# Quarkyonic matter and applications in neutron stars

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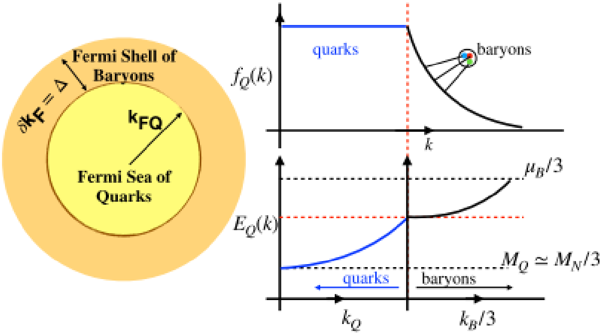
# Contents

In this presentation we will see the following :

- An introduction to quarkyonic state of matter
- The NDU quarkyonic model interacting via a momentum dependent potential
- Equations of state and sound velocity for our models
- Application in cold neutron stars (TOV equations) and first results, M(R) diagrams
- Discussion of results
- Perspectives and open questions of the theory

# Quarkyonic matter

Quarkyonic matter is a hybrid state of matter, which believed to occur at large densities and Fermi energies, where we consider particles as quasiparticles in the phase space. We assume that low momentum states are occupied by quarks which form a Fermi sea in the momentum space and baryons occupy greater momentum states forming a shell with a width $\delta k_F = \Delta$. This description can be illustrated in the following figure.



We assume that Chiral symmetry remains broken in our model so quark masses are obtained as $m_Q = m_N/N_c$, where $N_c$ is the number of colors. For the thickness of the Fermi shell where nucleons reside, we impose the following relation,

$$\Delta = \frac{\Lambda_{Qyc}^3}{\hbar^3 c^3 k_{F_N}^2} + \kappa_{Qyc} \frac{\Lambda_{Qyc}}{\hbar c N_c^2} \tag{1}$$

where $\Lambda_{Qyc}, \kappa_{Qyc}$ are parameters, $\Lambda_{Qyc} \approx \Lambda_{QCD}$ and $\kappa_{Qyc} \approx 0.2 - 0.3$. The number density for quarks and nucleons will be,

$$n_Q = \frac{g_s N_c}{2\pi^2} \sum_{Q=u,d} \int_0^{k_{F_Q}} k^2 dk \qquad , \tag{2}$$

$$n_N = \frac{g_s}{2\pi^2} \sum_{N=p,n} \int_{k_{F_N}-\Delta}^{k_{F_N}} k^2 dk \tag{3}$$

The energy density will be in the form,

$$\epsilon_Q = \frac{g_s N_c}{2\pi^2} \sum_{Q=u,d} \int_0^{k_{F_Q}} k^2 \sqrt{(\hbar c k)^2 + m_Q^2 c^4} dk \tag{4}$$

$$\epsilon_N = \frac{g_s}{2\pi^2} \sum_{N=p,n} \int_{k_{F_N}-\Delta}^{k_{F_N}} k^2 \sqrt{(\hbar c k)^2 + m_N^2 c^4} dk + V(n_N) \tag{5}$$

respectively. Initially, we consider that quarks are non-interacting and only nucleons interact via a potential $V$. The chemical potentials and Pressure are obtained from the thermodynamic relations,

$$\mu_i = \frac{\partial \epsilon}{\partial n_i} \tag{6}$$

and

$$P = -\epsilon + \sum_i \mu_i n_i \tag{7}$$

where $\epsilon$ is the total energy density.

# The NDU (neutrons, up and down quarks) quarkyonic model with momentum dependent interaction

In this model we assume the quarkyonic state of matter including neutrons, up and down quarks and we consider that neutrons interact via a momentum dependent interaction. Quarks are treated as non-interacting particles. The potential energy will be in the form,

$$
\begin{aligned}
V_{\rm int}(n_n, k_{F_n}) &= \frac{1}{3} A n_0 (1 + x_0) u^2 + \frac{\frac{2}{3} B n_0 (1 - x_3) u^{\sigma+1}}{1 + \frac{2}{3} B' n_0 (1 - x_3) u^{\sigma-1}} \\
&+ u \sum_{i=1,2} \frac{1}{5} \left[ 6C_i - 8Z_i \right] \mathcal{J}_n^i
\end{aligned} \tag{8}
$$

where $u = \frac{n_n}{n_0}$, $n_0$ is the saturation density and

$$
\begin{aligned}
\mathcal{J}_n &= \frac{2}{(2\pi)^3} \int d^3k \mathrm{g}(n, \Lambda_i) f_\tau \\
&= \frac{2}{(2\pi)^3} \int_{k_{F_N}-\Delta}^{k_{F_N}} 4\pi \left[ 1 + \left( \frac{k}{\Lambda_i} \right)^2 \right]^{-1} k^2 dk
\end{aligned} \tag{9}
$$

The first two terms of the potential energy (equation 8) are momentum independent. The first corresponds to an attractive interaction and the second one corresponds to a repulsive interaction and dominates for high density values. The third term is the momentum dependent part, express an attractive interaction and has been introduced to include finite range interactions.

The total energy density will be in the form,

$$\epsilon_{tot} = \epsilon_n + \epsilon_Q = \frac{g_s}{2\pi^2}\int_{k_{F_n}-\Delta}^{k_{F_n}} k^2\sqrt{(\hbar ck)^2 + m_n^2c^4}dk + V_{int}(n_n, k_{F_n})$$

$$+\sum_{i=u,d}\frac{g_sN_c}{2\pi^2}\int_0^{k_{F_i}} k^2\sqrt{(\hbar ck)^2 + m_Q^2c^4}dk \qquad (10)$$

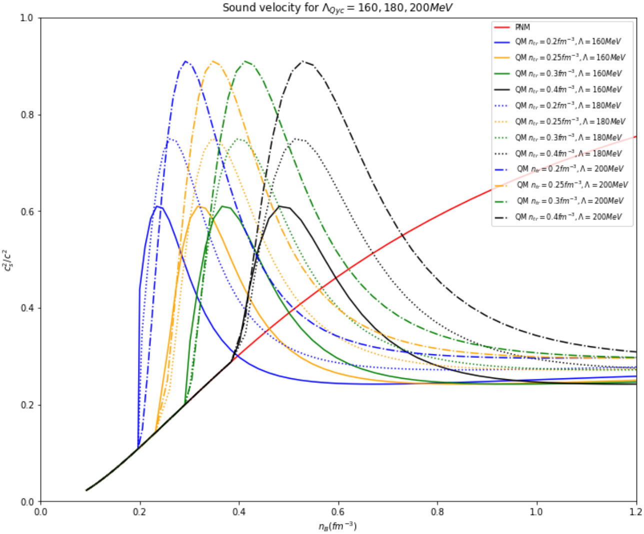
We set up the parameters of the potential to be:

$\Lambda_1 = 1.5k_{F_N},\ \ \Lambda_2 = 3k_{F_N},\ \ A = -46.65,\ \ B = 39.45,\ \ B' = 0.3,\ \ \sigma = 1.663,\ \ C_1 = -83.84,\ \ C_2 = 23,\ \ \chi_0 = 1.654,\ \ \chi_3 = -1.112,\ \ Z_1 = 3.81,\ \ Z_2 = 13.16$
and
$\kappa_{Qyc} = 0.3$, $N_c = 3$, $g_s = 2$.

We constructed equations of state for several values for parameter $\Lambda_{Qyc}$ and transition densities ($n_{tr}$). We calculate the total pressure, the total energy density and we compute the sound velocity from:

$$\frac{c_s^2}{c^2} = \frac{\partial P}{\partial \epsilon} \tag{11}$$

Our results are illustrated in the following figure.




The sound velocity for NDU quarkyonic model interacting via a momentum dependent interaction, for $n_{tr} = 0.2, 0.25, 0.3, 0.4 \ fm^{-3}$ (blue, yellow, green and black lines respectively) and for $\Lambda_{Qyc} = 160, 180, 200 \ MeV$(solid, dotted and dashed - dotted lines respectively). The solid red line corresponds to pure neutron matter(PNM).

# Applications in neutron stars.Tolman-Oppenheimer-Volkoff equations

After the construction of equations of state for pure neutron and quarkyonic matter, we apply them in Einstein equations, for a static , spherically symmetric neutron star. These equations together with the equation of state give us the so called Tolman-Oppenheimer-Volkoff equations (TOV) which are in the following form,

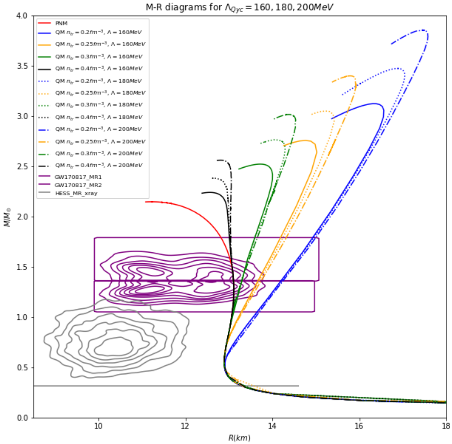
$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r),$$

$$\frac{dP(r)}{dr} = \rho(r)c^2 \big(1 + \frac{P(r)}{\rho(r)c^2}\big) \frac{d\phi(r)}{dr},$$

$$\frac{d\phi(r)}{dr} = \frac{Gm(r)}{c^2 r^2} \big(1 + \frac{4\pi P(r) r^3}{m(r)c^2}\big) \big(1 - \frac{2Gm(r)}{rc^2}\big)^{-1}, \qquad (12)$$

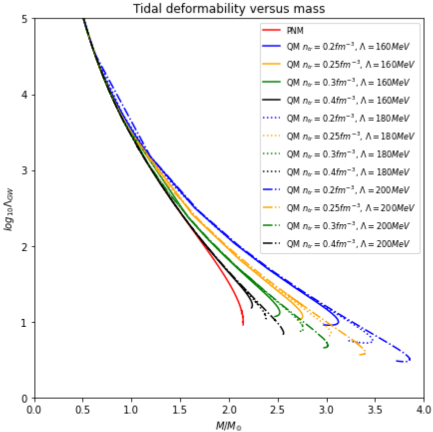
where G is the gravitational constant, c is the speed of light, $P(r)$ is the total pressure, $m(r)$ is the enclosed mass of the star, $\rho(r)$ is the total mass density, and $\phi(r)$ is the gravitational field.

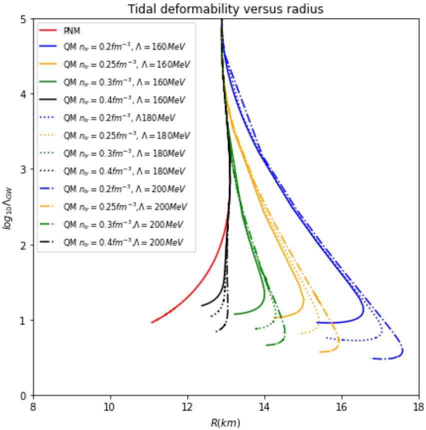
# Mass-Radius diagrams




Mass-Radius diagram for NDU quarkyonic model interacting via a momentum dependent interaction, for $n_{tr} = 0.2, 0.25, 0.3, 0.4\ fm^{-3}$ (blue, yellow, green and black lines respectively) and for $\Lambda_{Qyc} = 160, 180, 200\ MeV$(solid, dotted and dashed - dotted lines respectively). The solid red line corresponds to pure neutron matter(PNM).

# Tidal deformability




Tidal deformability versus mass (left figure) and versus radius (right figure) for NDU quarkyonic model interacting via a momentum dependent interaction, for $n_{tr} = 0.2, 0.25, 0.3, 0.4\ fm^{-3}$ (blue, yellow, green and black lines respectively) and for $\Lambda_{Qyc} = 160, 180, 200\ MeV$(solid, dotted and dashed - dotted lines respectively). The solid red line corresponds to pure neutron matter(PNM).

# Discussion of results

After this initial effort we can note some interesting features of quarkyonic matter.

- Quarkyonic matter provides the sound speed as a non - monotonic function of the baryon density. We obtain a rapid increase up to a maximum value, after it decreases and eventually it is reaching asymptotically the value of $\frac{1}{\sqrt{3}}$ (conformal limit).
- Quarkyonic matter can predict more massive neutron stars (about 2.5 - 3.5 $M_{\odot}$), without violating the causality in contrast with pure hadronic matter. Also, for neutron stars with mass $1.4M_{\odot}$ predict radii about 13 - 14 km, a little greater compared to the pure neutron case.
- We constrained the value of parameter $\Lambda_{Qyc} < 210\ MeV$ from causality and also the value of transition density to be $n_{tr} \geq 0.25\ fm^{-3}$ from observational data resulting from LIGO and HESS experiments.
- This state of matter may be a consistent way to explain the phase transition in the interior of a neutron star.

# Perspectives and open questions of the theory

- Is there any fundamental theory which can provide quarkyonic state of matter with a consistent way?
- Can we apply this theory in a finite temperature neutron star?
- How will change our results if we insert protons and electrons to impose $\beta$- equilibrium?
- Can we insert strange matter in this theory and which will be the consequence in the equation of state?
- Do we have a Chiral symmetry restoration after the phase transition or it remains broken?
- Can we constrain further the microscopic parameters of our model from new gravitational waves observations?

# Acknowledgments

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