Importance of elastic scattering in nucleon-exotic nucleus experiments

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Collaborators (g folding and MCAS)

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Introduction

Subject: quasi-free scattering with RIBs between $100A$ MeV to $200A$ MeV.

Problem: definition of the interactions between projectile and target at these energies, and how to constrain them.

Back to basics: elastic scattering.

Elastic scattering constrains the underlying optical potential and hence the relative wave functions between projectile and target.

Formal theory, optical potential

Optical potential: associated with elastic scattering only. We split the Hilbert space:

1. P projects onto the elastic scattering channel;
2. Q projects onto everything else.

Thus: $P + Q = 1$, $PQ = QP = 0$, $Q \left| \Phi_{\text{gs}} \right\rangle = 0$, etc. This leads to (with $H_{AB} \equiv AHB$):

$$\left(E - H_{PP}\right) \left|\psi^{+}\right\rangle = H_{PQ} \left|\psi^{+}\right\rangle$$

$$\left(E - H_{QQ}\right) \left|\psi^{+}\right\rangle = H_{QP} \left|\psi^{+}\right\rangle$$

By recoupling, the S.E. for the *projectile* wave function is

$$\left\{E - H_0 - \left\langle \Phi_{\text{gs}} \left| V \right| \Phi_{\text{gs}} \right\rangle - \left\langle \Phi_{\text{gs}} \left| V G_{QQ}^{(+)} V \right| \Phi_{\text{gs}} \right\rangle \right\} \left| \chi^{(+)} \right\rangle = 0$$

where, $G_{QQ}^{(+)} = \left[E - H_{QQ} + i\varepsilon\right]^{-1}$.

Definition of the optical potential: $$U = \langle \Phi_{\rm gs} | V | \Phi_{\rm gs} \rangle + \langle \Phi_{\rm gs} \left| V G_{QQ}^{(+)} V \right| \Phi_{\rm gs} \rangle$$

It is a many-body problem…

It is complex, nonlocal, target-, and **energy-** dependent.

Low energy: $E \leqslant 10$ MeV.

Coupled-channel methods, formation of states in the compound nucleus in elastic scattering.

MCAS: Multi-Channel Algebraic Scattering.
(Steven Karataglidis, Ken Amos, Paul R. Fraser, and Luciano Canton, *A new development at the intersection of nuclear structure and reaction theory,* Springer-Nature 2019.)

Intermediate energy: $25 \leqslant E \leqslant 300$ MeV.

Folding NN potentials with target structures to form optical potentials.

e.g. Melbourne *g*-folding model, K. Amos *et al.,* Adv. Nucl. Phys. **25**, 275 (2000).

Melbourne g-folding optical potential: g matrix

(A refresher for intermediate energy nucleon-nucleus scattering.)

Effective NN potential - Melbourne model.

- Effective NN interaction obtained from g matrices.
- Bonn-B interaction used for the current examples.
- Momentum-space effective interaction mapped to coordinate space.
- Densities obtained from credible models of structure.

The g matrix is a solution of the Bethe-Goldstone equation:

$$g(\mathbf{q},\mathbf{q}';\mathbf{K}) = V(\mathbf{q}',\mathbf{q}) + \int V(\mathbf{q}',\mathbf{k}) \frac{Q(\mathbf{k}',\mathbf{K};k_F)}{E(\mathbf{k},\mathbf{K}) - E(\mathbf{k}',\mathbf{K})} g(\mathbf{k}',\mathbf{q};\mathbf{K}) d\mathbf{k}'$$

where Q is a Pauli operator and the energy denominator accounts for medium effects and is dependent on auxiliary potentials (eg. effective mass operators).

Construction of the optical potential

In coordinate space, the OMP for elastic scattering is

$$U(\mathbf{r},\mathbf{r}';E) = \delta(\mathbf{r}-\mathbf{r}')\sum_i n_i \int \varphi_i^*(\mathbf{s})\, g_D(\mathbf{r},\mathbf{s};E)\, \varphi_i(\mathbf{s})\; d\mathbf{s}$$

$$+ \sum_i n_i \varphi_i^*(\mathbf{r}')\, g_E(\mathbf{r},\mathbf{r}';E)\, \varphi_i(\mathbf{r})$$

$$= U_D(\mathbf{r},E)\,\delta(\mathbf{r}-\mathbf{r}') + U_E(\mathbf{r},\mathbf{r}';E)\,.$$

- First term is the "$g\rho$" (direct) potential.
- Second term is the exchange term.
- All nonlocality stems from the exchange term.
- Structure enters through the single particle wave functions and occupation numbers, n_i.
- For non-zero spin targets, terms with non-zero spin coupling may be included via the DWA.

Structure of the target is critical.

Nuclear structure

One-body density matrix elements are required, and obtained from the Shell model.

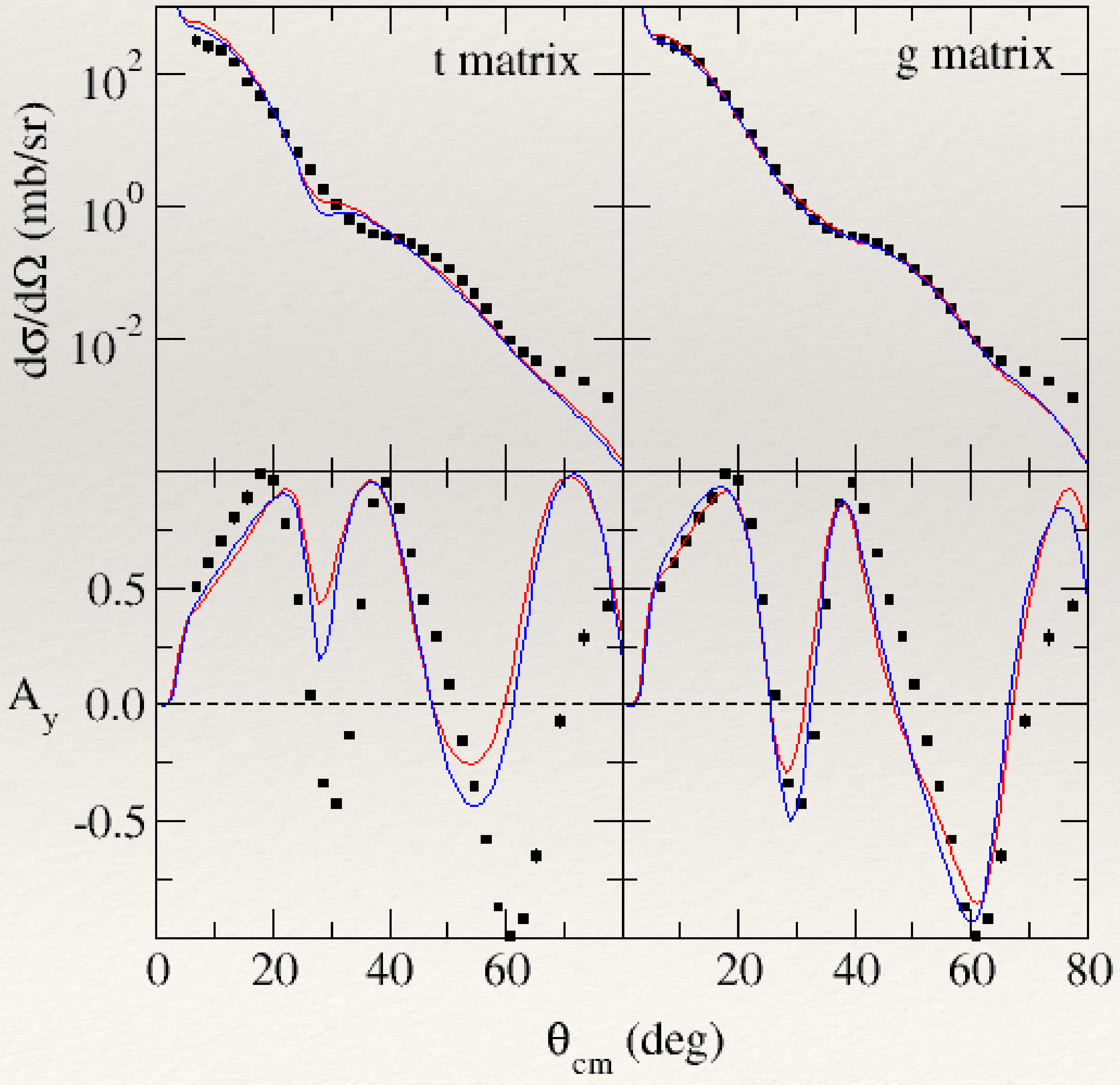
$$S_{j_1 j_2 J} = \langle J_f \parallel \left[a^\dagger_{j_2} \times \tilde{a}_{j_1}\right]^J \parallel J_i \rangle$$

Single particle wave functions are set and consistent with nucleon separation energies.

- HO: (naive shell model) gives skin attributes.
- WS: binding energies set to the single-nucleon separation energy gives
  appropriate extension of the nucleon (proton or neutron) density consistent with a halo or skin.

Testing the OMP: ^{12}C, and $^{6,8}He$

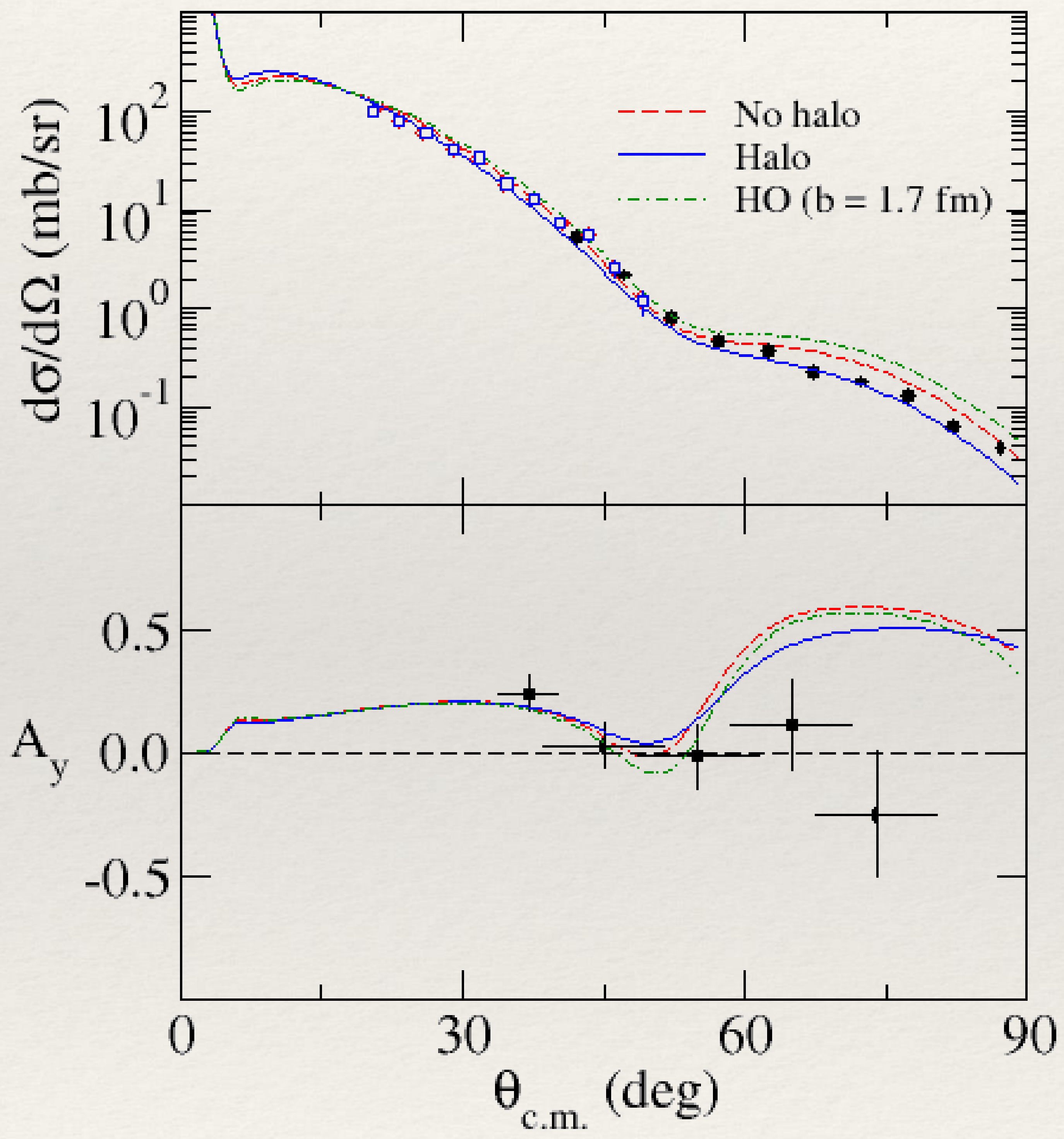
The interaction between projectile and target, both assumed hadronic, is encoded in the optical potential. The main test of that potential is through ***elastic*** scattering, and through the use of the differential cross section and analysing power… That optical potential is required to define the relative wave functions for use in quasi-elastic scattering.



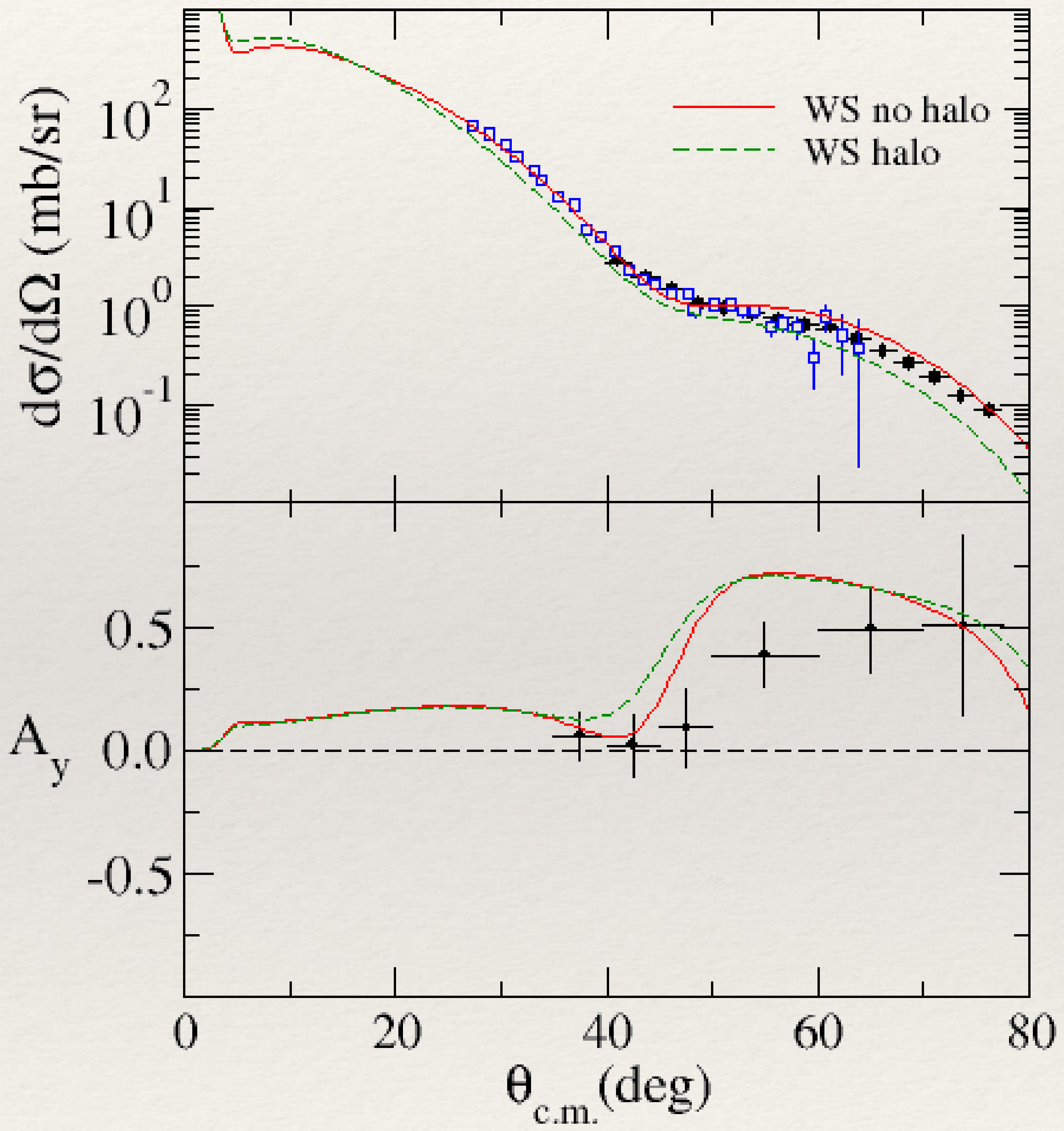
$^{12}C(p,p)$ at 200 MeV. Red: oscillator single-particle wave functions; blue: WS functions. (Ref: a certain speaker's PhD…)

^{6,8}*He proton elastic scattering*

(From S. Karataglidis and K. Amos, Phys. Rev. C **87**, 054623 (2013); data from S. Sakaguchi et al. Phys. Rev. C **84**, 024604 (2011), and Phys. Rev. C **87**, 021601(R) (2013).)

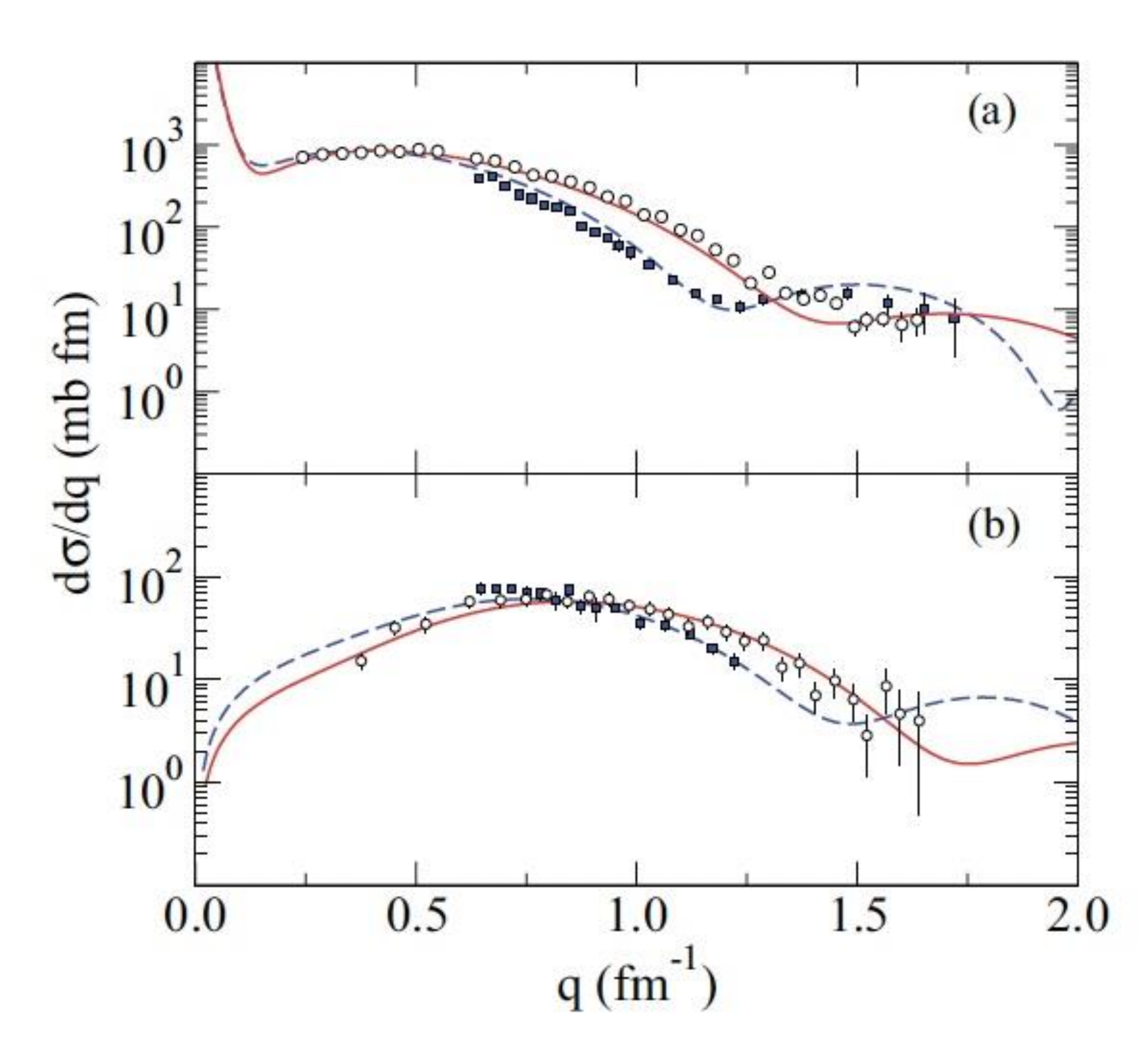


^{6}He(p,p) at 71*A* MeV



^{8}He(p,p) at 71*A* MeV

Energy dependence (^{6}He-p scattering):



Elastic scattering

Inelastic scattering to the 2^+ state

(Data from GANIL and JINR, SK and K. Amos, Phys. Rev. C **87**, 054623 (2013).)

The results of those calculations came from….

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Alternative evaluations of halos in nuclei

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Data for the scattering of ^{6}He, ^{8}He, ^{9}Li, and ^{11}Li from hydrogen are analyzed within a fully microscopic folding model of proton-nucleus scattering. Current data suggest that of these only ^{11}Li has a noticeable halo. For ^{6}He, we have also analyzed the complementary reaction $^{6}\mathrm{Li}(\gamma,\pi^{+})^{6}\mathrm{He}_{\mathrm{g.s.}}$. The available data for that reaction support the hypothesis that ^{6}He may not be a halo nucleus. However, those data are scarce and there is clearly a need for more to elicit the microscopic structure of ^{6}He.

PACS number(s): 21.10.Gv, 24.10.−i, 25.40.Ep, 27.20.+n

Low-energy scattering: MCAS

We seek to obtain S matrices and evaluate:

Total elastic scattering cross sections:

$$\sigma_{\mathrm{el}} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left| S_{ll+\frac{1}{2}}(k) - 1 \right|^2 + l \left| S_{ll-\frac{1}{2}}(k) - 1 \right|^2 \right\}$$

Total reaction cross sections:

$$\sigma_{\mathrm{R}} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left[1 - \left| S_{ll+\frac{1}{2}}(k) \right|^2 \right] + l \left[1 - \left| S_{ll-\frac{1}{2}}(k) \right|^2 \right] \right\}$$

The MCAS approach is built upon:

1. Finite-rank separable representations of realistic interactions;
2. Scattering matrices for separable Schrödinger interactions;
3. Sturmian functions (Weinberg states) to define form factors.

Multi-channel T matrices

Solution of coupled Lippmann-Schwinger equations:

$$T_{cc'}(p,q;E) = V_{cc'}(p,q) - \mu \sum_{c'=1}^{\text{closed}} \int_0^\infty V_{cc''}(p,x) \frac{1}{h_{c''}^2 + x^2} T_{c''c'}(x,q;E)\, x^2 dx$$

$$+ \mu \sum_{c'=1}^{\text{open}} \int_0^\infty V_{cc''}(p,x) \frac{1}{k_{c''}^2 - x^2 + i\varepsilon} T_{c''c'}(x,q;E)\, x^2 dx$$

Expand the potential matrix:

$$V_{cc'}(p,q) \sim V_{cc'}^{(N)}(p,q) = \sum_{n=1}^{N} \hat{\chi}_{cn}(p) \eta_n^{-1} \hat{\chi}_{c'n}(q)$$

Optimal functions, $\hat{\chi}_{cn}(q)$, involve Sturmians $|\Phi_{c'n}\rangle$:

$$|\hat{\chi}_{cn}\rangle = \sum_{c'} V_{cc'}\, |\Phi_{c'n}\rangle$$

$$\sum_{c'} G_c^{(0)} V_{cc'}\, |\Phi_{c'n}\rangle = -\eta_n\, |\Phi_{cn}\rangle$$

Multi-channel S matrices

Separable expansion of multi-channel $V_{cc'}$ ⇒multi-channel S matrix (cc' are open channels, specified by J^{π}):

$$S_{cc'} = \delta_{cc'} - i\pi\mu\sqrt{k_c k_{c'}} T_{cc'}$$

$$= \delta_{cc'} - i\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c}\chi_{cn}(k_c) \left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1} \right)_{nn'} \chi_{c'n'}(k_{c'}) \sqrt{k_{c'}}$$

Matrix elements (Sturmian basis) $[\boldsymbol{\eta}]_{nn'} = \eta_n \delta_{nn'}$, with:

$$[\mathbf{G}_0]_{nn'} = \mu \sum_{c=1}^{\text{open}} \int_0^{\infty} \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\varepsilon} \hat{\chi}_{cn'}(x) dx - \mu \sum_{c=1}^{\text{closed}} \int_0^{\infty} \hat{\chi}_{cn} \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'} dx$$

Collective model for Vcc'(r)

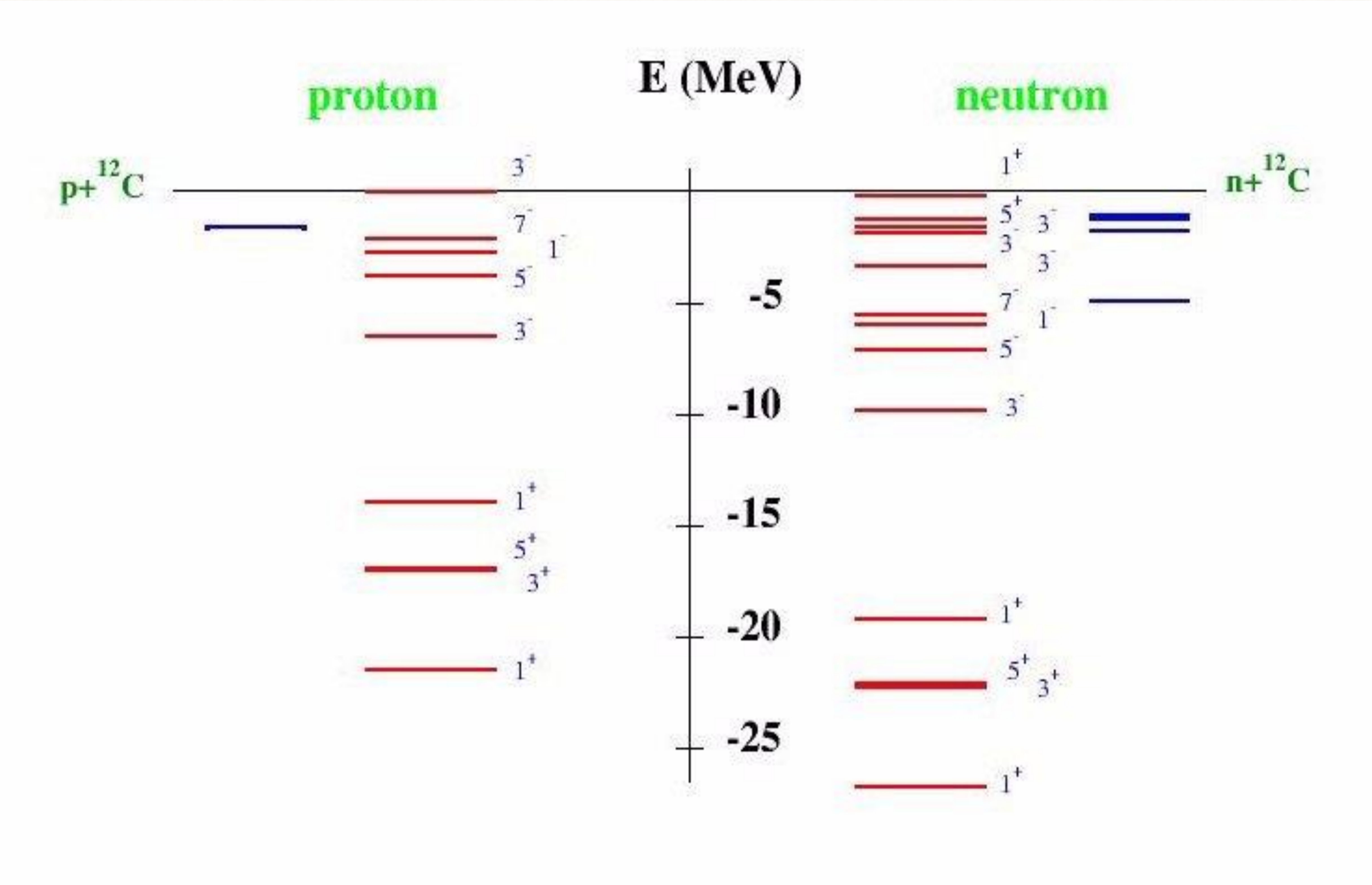
$$V_{cc'}(r) = ((l's)\,J'I';J^\pi\,|f(r,\mathbf{R})|\,(ls)\,JI;J^\pi)$$

Deformation is included via a rotational or vibrational model with the nuclear surface defined by:

$$R = R_0(1+\varepsilon);\varepsilon = \sum_{L\geq 2}\sqrt{\frac{4\pi}{2L+1}}\beta_L\left[\mathcal{Y}_L(\hat{r})\cdot\mathcal{Y}_L(\hat{r}')\right]$$

$$\begin{aligned} f(r,\mathbf{R}) &= f_0(r) + \varepsilon\left[\frac{df(r)}{d\varepsilon}\right]_0 + \frac{1}{2}\varepsilon^2\left[\frac{d^2 f(r)}{d\varepsilon^2}\right]_0 \\ &= f_0(r) + \frac{4\pi}{2L+1}\beta_L\left[\mathcal{Y}_L\cdot\mathcal{Y}_L\right]\frac{df_0(r)}{dr} \\ &\quad + \frac{1}{2}\beta_L^2(2L+1)\sum_{l,\text{even}}^{2L}\frac{1}{2L}\langle L\,0\,L\,0\,|l\,0\rangle\left[\mathcal{Y}_L\cdot\mathcal{Y}_L\right]\frac{d^2 f_0(r)}{dr^2} \end{aligned}$$

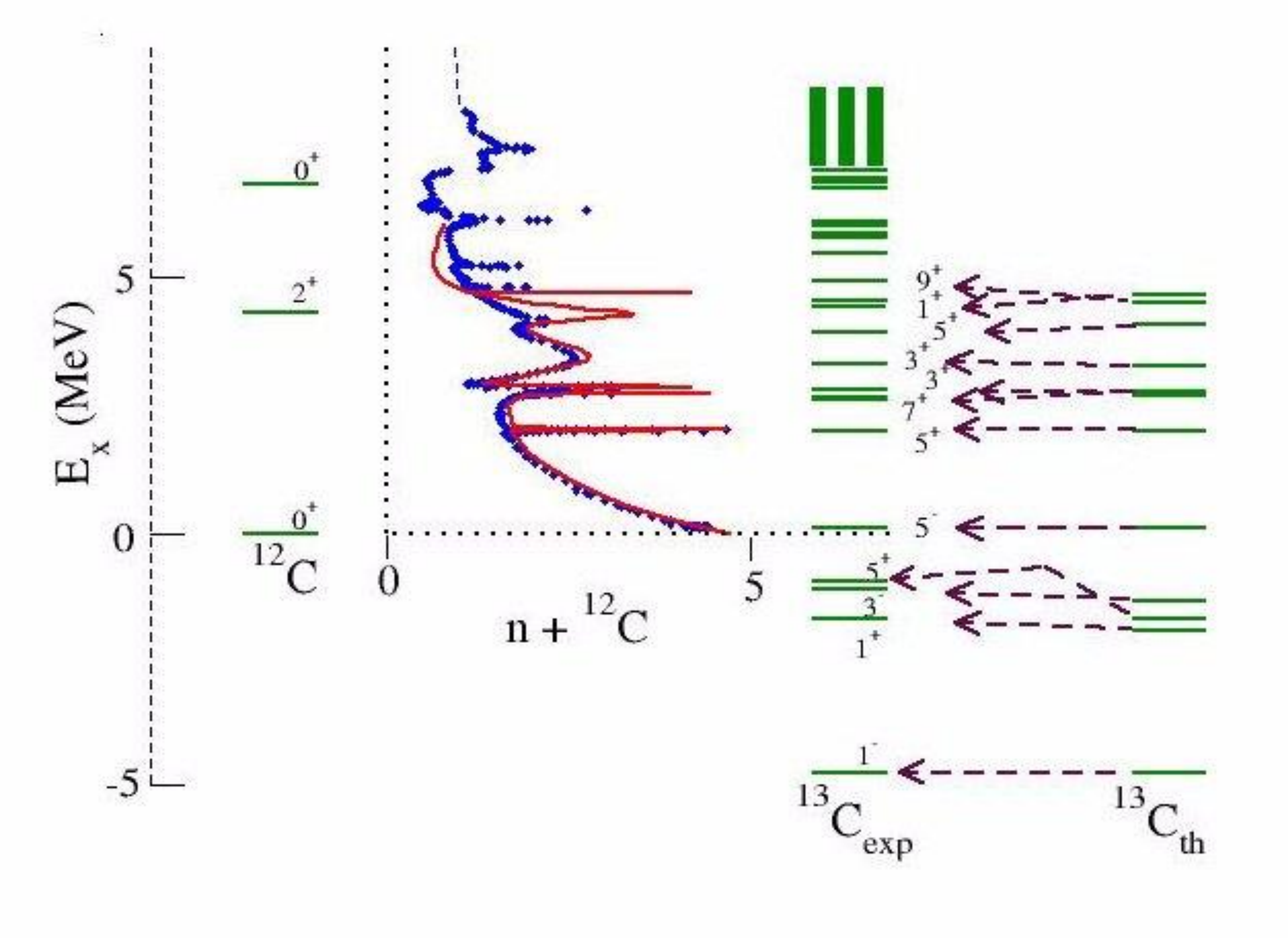
$^{12}C(p,p)$ low energy, mass-13 bound states



Energies in reference to p+^{12}C and n+^{12}C thresholds.

Problem! Pauli principle is violated.

OPP correction for Pauli



Conclusions

1. Presented optical models and at low and intermediate energies for NA scattering.
2. Potentials found are complex, nonlocal, target- and energy-dependent.
3. For quasi-free scattering with radioactive nuclei at intermediate energies, such potentials are necessary to obtain relative wave functions of the projectile and target.
4. Elastic scattering is essential to constrain the optical potentials used in the analyses of data coming from such experiments.