Isovector properties of finite nuclei: constraints from neutron stars observations

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Outline of the talk

- The basic formalism of finite nuclei
- Neutron skin thickness
- Equation of state of neutron star matter
- The basic formalism of neutron star structure
- Tidal effects
- Presentation of the results
- Conclusions

The basic formalism for finite nuclei

Bethe-Weizsacker formula the binding energy of a finite nucleus

$$BE(A,Z) = -a_V A + a_S A^{2/3} + a_C \frac{Z(Z-1)}{A^{1/3}} + a_A \frac{(N-Z)^2}{A} + E_{\rm add}$$

The symmetry energy coefficient a_A can be expanded as

$$a_A^{-1} = (a_A^V)^{-1} + (a_A^S)^{-1} A^{-1/3}$$

The total energy of the nucleus in terms of an energy density and asymmetry $\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is given by

$$E = \int_{\mathcal{V}} \mathcal{E}\left(\rho(r), \alpha(r)\right) d^3 r$$

In the local density approximation, the coefficient a_A is defined by the integral

$$a_A = \frac{A}{(N-Z)^2} \int_{\mathcal{V}} \rho(r) S(\rho) \alpha^2(r) d^3 r$$

Neutron skin thickness

The neutron skin thickness (related with the isovector character of the nuclear forces) defined as

$$\Delta R_{\rm skin} = R_n - R_p$$

with

$$R_n = \left(\frac{1}{N}\int_{\mathcal{V}} r^2 \rho_n d^3 r\right)^{1/2} = \left(\frac{1}{N}\int_{\mathcal{V}} r^2 \frac{\rho(1+\alpha)}{2} d^3 r\right)^{1/2}$$

and

$$R_p = \left(\frac{1}{Z}\int_{\mathcal{V}} r^2 \rho_p d^3 r\right)^{1/2} = \left(\frac{1}{Z}\int_{\mathcal{V}} r^2 \frac{\rho(1-\alpha)}{2} d^3 r\right)^{1/2}$$

It is worth mentioning that $\Delta R_{\rm skin}$ is not directly dependent on $S(\rho)$, compared to the case of a_A. However, it is dependent indirectly via the asymmetry function $\alpha(r)$. We expect $\Delta R_{\rm skin}$ and a_A, a_A^S, and a_A^V to be strong indicators of the isospin character of the nuclear interaction.
The experimental values of the skin of ^{208}Pb reported by PREX-2

$$\Delta R_{\rm skin} = (0.283 \pm 0.071)\ {\rm fm}$$

The equation of state of nuclear matter

Energy per particle of asymmetric nuclear matter

$$E(n,\alpha) = E_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + S(n)\alpha^2$$

Nuclear symmetry energy $S(n)$ in a series around the saturation density

$$S(n) = J + \frac{L}{3n_0}(n-n_0) + \frac{K_{\rm sym}}{18n_0^2}(n-n_0)^2 + \cdots$$

The slope parameter L and $K_{\rm sym}$

$$L = 3n_0\left(\frac{dE_{\rm sym}(n)}{dn}\right)_{n=n_0}, \quad K_{\rm sym} = 9n_0^2\left(\frac{dE^2_{\rm sym}(n)}{d^2n}\right)_{n=n_0}$$

The parametrization of the equation of state

$$\eta = (K_0 L^2)^{1/3}$$

The basic formalism of neutron stars

The pressure of the baryons

$$P_b \equiv n^2 \frac{d(\mathcal{E}/n)}{dn} = \frac{K_0}{9n_0^2} n^2 (n - n_0) + \alpha^2 \frac{L}{3n_0} n^2$$

The total energy density and pressure (equation of state (EOS))

$$\mathcal{E}_{\rm tot} = \mathcal{E}_b + \mathcal{E}_e, \qquad P_{\rm tot} = P_b + P_e$$

The mechanical equilibrium of the star matter (as input the EoS $\mathcal{E} = \mathcal{E}(P)$)

$$\frac{dP(r)}{dr} = -\frac{G\mathcal{E}(r)M(r)}{c^2 r^2} \left(1 + \frac{P(r)}{\mathcal{E}(r)}\right) \times \left(1 + \frac{4\pi P(r) r^3}{M(r)c^2}\right) \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1},$$

$$\frac{dM(r)}{dr} = \frac{4\pi r^2}{c^2} \mathcal{E}(r)$$

Neutron star structure and tidal effects

Tidal effects is given

$$Q_{ij} = -\frac{2}{3} k_2 \frac{R^5}{G} E_{ij} \equiv -\lambda E_{ij}$$

The tidal Love number k_2 is given by

$$\begin{aligned}
k_2 &= \frac{8\beta^5}{5} (1-2\beta)^2 \left[2 - y_R + (y_R - 1) 2\beta\right] \\
&\times \Big[2\beta \left(6 - 3y_R + 3\beta(5y_R - 8)\right) \\
&+ 4\beta^3 \left(13 - 11y_R + \beta(3y_R - 2) + 2\beta^2(1 + y_R)\right) \\
&+ 3 (1-2\beta)^2 \left[2 - y_R + 2\beta(y_R - 1)\right] \ln(1 - 2\beta) \Big]^{-1}
\end{aligned}$$

The dimensionless tidal deformability Λ, defined as

$$\Lambda = \frac{2}{3} k_2 \left(\frac{c^2 R}{GM}\right)^5 = \frac{2}{3} k_2 (1.473)^{-5} \left(\frac{R}{\mathrm{Km}}\right)^5 \left(\frac{M_\odot}{M}\right)^5$$

In spiral and merger of a binary neutron star system

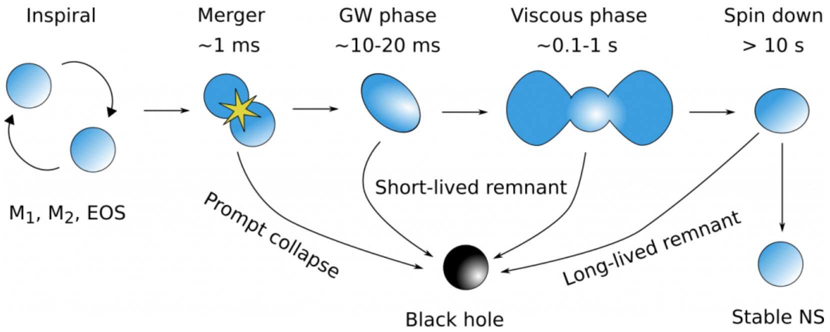

Figure: In spiral and final stage possibilities of a binary neutron star system[1]

[1] The figure taken from D.Radice, S.Bernuzzi,and A. Perego, Annu. Rev. Nucl. Part. Sci. 2020. 70:95–119

Equations of state and M-R diagramma[2]

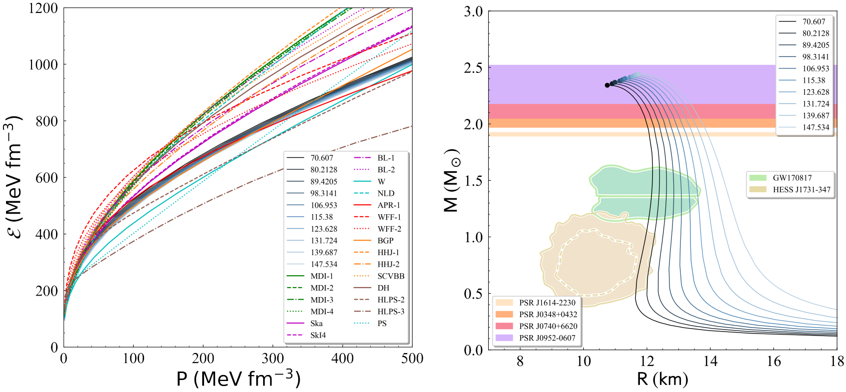

Figure: left: EOS according to the parametrization η. right:The mass-radius (M-R) dependence for various EoSs depending on the parameter η

[2] M. Divaris, A. Kanakis-Pegios, and Ch.C.M., Phys. Rev. C 109, 055805 (2024)

Effects on tidal deformability

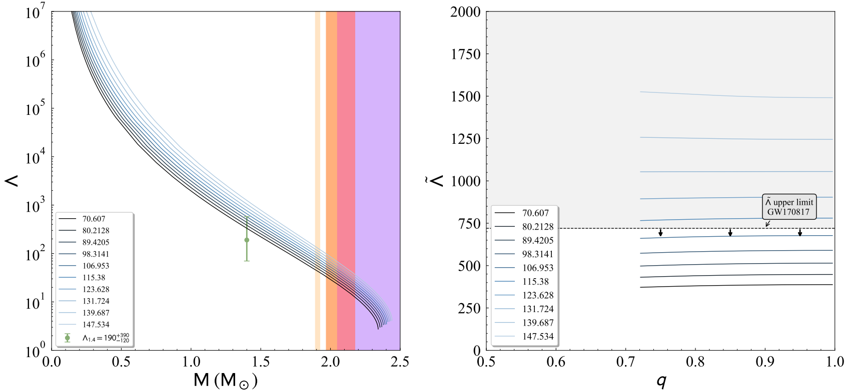

Figure: left: The dimensional tidal deformability Λ as a function of the mass for various EoSs corresponding to the selected values of the parameter η. right: The effective tidal deformability $\tilde{\Lambda}$ vs the binary mass ratio q for all the cases of EoS, applied to the GW170817 event

Tidal dependence on η and correlation with ΔR_{skin}

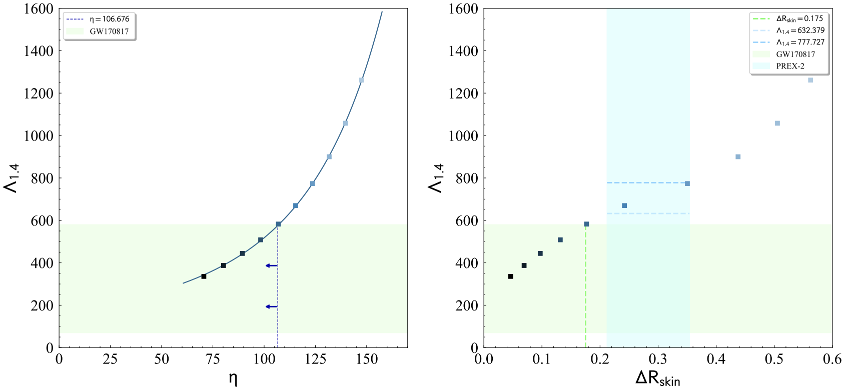

Figure: left: The tidal deformability $\Lambda_{1.4}$ of a $1.4\ M_{\odot}$ neutron star related to the parameter η. The green shaded area indicates the observational constraints from GW170817. right: The tidal deformability $\Lambda_{1.4}$ of a $1.4\ M_{\odot}$ neutron star related to the neutron skin R_{skin} (in fm). The green shaded area indicates the observational constraints on $\Lambda_{1.4}$ by GW170817, while the blue one indicates the PREX-2 estimation for R_{skin}.

Correlation of $\Lambda_{1.4}$ with the asymmetry coefficients

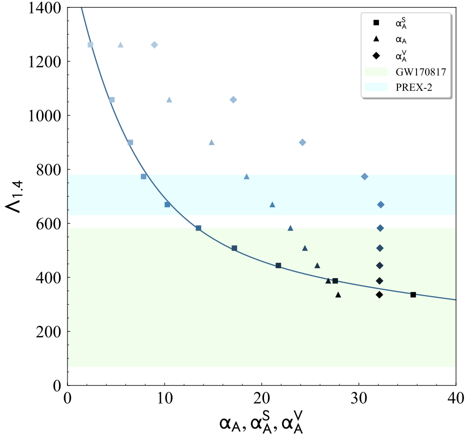

Figure: $\Lambda_{1.4}$ of a $1.4\ \mathrm{M}_\odot$ neutron star related to the asymmetry coefficient α_{A} (in MeV) and the surface (volume) coefficient $\alpha_{\mathrm{A}}^{\mathrm{S}}$ ($\alpha_{\mathrm{A}}^{\mathrm{V}}$) (in MeV). The green shaded area indicates the observational constraints on $\Lambda_{1.4}$ by GW170817, while the blue one indicates the corresponding PREX-2 estimation for $\Lambda_{1.4}$.

Conclusions

- The neutron skin thickness and the coefficients a_A, a_A^S and a_A^V are sensitive on the parameter η which characterizes the stiffness of the equation of state. The effect is very dramatic especially for high values of η ($\eta > 120$ MeV) leading to abnormal values for these parameters.
- For the neutron skin thickness, in order to be compatible with the results of the PREX-2 experiment, the range of the parameter η must be in the range 110 MeV $\lesssim \eta \lesssim$ 125 MeV.
- The effects of the parameter η are also very pronounced in neutron stars properties. In particular, the increasing of η affects more the radius compared to the $\mathrm{M}_{\mathrm{max}}$. The EoSs with the highest η lie outside of the GW170817 observation (green shaded contours), while only the EoS with the lowest value of η can predict the HESS observation.
- By applying the observational limits of $\Lambda_{1.4}$, provided by LIGO, we extracted an upper value of $\eta_{\mathrm{max}} \simeq 106.676$ MeV so that all the EoSs with $\eta \leq \eta_{\mathrm{max}}$ fulfill the observational constraints of GW170817.

Conclusions

- The combination of these two constraints, originated from observational and experimental data, lead to different directions. The gravitational-wave origin leads to smaller values of the neutron skin, while the PREX-2 favors higher values. This contradiction arises from the softness of the EoS that the GW170817 imposes, while the PREX-2 requires a stiffer EoS.
- We present for a first time constraints for the other three microscopic parameters, α_A, α_A^S, and α_A^V with the help of recent observations (related mainly with the tidal deformability). We conclude that if we define the tidal deformability or even more the radius of a neutron star more precisely, we will also be able to define even more precisely the range of these coefficients.
- A final comment: We use a simple model to simultaneously describe finite nuclei and neutron stars. The results show that although the difference in their dimensions is huge (from a few fm to a few Km), they can be directly connected due to the common isovector dependence of their properties. Future precise measurements of the properties of neutron stars will lead to a more precise determination of the microscopic structure of finite nuclei, especially those that are neutron-rich, and vice versa.

THANK FOR YOUR ATTENTION