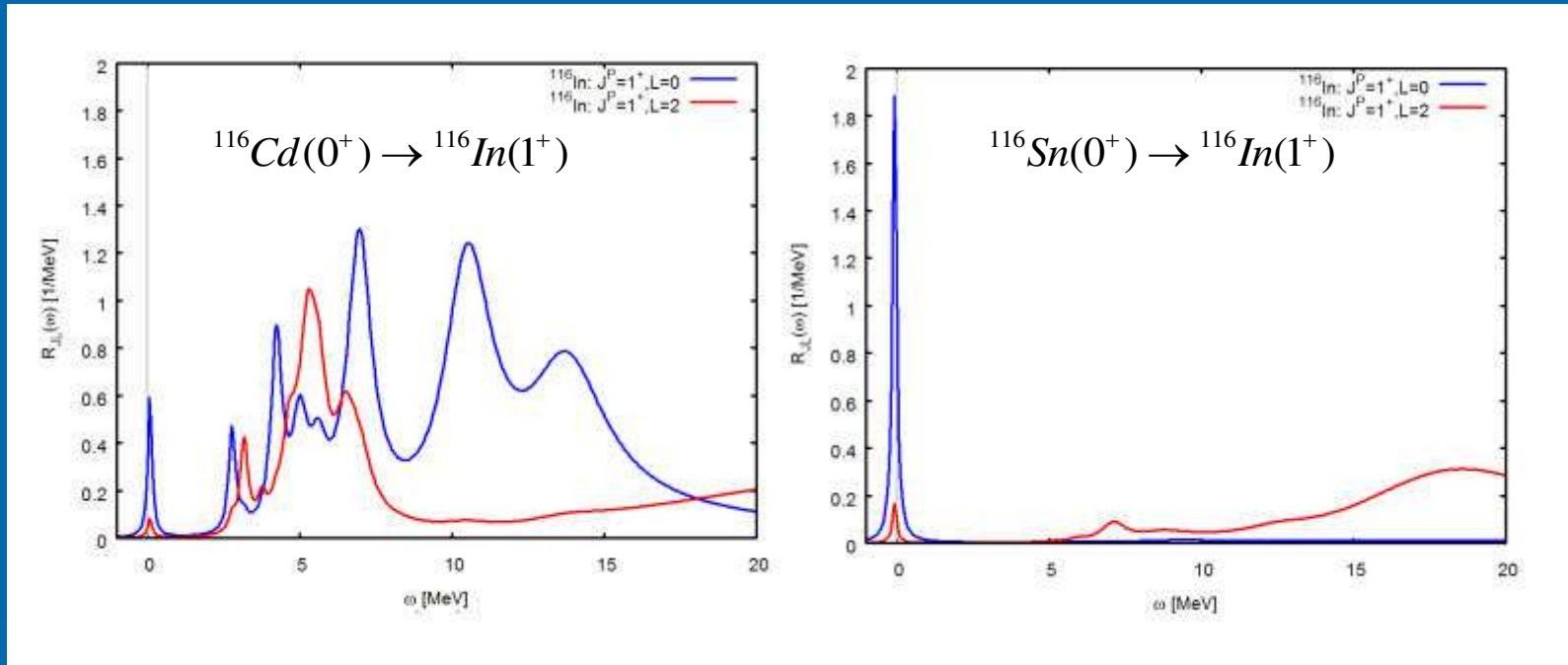
$$U_{\alpha\beta}(\mathbf{r}_{\alpha}, \mathbf{r}_{\beta}) = \int \frac{d^3 p_{\beta}}{(2\pi)^3} \int \frac{d^3 p_{\alpha}}{(2\pi)^3} e^{ip_{\beta} \cdot \mathbf{r}_{\beta}} e^{ip_{\alpha} \cdot \mathbf{r}_{\alpha}} K_{\alpha\beta}(\mathbf{p}_{\alpha}, \mathbf{p}_{\beta})$$

$$\begin{aligned} \mathcal{K}_{\alpha\beta}(\mathbf{p}_{\alpha}, \mathbf{p}_{\beta}) = & \int \frac{d^3 k}{(2\pi)^3} D_{\pi^0}(k) \int \frac{d^3 k_1}{(2\pi)^3} D_{\pi^-}(k_1) \int \frac{d^3 k_2}{(2\pi)^3} D_{\pi^-}(k_2) \\ & \delta(\mathbf{k}_1 + \mathbf{k}_2 - (\mathbf{p}_{\alpha} - \mathbf{p}_{\beta})) \\ & T_{\pi N}^{(1)}(\mathbf{k} - \mathbf{k}_1) T_{\pi N}^{(2)}(\mathbf{k}_2 - \mathbf{k}) \\ & \langle B | \mathcal{J}_+^{(1)}(\mathbf{k} - \mathbf{k}_1) \mathcal{J}_+^{(2)}(\mathbf{k} + \mathbf{k}_2 | A \rangle \langle b | \mathcal{S}_{--}(\mathbf{k}_1), \mathbf{k}_2 | a \rangle \end{aligned}$$



CC Response Functions

$^{116}\text{Cd} \rightarrow ^{116}\text{In}(1^+)$ and $^{116}\text{Sn} \rightarrow ^{116}\text{In}(1^+)$



Nuclear Response Functions: Continuum-ccQRPA