Vector boson model with broken proxy-SU(3) symmetry

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Contents

The vector-bosons as elementary nuclear collective excitations [P. Raychev, R. Roussev, Sov. J. Nucl. Phys. 27, 792 (1978)]

Assumption:
Certain class of nuclear collective properties can be described by two types of elementary excitations created by the operators
$\boldsymbol{\xi}^{+}$, $\boldsymbol{\eta}^{+}$
$\rightarrow$ vectors defined in Fock space

$$\xi_\nu = (-1)^\nu \partial/\partial\xi^+_{-\nu} \; ; \quad \eta_\nu = (-1)^\nu \partial/\partial\eta^+_{-\nu} \; , \qquad \nu = 1, 0, -1$$

$$\xi^\mu = (-1)^\mu \xi_{-\mu} \; ; \quad \eta^\mu = (-1)^\mu \eta_{-\mu}$$

$\rightarrow$ closing boson commutation relations

$$[\xi^\mu, \xi^+_\nu] = [\eta^\mu, \eta^+_\nu] = \delta_{\mu\nu} \; , \qquad \mu, \nu = 1, 0, -1$$

Vector-boson realization of SU(3) algebra

Angular momentum

$$L_m = -\sqrt{2}\sum_{\mu,\nu} C^{1m}_{1\mu 1\nu}(\xi^+_\mu \xi_\nu + \eta^+_\mu \eta_\nu) \ , \qquad m = 0, \pm 1$$

Quadrupole momentum

$$Q_k = \sqrt{6}\sum_{\mu,\nu} C^{2k}_{1\mu 1\nu}(\xi^+_\mu \xi_\nu + \eta^+_\mu \eta_\nu) \ , \qquad k = 0, \pm 1, \pm 2$$

$\rightarrow$ closing SU(3) algebra

$$\begin{aligned}
[L_m, L_n] &= -\sqrt{2}C^{1m+n}_{1m1n} L_{m+n} \\
[L_m, Q_n] &= \sqrt{6}C^{2m+n}_{1m2n} Q_{m+n} \\
[Q_m, Q_n] &= 3\sqrt{10}C^{1m+n}_{2m2n} L_{m+n}
\end{aligned}$$

SU(3) irreducible representations (irreps) and Casimir operators

$U(n) \supset U(3) \supset SU(3)$

$U(3)$ representation $[f_1, f_2, f_3]$, $\quad (f_1 \geq f_2 \geq f_3)$

$SU(3)$ irreps: (λ, μ), $\quad \lambda = f_1 - f_2,\ \mu = f_2 - f_3$

$SU(3)$ **invariants (Casimir operators):**

$\hat{C}_2 \equiv \sum_i \hat{F}_i \hat{F}_i \simeq \frac{1}{4} \hat{Q} \cdot \hat{Q} + \frac{3}{4} \hat{L}^2$

$\hat{C}_3 \equiv \sum_{ijk} \hat{F}_i \hat{F}_j \hat{F}_k$

Eigenvalues (calc. in SU(3) matr. repr., Baird, Biedenharn, JMP 1963)

$\langle \hat{C}_2 \rangle \sim \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)$

$\langle \hat{C}_3 \rangle \sim (\lambda - \mu)(\lambda + 2\mu + 3)(2\lambda + \mu + 3)$

$dim(\lambda, \mu) = 1/2(\lambda + 1)(\mu + 1)(\lambda + \mu + 2)$

$\boldsymbol{\xi}^+, \boldsymbol{\eta}^+ \rightarrow$ O(3) vectors transforming under two independent $(\lambda, \mu) = (1, 0)$ irreps

SU(3) breaking. VBM Hamiltonian and basis.

Rotation invariants reducing $SU(3)$ to $O(3)$:

$$L^2 = \sum_m (-1)^m L_m L_{-m}$$

$$L \cdot Q \cdot L = \sum_{M,m,m'} (-1)^M C^{2M}_{1m1m'} Q_{-M} L_m L_{m'}$$

$$A^+ A, \qquad A^+ = (\boldsymbol{\xi}^+)^2 (\boldsymbol{\eta}^+)^2 - (\boldsymbol{\xi}^+ \cdot \boldsymbol{\eta}^+)^2$$

Vector-boson Hamiltonian with broken SU(3) symmetry

$$H_{VBM} = g_1 L^2 + g_2 L \cdot Q \cdot L + g_3 A^+ A$$

$$SU(3) \supset O(3) \supset O(2)$$

Basis [V. Bargmann and M. Moshinsky, Nucl. Phys. **23**, 177 (1961)]

$$\left| \begin{array}{c} (\lambda, \mu) \\ \alpha, L, M \end{array} \right\rangle = P^{(\lambda,\mu,\alpha,L,M)}(\xi^+_\nu, \eta^+_\nu)|0\rangle, \quad \alpha \ - \ O(3) \text{ multiplicity q.n.}$$

Diagonalization and spectrum [N.M. et al, PRC 55 2345 (1997)]

$$\left| \begin{array}{c} (\lambda,\mu) \\ \omega_i^L, L, L \end{array} \right\rangle = \sum_{j=1}^{d_L} C_{i,j}^L \left| \begin{array}{c} (\lambda,\mu) \\ \alpha_j, L, L \end{array} \right\rangle$$

L: $\{\alpha_j\}_{j=1\div d_L}$ $(\alpha_j < \alpha_{j+1})$

$\max\{0, \frac{1}{2}(\mu - L)\} \le \alpha \le \min\{\frac{1}{2}(\mu-\beta), \frac{1}{2}(\lambda+\mu-L-\beta)\}$

$$\beta = \begin{cases} 0, & \lambda+\mu-L \text{ even} \\ 1, & \lambda+\mu-L \text{ odd} \end{cases}$$

λ, μ even, $\lambda > \mu$: $\alpha = 0,\ 1,\ 2,\ \ldots\ \mu/2 = \alpha_{max}$

$(\lambda,\mu) \to \{(\alpha_i, L_{\alpha_i})\} \to SU(3)$ multiplet

$K = \mu - 2\alpha$, $\quad N = \lambda + 2\mu$ – number of vector bosons

Energy bands

$\alpha_{max} = \mu/2$: $L = 0^+,\ 2^+,\ 4^+,\ \ldots,\ L_{max} = \lambda$ gsb
$\alpha_{max} - 1$: $L = 2^+,\ 3^+,\ 4^+,\ \ldots,\ L_{max} = \lambda + 2$ γ band
$\alpha_{max} - 2$: $L = 4^+,\ 5^+,\ 6^+,\ \ldots,\ L_{max} = \lambda + 4$ $K^\pi = 4^+$ band; ...
$\alpha = 0$: $L = \mu,\ \mu+1,\ \mu+2,\ \ldots,\ L_{max} = \lambda + \mu$ $K = \mu$ band

Transition rates

$$B(E2;\omega_i^L \to \omega_{i'}^{L+k}) = \frac{1}{2L+1}\begin{pmatrix} L+k & 2 & L \\ -L & 0 & L \end{pmatrix}^{-2} \times \left| \left\langle \begin{array}{c} (\lambda,\mu) \\ \omega_{i'}^{L+k}, L+k, L \end{array} \middle| Q_0 \middle| \begin{array}{c} (\lambda,\mu) \\ \omega_i^L, L, L \end{array} \right\rangle \right|^2$$

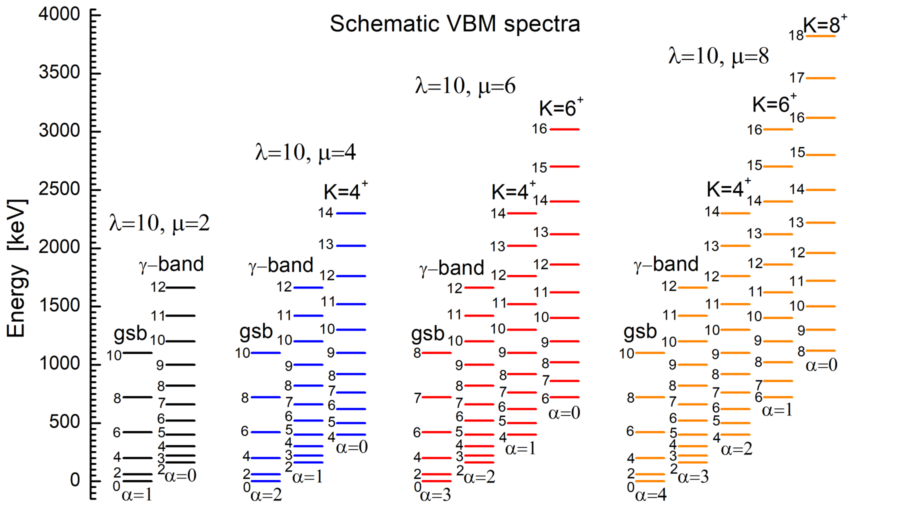
$$R_1(L) = \frac{B(E2; L_\gamma \to L_g)}{B(E2; L_\gamma \to (L-2)_g)}, \quad \text{L even}$$

$$R_2(L) = \frac{B(E2; L_\gamma \to (L+2)_g)}{B(E2; L_\gamma \to L_g)}, \quad \text{L even}$$

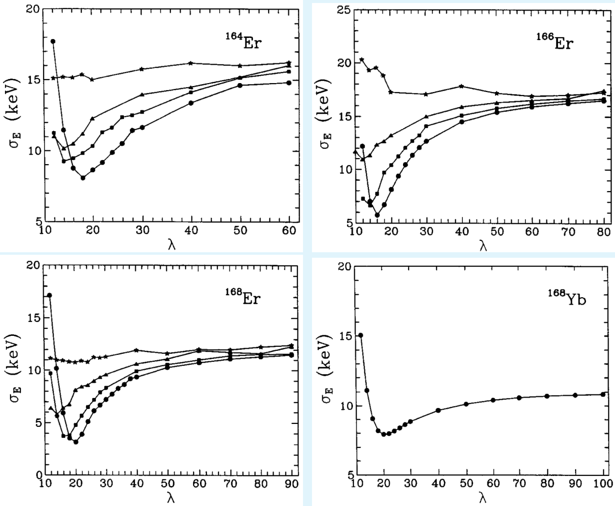
$$R_3(L) = \frac{B(E2; L_\gamma \to (L+1)_g)}{B(E2; L_\gamma \to (L-1)_g)}, \quad \text{L odd}$$

$$R_4(L) = \frac{B(E2; L_g \to (L-2)_g)}{B(E2; (L-2)_g \to (L-4)_g)}, \quad \text{L odd}$$

VBM spectra for different SU(3) irreps: SU(3) multiplets

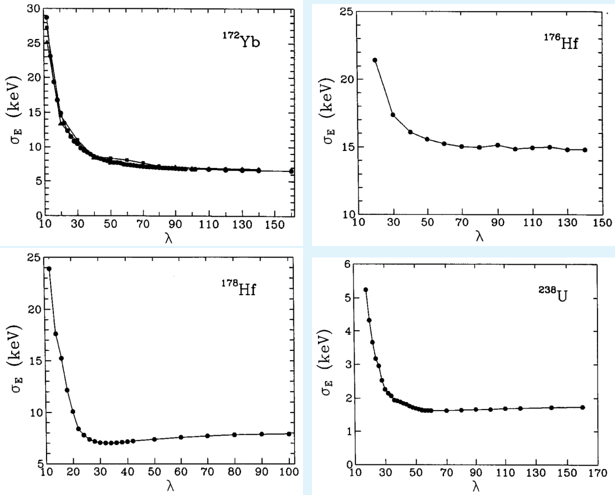


Favored SU(3) multiplets



$\bullet$ $\mu = 2$, ■ $\mu = 4$, ▲ $\mu = 6$, ★ $\mu = 8$

Favored SU(3) multiplets



Favored SU(3) multiplets [PRC 55 2345 (1997)])

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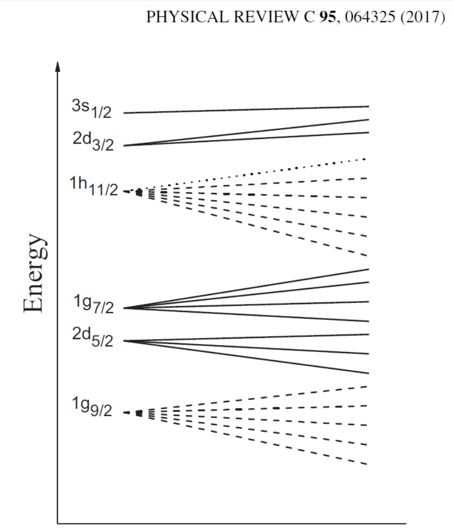
TABLE II. The parameters of the fits of the energy levels and the transition ratios [Eqs. (20) and (21)] of the nuclei investigated are listed for the (λ,μ) multiplets which provide the best model descriptions. The Hamiltonian parameters g_1, g_2, and g_3 [Eq. (5)] are given in keV. The quantities σ_E (in keV) and σ_B (dimensionless) represent the energy [Eq. (44)] and the transition [Eq. (45)] rms factors, respectively. The splitting ratios ΔE_2 [Eq. (46), dimensionless] and the vector-boson numbers N [Eq. (9)] are also given.

Nucl	ΔE_2	λ,μ	σ_E	σ_B	g_1	g_2	g_3	N
^{164}Dy	9.4	16,2	14.1	0.52	-1.159	-0.321	-0.590	20
^{164}Er	8.4	18,2	8.1	0.14	3.625	-0.238	-0.513	22
^{166}Er	8.8	16,2	5.8	0.47	2.942	-0.235	-0.572	20
^{168}Er	9.3	20,2	3.2	0.28	4.000	-0.181	-0.401	24
^{168}Yb	10.2	20,2	7.9	0.27	0.500	-0.271	-0.501	24
^{172}Yb	17.6	$\geqslant$80,2	6.8	0.12	9.875	-0.017	-0.052	84
^{176}Hf	14.2	$\geqslant$70,2	15.0	0.17	9.547	-0.030	-0.062	74
^{178}Hf	11.6	34,2	7.0	0.86	8.322	-0.083	-0.213	38
^{238}U	22.6	$\geqslant$60,2	1.6	0.08	-37.697	-0.360	-0.098	64

$$\Delta E_2 = (E_{2_2^+} - E_{2_1^+})/E_{2_1^+}$$

Proxy SU(3) mapping (D. Bonatsos et al)



Proxy SU(3) irreps (D. Bonatsos et al)

ANALYTIC PREDICTIONS FOR NUCLEAR SHAPES, . . . PHYSICAL REVIEW C **95**, 064326 (2017)

TABLE II. Most leading SU(3) irreps [34,35] for nuclei with protons in the 50-82 shell and neutrons in the 82-126 shell. Boldface numbers indicate nuclei with $R_{4/2} = E(4_1^+)/E(2_1^+) \geqslant 2.8$, while * denotes nuclei with $2.8 > R_{4/2} \geqslant 2.5$, and ** labels a few nuclei with $R_{4/2}$ ratios slightly below 2.5, shown for comparison, while no irreps are shown for any other nuclei with $R_{4/2} < 2.5$. For the rest of the nuclei shown (using normal fonts and without any special signs attached) the $R_{4/2}$ ratios are still unknown [46]. Irreps corresponding to oblate shapes are underlined.

N	$N_{\rm val}$	Z	Ba	Ce	Nd	Sm	Gd	Dy	Er	Yb	Hf	W	Os	Pt
		$Z_{\rm val}$	56	58	60	62	64	66	68	70	72	74	76	78
		irrep	6	8	10	12	14	16	18	20	22	24	26	28
			(18,0)	(18,4)	(20,4)	(24,0)	(20,6)	(18,8)	(18,6)	(20,0)	(12,8)	(6,12)	(2,12)	(0,8)
88	6	(24,0)	(42,0)*	(42,4)*	(44,4)*									
90	8	(26,4)	**(44,4)**	**(44,8)**	**(46,8)**	**(50,4)**	**(46,10)**	**(44,12)**	(44,10)*	(46,4)*	(38,12)*			
92	10	(30,4)	**(48,4)**	**(48,8)**	**(50,8)**	**(54,4)**	**(50,10)**	**(48,12)**	**(48,10)**	**(50,4)**	(42,12)*			
94	12	(36,0)	(54,0)	**(54,4)**	**(56,4)**	**(60,0)**	**(56,6)**	**(54,8)**	**(54,6)**	**(56,0)**	**(48,8)**	**(42,12)**	(38,12)*	
96	14	(34,6)	(52,6)	(52,10)	**(54,10)**	**(58,6)**	**(54,12)**	**(52,14)**	**(52,12)**	**(54,6)**	**(46,14)**	**(40,18)**	(36,18)*	
98	16	(34,8)	(52,8)	(52,12)	(54,12)	**(58,8)**	**(54,14)**	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	(36,20)*	
100	18	(36,6)	(54,6)	(54,10)	(56,10)	(60,6)	**(56,12)**	**(54,14)**	**(54,12)**	**(56,6)**	**(48,14)**	**(42,18)**	**(38,18)**	(36,14)*
102	20	(40,0)	(58,0)	(58,4)	(60,4)	(64,0)	**(60,6)**	**(58,8)**	**(58,6)**	**(60,0)**	**(52,8)**	**(46,12)**	**(42,12)**	(40,8)*
104	22	(34,8)	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	**(36,20)**	(34,16)*
106	24	(30,12)	(48,12)	(48,16)	(50,16)	(54,12)	(50,18)	(48,20)	**(48,18)**	**(50,12)**	**(42,20)**	**(36,24)**	**(32,24)**	(30,20)*
108	26	(28,12)	(46,12)	(46,16)	(48,16)	(52,12)	(48,18)	(46,20)	(46,18)	**(48,12)**	**(40,20)**	**(34,24)**	**(30,24)**	(28,20)*
110	28	(28,8)	(46,8)	(46,12)	(48,12)	(52,8)	(48,14)	(46,16)	(46,14)	(48,8)	**(40,16)**	**(34,20)**	**(30,20)**	(28,16)*
112	30	(30,0)	(48,0)	(48,4)	(50,4)	(54,0)	(50,6)	(48,8)	(48,6)	(50,0)	**(42,8)**	**(36,12)**	**(32,12)**	(30,8)**
114	32	(20,10)	(38,10)	(38,14)	(40,14)	(44,10)	(40,16)	(38,18)	(38,16)	(40,10)	(32,18)	**(26,22)**	**(22,22)**	(20,18)**
116	34	(12,16)	(30,6)	(30,10)	(32,10)	(36,6)	(32,12)	(30,14)	(30,12)	(32,6)	(24,14)	<u>(18,28)*</u>	<u>**(14,28)**</u>	<u>(12,24)**</u>
118	36	(6,18)	(24,18)	(24,22)	(26,22)	(30,18)	(26,24)	(24,16)	(24,24)	(26,18)	<u>(18,26)</u>	<u>(12,30)</u>	<u>(8,30)*</u>	<u>(6,26)**</u>
120	38	(2,16)	(20,16)	(20,20)	(22,20)	(26,16)	(22,22)	<u>(20,24)</u>	<u>(20,22)</u>	(22,16)	<u>(14,24)</u>	<u>(8,28)</u>	<u>(4,28)*</u>	<u>(2,24)**</u>

Proxy SU(3) irreps [S. Sarantopoulou, D. Bonatsos et al, BJP 44, 417 (2017)]

TABLE II: Highest weight SU(3) irreps for nuclei with protons in the 82-126 shell and neutrons in the 126-184 shell.

			Rn	Ra	Th	U	Pu	Cm	Cf	Fm	No	Rf	Sg	Hs	Ds	Cn	Fl	Lv	Og		
		Z	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118	120	122
		Z_{val}	4	6	8	10	12	14	16	18	20	22	24	26	28						
N	N_{val}	irrep	(16,2)	(24,0)	(26,4)	(30,4)	(36,0)	(34,6)	(34,8)	(36,6)	(40,0)	(34,8)	(30,12)	(28,12)	(28,8)	(30,0)	(20,10)	(12,16)	(6,18)	(2,16)	(0,10)
130	4	(20,2)	(36,4)	(44,2)	(46,6)	(50,6)	(56,2)	(54,8)	(54,10)	(56,8)	(60,2)	(54,10)	(50,14)	(48,4)	(48,10)	(50,2)	(40,12)	(32,18)	(26,20)	(22,18)	(20,12)
132	6	(30,0)	(46,2)	(54,0)	(56,4)	(60,4)	(66,0)	(64,6)	(64,8)	(66,6)	(70,0)	(64,8)	(60,12)	(58,12)	(58,8)	(60,0)	(50,10)	(42,16)	(36,18)	(32,16)	(30,10)
134	8	(34,4)	(50,6)	(58,4)	(60,8)	(64,8)	(70,4)	(68,10)	(68,12)	(70,10)	(74,4)	(68,12)	(64,16)	(62,16)	(62,14)	(64,4)	(54,14)	(46,20)	(40,22)	(36,20)	(34,14)
136	10	(40,4)	(56,6)	(64,4)	(66,8)	(70,8)	(76,4)	(74,10)	(74,12)	(76,10)	(80,4)	(74,12)	(70,16)	(68,16)	(68,12)	(70,4)	(60,14)	(52,20)	(46,22)	(42,20)	(40,14)
138	12	(48,0)	(64,2)	(72,0)	(74,4)	(78,4)	(84,0)	(82,6)	(82,8)	(84,6)	(88,0)	(82,8)	(78,12)	(76,12)	(76,8)	(78,0)	(68,10)	(60,16)	(54,18)	(50,16)	(48,10)
140	14	(48,6)	(64,8)	(72,6)	(74,10)	(78,10)	(84,6)	(82,12)	(82,14)	(84,12)	(88,6)	(82,14)	(78,18)	(76,18)	(76,14)	(78,6)	(68,16)	(60,22)	(54,24)	(50,22)	(48,16)
142	16	(50,8)	(66,10)	(74,8)	(76,12)	(80,12)	(86,8)	(84,14)	(84,16)	(86,14)	(90,8)	(84,16)	(80,20)	(78,20)	(78,16)	(80,8)	(70,18)	(62,24)	(56,26)	(52,24)	(50,18)
144	18	(54,6)	(70,8)	(78,6)	(80,10)	(84,10)	(90,6)	(88,12)	(88,14)	(90,12)	(94,6)	(88,14)	(84,18)	(82,18)	(82,14)	(84,6)	(74,16)	(66,22)	(60,24)	(56,22)	(54,16)
146	20	(60,0)	(76,2)	(84,0)	(86,4)	(90,4)	(96,0)	(94,6)	(94,8)	(96,6)	(100,0)	(94,8)	(90,12)	(88,12)	(88,8)	(90,0)	(80,10)	(72,16)	(66,18)	(62,16)	(60,10)
148	22	(56,8)	(72,10)	(80,8)	(82,12)	(86,12)	(92,8)	(90,14)	(90,16)	(92,14)	(96,8)	(90,16)	(86,20)	(84,20)	(84,16)	(86,8)	(76,18)	(68,24)	(62,26)	(58,24)	(56,18)
150	24	(54,12)	(70,14)	(78,12)	(80,16)	(84,16)	(90,12)	(88,18)	(88,20)	(90,18)	(94,12)	(88,20)	(84,24)	(82,24)	(82,20)	(84,12)	(74,22)	(66,28)	(60,30)	(56,28)	(54,22)
152	26	(54,12)	(70,14)	(78,12)	(80,16)	(84.16))	(90,12)	(88,18)	(88,20)	(90,18)	(94,12)	(88,20)	(84,24)	(82,24)	(82,20)	(84,12)	(74,22)	(66,28)	(60,30)	(56,28)	(54,22)
154	28	(56,8)	(72,10)	(80,8)	(82,12)	(86,12)	(92,8)	(90,14)	(90,16)	(92,14)	(96,8)	(90,16)	(86,20)	(84,20)	(84,16)	(86,8)	(76,18)	(68,24)	(62,26)	(58,24)	(56,18)
156	30	(60,0)	(76,2)	(84,0)	(86,4)	(90,4)	(96,0)	(94,6)	(94,8)	(96,6)	(100,0)	(94,8)	(90,12)	(88,12)	(88,8)	(90,0)	(80,10)	(72,16)	(66,18)	(62,16)	(60,10)
158	32	(52,10)	(68,12)	(76,10)	(78,14)	(82,14)	(88,10)	(86,16)	(86,18)	(88,16)	(92,10)	(86,18)	(82,22)	(80,22)	(80,18)	(82,10)	(72,20)	(64,26)	(58,28)	(54,26)	(52,20)
160	34	(46,16)	(62,18)	(70,16)	(72,20)	(76,20)	(82,16)	(80,22)	(80,24)	(82,22)	(86,16)	(80,24)	(76,28)	(74,28)	(74,24)	(76,16)	(66,26)	(58,32)	(52,34)	(48,32)	(46,26)
162	36	(42,18)	(58,20)	(66,18)	(68,22)	(72,22)	(78,18)	(76,24)	(76,26)	(78,24)	(82,18)	(76,26)	(72,30)	(70,30)	(70,26)	(72,18)	(62,28)	(54,34)	(48,36)	(44,34)	(42,28)
164	38	(40,16)	(56,18)	(64,16)	(66,20)	(70,20)	(76,16)	(74,22)	(74,24)	(76,22)	(80,16)	(74,24)	(70,28)	(68,28)	(68,24)	(70,16)	(60,26)	(52,32)	(46,24)	(42,32)	(40,26)
166	40	(40,10)	(56,12)	(64,10)	(66,14)	(70,14)	(76,10)	(74,16)	(74,18)	(76,16)	(80,10)	(74,18)	(70,22)	(68,22)	(68,18)	(70,10)	(60,20)	(52,26)	(46,28)	(42,26)	(40,20)
168	42	(42,0)	(58,2)	(66,0)	(68,4)	(72,4)	(78,0)	(76,6)	(76,8)	(78,6)	(82,0)	(76,8)	(72,12)	(70,12)	(70,8)	(72,0)	(62,10)	(54,16)	(48,18)	(44,16)	(42,10)
170	44	(30,12)	(46,14)	(54,12)	(56,16)	(60,16)	(66,12)	(64,18)	(64,20)	(66,18)	(70,12)	(64,20)	(60,24)	(58,24)	(58,20)	(60,12)	(50,22)	(42,28)	(36,30)	(32,28)	(30,22)
172	46	(20,20)	(36,22)	(44,20)	(46,24)	(50,24)	(56,20)	(54,26)	(54,28)	(56,26)	(60,20)	(54,28)	(50,32)	(48,32)	(48,28)	(50,20)	(40,30)	(32, 36)	(26, 38)	(22, 36)	(20, 30)
174	48	(12,24)	(28,26)	(36,24)	(38,28)	(42,28)	(48,24)	(46,30)	(46,32)	(48,30)	(52,24)	(46,32)	(42,36)	(40,36)	(40,32)	(42,24)	(32, 34)	(24, 40)	(18, 42)	(14, 40)	(12, 34)
176	50	(6,24)	(22,26)	(30,24)	(32,28)	(36,28)	(42,24)	(40,30)	(40,32)	(42,30)	(46,24)	(40,32)	(36,36)	(34,36)	(34,32)	(36,24)	(26, 34)	(18, 40)	(12, 42)	(8, 40)	(6, 34)
178	52	(2,20)	(18,22)	(26,20)	(28,24)	(32,24)	(38,20)	(36,26)	(36,28)	(38,26)	(42,20)	(36,28)	(32,32)	(30,32)	(30,28)	(32,20)	(22, 30)	(14, 36)	(8, 38)	(4, 36)	(2, 30)
180	54	(0,12)	(16,14)	(24,12)	(26,16)	(30,16)	(36,12)	(34,18	(34,18)	(36,18)	(40,12)	(34,20)	(30,24)	(28,24)	(28,20)	(30,12)	(20, 22)	(12, 28)	(6, 30)	(2, 28)	(0, 22)

Favored and proxy SU(3) multiplets

Nucl	ΔE_2	λ, μ	σ_E	σ_B	g_1	g_2	g_3	N
^{164}Dy	9.4	16, 2	14.1	0.52	−1.159	−0.321	−0.590	20
	9.4	52, 16	19.8	0.46	−18.558	−0.194	−0.052	84
^{164}Er	8.4	18, 2	8.1	0.14	3.625	−0.238	−0.513	22
	8.4	52, 12	18.5	0.15	−8.805	−0.158	−0.059	76
^{166}Er	8.8	16, 2	5.8	0.47	2.942	−0.235	−0.572	20
	8.8	52, 14	19.1	0.43	−11.081	−0.235	−0.153	80
^{168}Er	9.3	20, 2	3.2	0.28	4.000	−0.181	−0.401	24
	9.3	54, 12	12.8	0.21	−7.799	−0.136	−0.053	78
^{168}Yb	10.2	20, 2	7.9	0.27	0.500	−0.271	−0.501	24
	10.2	54, 8	10.9	0.24	−6.536	−0.151	−0.071	70
^{172}Yb	17.6	$\geq$ 80, 2	6.8	0.12	9.875	−0.017	−0.052	84
	17.6	60, 2	7.4	0.12	9.531	−0.024	−0.091	64
^{176}Hf	14.2	$\geq$ 70, 2	15.0	0.17	9.547	−0.030	−0.062	74
	14.2	46, 16	15.5	0.16	−28.637	−0.262	−0.106	78
^{178}Hf	11.6	34, 2	7.0	0.86	8.322	−0.083	−0.213	38
	11.6	42, 20	7.6	0.86	−43.408	−0.354	−0.102	82
^{238}U	22.6	$\geq$ 60, 2	1.7	0.08	−38.112	−0.363	−0.098	64
	22.6	90, 4	1.7	0.08	−32.992	−0.215	−0.042	98

Concluding remarks

- The VBM algorithm is applicable with the use of proxy SU(3) defined multiplets.
- The proxy SU(3) symmetry acquires further physical significance in the structure of nuclear collective excited spectra (ground and γ bands, in particular)
- Possible application of the VBM with broken SU(3) symmetry for further detailed analysis and systematics of spectra and transition rates in wide ranges of heavy even-even deformed nuclei.