Nuclear structure studies within the realistic shell-model framework

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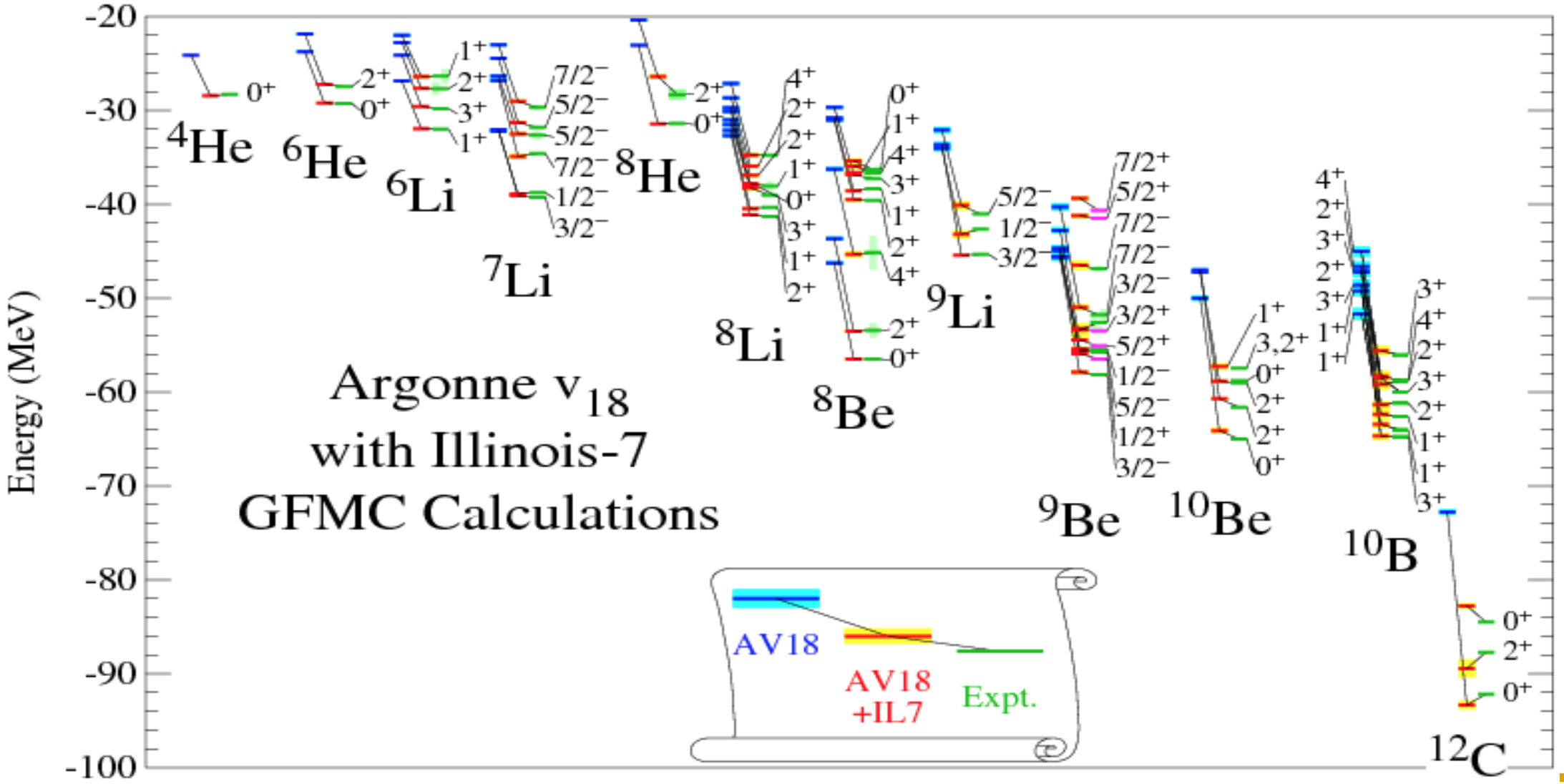
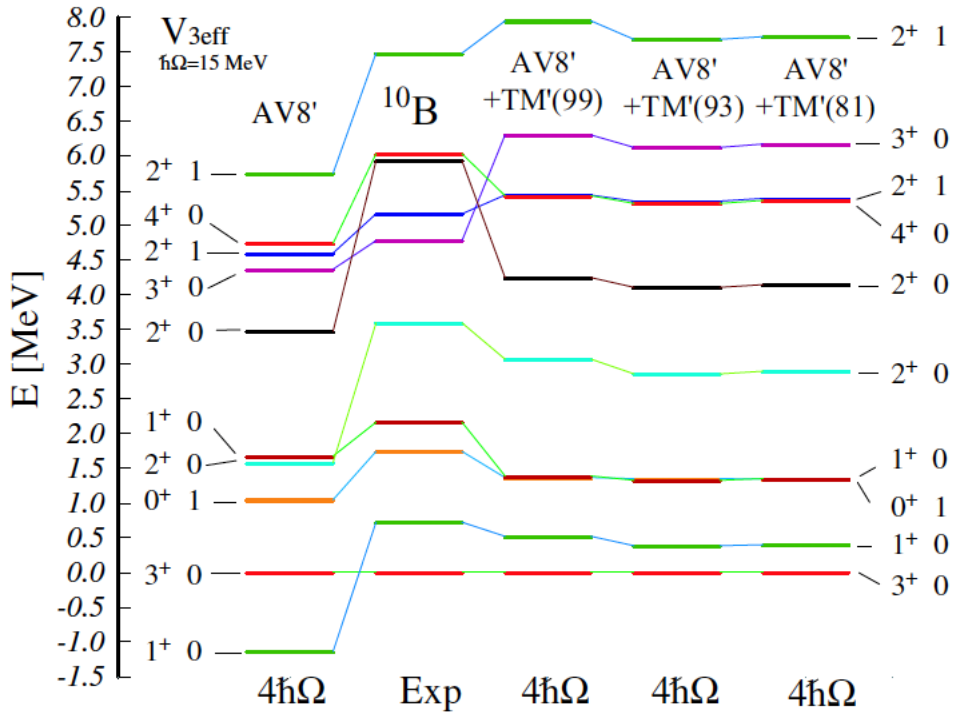
L. Coraggio et al., Prog.Part.Nucl.Phys. 134 (2024) 104079

The quantitative role of 3-body force has well established in light nuclei, A>=12, with “exact” calculations



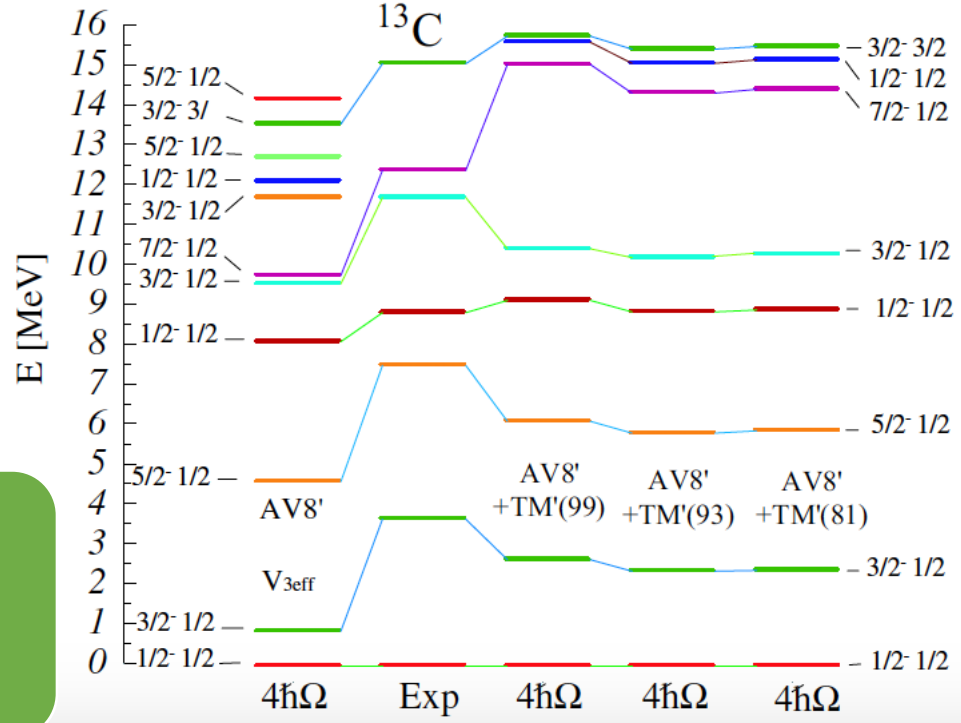
Carlson et al. Rev. Mod. Phys. 87 1067 (2015)



No core shell-model
results with AV8' NN potential + Tucson Melbourne 3N interactions

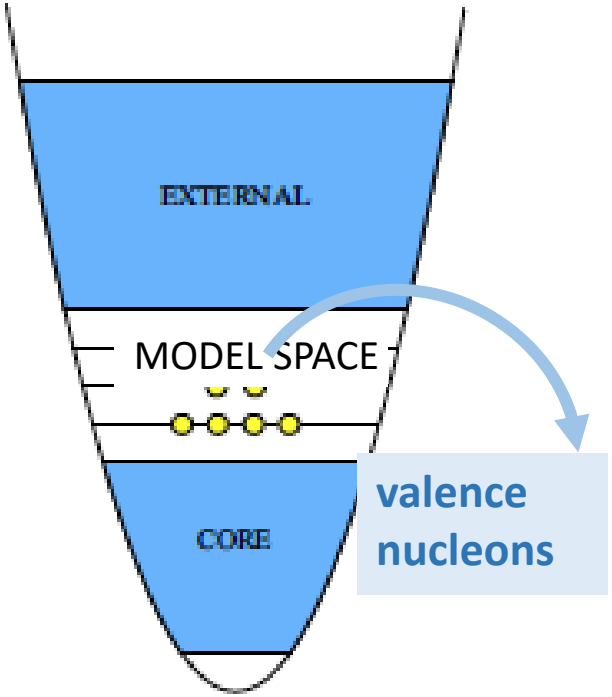
All considered 3N forces give similar results, but dramatically different compared to the calculations with only the NN potential

3N forces are necessary to give
- the 1^+ - 0^+ ground-state spin inversion in ^{10}B
- the correct splitting of the 1/2- - 3/2- levels in ^{13}C



P. Navratil et al, Eur. Phys. J. A 25, 481 (2005)

Shell model and realistic *NN* interactions



$$H_{\text{eff}} = \sum_a \epsilon_a N_a - \frac{1}{4} \sum_{abcdJ} \langle ab; J \mid V_{\text{eff}} \mid cd; J \rangle (-1)^J [a_a^\dagger a_b^\dagger]^J \cdot [\tilde{a}_c \tilde{a}_d]^J ,$$

defined in the model space for only valence nucleons

SM effective interactions derived from realistic *NN* potentials

$$V_{NN} \Longrightarrow \text{many} - \text{body theory} \Longrightarrow \text{H}_{\text{eff}}$$

show some deficiencies in reproducing nuclear spectroscopy, especially when moving far from closed shells

phenomenological corrections to H_{eff}

Monopole modified SM effective interactions

initiated in Eduardo Pasquini's Ph.D. thesis (1976) and pursued by the Strasbourg-Madrid group

H_{eff}'s from realistic *NN* potential need to be modified only in their monopole components

Monopole component

$$H_{\rm mon} = \sum_{a\tau} \epsilon_{a\tau} N_{a\tau} + \frac{1}{2} \sum_{ab\tau\tau'} \frac{\bar{V}^{\tau\tau'}_{ab} N_{a\tau}(N_{b\tau'} - \delta_{ab}\delta_{\tau\tau'})}{\sum_J \hat{J}^2}$$

Centroid

$$\bar{V}^{\tau\tau'}_{ab} = \frac{\sum_J \hat{J} \langle a\tau b\tau'; J \mid V_{\rm eff} \mid a\tau b\tau'; J \rangle}{\sum_J \hat{J}}$$

$$\mathrm{ESPE}(a\tau) = \epsilon_{a\tau} + \sum_{b\tau'} \bar{V}^{\tau\tau'}_{ab} n^{\tau'}_{b}$$

Responsible for the evolution of the SP energies

- Only centroids should be fitted to obtain results of a quality comparable with that provided by phenomenological interactions

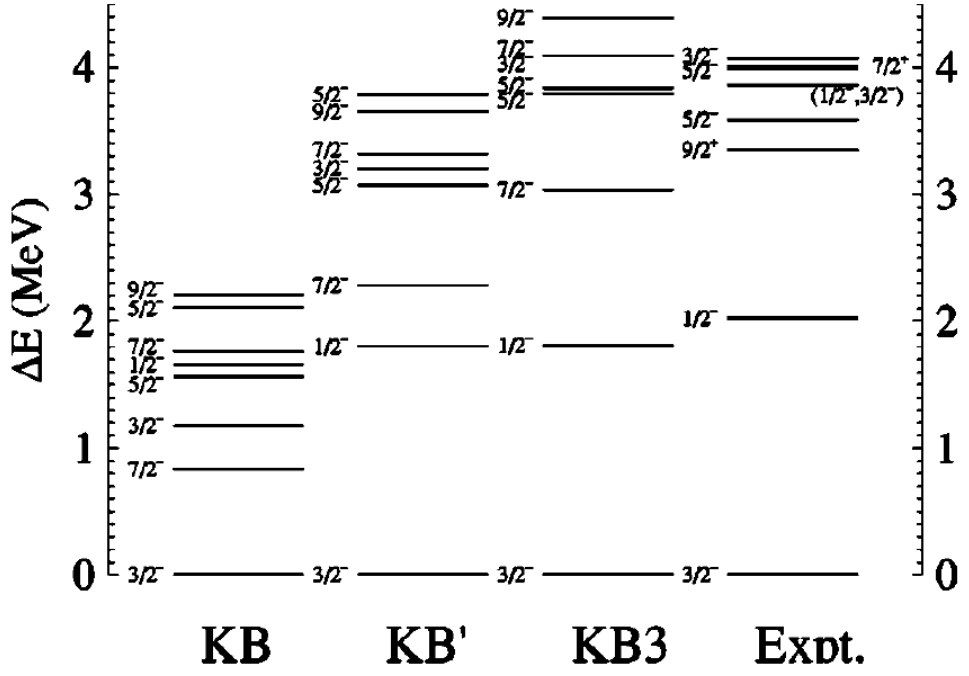
- Deficiencies in centroids are related to the bad saturation and shell formation properties of the NN interaction and can be traced back to the lack of 3N forces

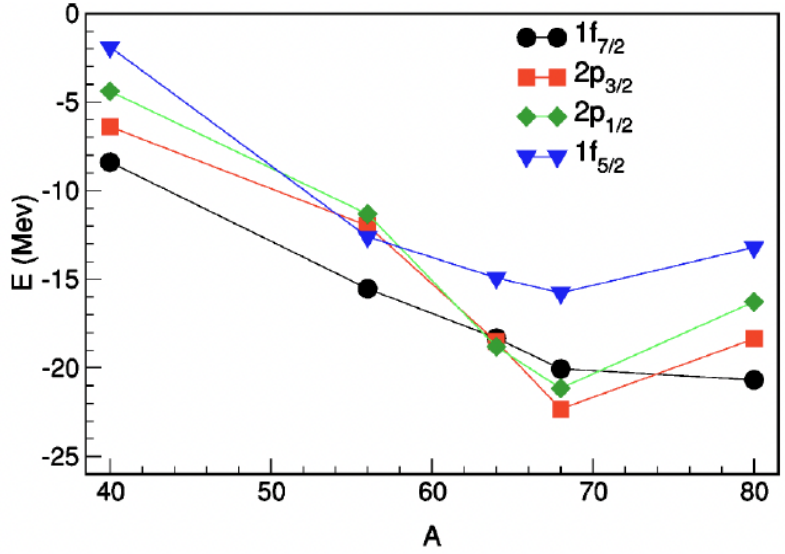
A. Poves, A. Zuker, Phys. Rep. 70 (1981) 235
E. Caurier et al, PRC C 50 (1994) 225
G.. Martínez-Pinedo, et al, PRC 55, 1871997
A. Poves et al, NPA 694, 157 (2001)
E. Caurier et al, Rev. Mod. Phys. 77 (2005) 427
S. M. Lenzi et al,PRC 82 (2010) 054301

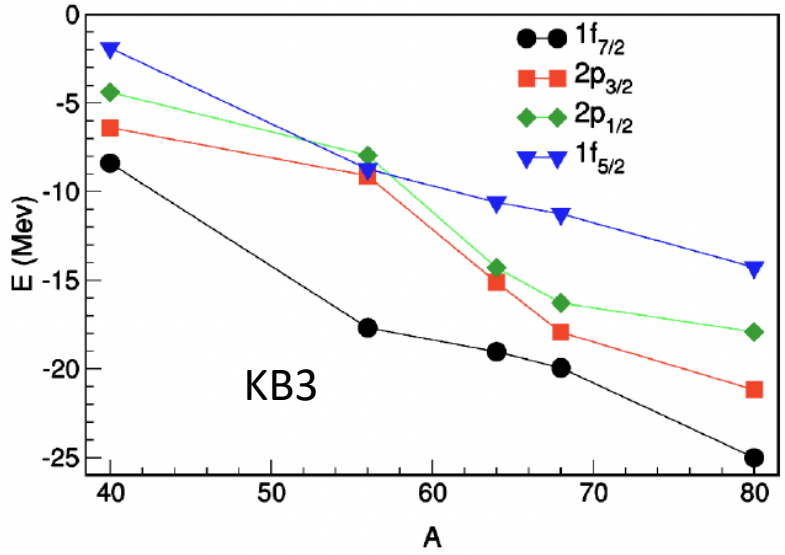
The case of ^{49}Ca

KB: Kuo-Brown interaction for the *fp* valence space
KB', KB3: monopole modified interactions from KB

^{49}Ca







E. Caurier et al. Rev. Mod. Phys. 77 (2005) 427

3N forces and many-body systems

The explicit inclusion of 3NFs has historically been neglected in SM treatment due to

little knowledge of a mechanism providing many-body forces consistently with the nature of the NN interaction

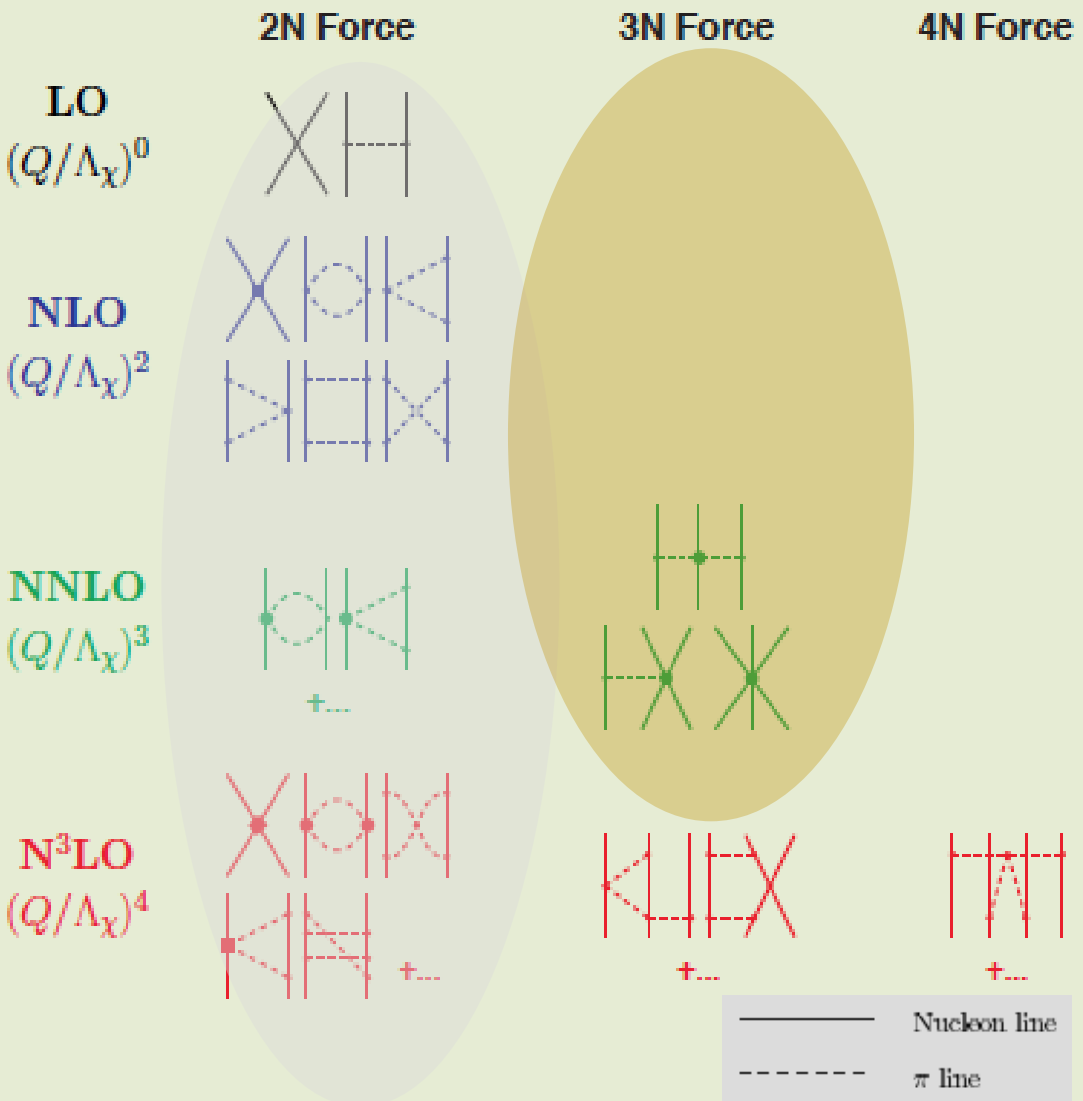
difficulty in handling a 3N term in many-body systems whose solution requires formalisms that are computationally extremely demanding

Chiral potentials derived in ChPT

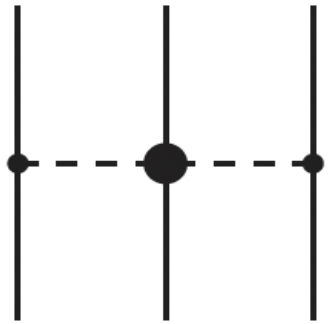
- Nucleons interact via pion exchanges and short-range contact interactions. The long-range forces are ruled by the symmetries of QCD, while short-range forces - which are not resolved - are absorbed into contacts terms proportional to low-energy constants (LEC)

- Chiral potentials are organized in a systematic low-momentum expansion, where two- and many-body forces are generated on an equal footing

- Most interaction vertices that appear in the 3N force also occur in the *NN* force → consistency requires that the parameters LECs carried by these vertices have the same values for *NN* and 3N terms



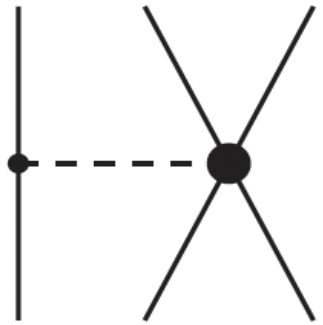
3N chiral potential at N^3LO



two-pion exchange

No new parameters with respect to those appearing

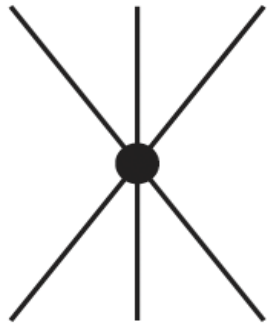
in the expression of the N^3LO 2NF



one-pion exchange +

NN-contact interaction

New LEC parameter: c_D



pure 3N-contact interaction

New LEC parameter: c_E

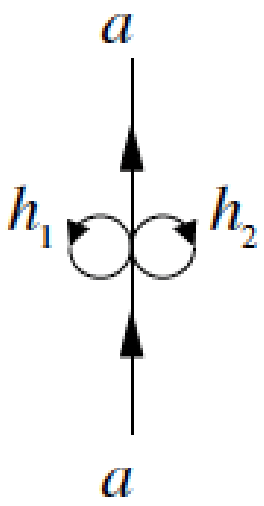
c_D and c_E may be fixed by an optimal over-all fit of the properties of light nuclei

[see for instance P. Navrátil et al, PRL 99 (2007) 042501]

Normal-ordered decomposition of the 3N component of H

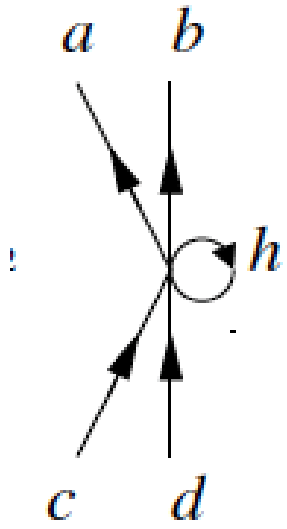
Starting from a reference state and using the Wick theorem, the three-body component of the nuclear Hamiltonian can be re-arranged into a sum of zero-, one-, two-, and three-body terms → only normal-ordered one- and two-body parts of 3N forces are included

normal-ordered one-body part of the 3N force



$$\epsilon_a^{(3\mathrm{NF})} = \sum_{\substack{h_1,h_2 \\ J_{12}J}} \frac{\hat{J}^2}{2\hat{j}_a{}^2} {}_A\langle (h_1,h_2),a;J_{12}J\,|V_{3N}|\,(h_1,h_2),a;J_{12}J\rangle_A$$

normal-ordered two-body part of the 3N force

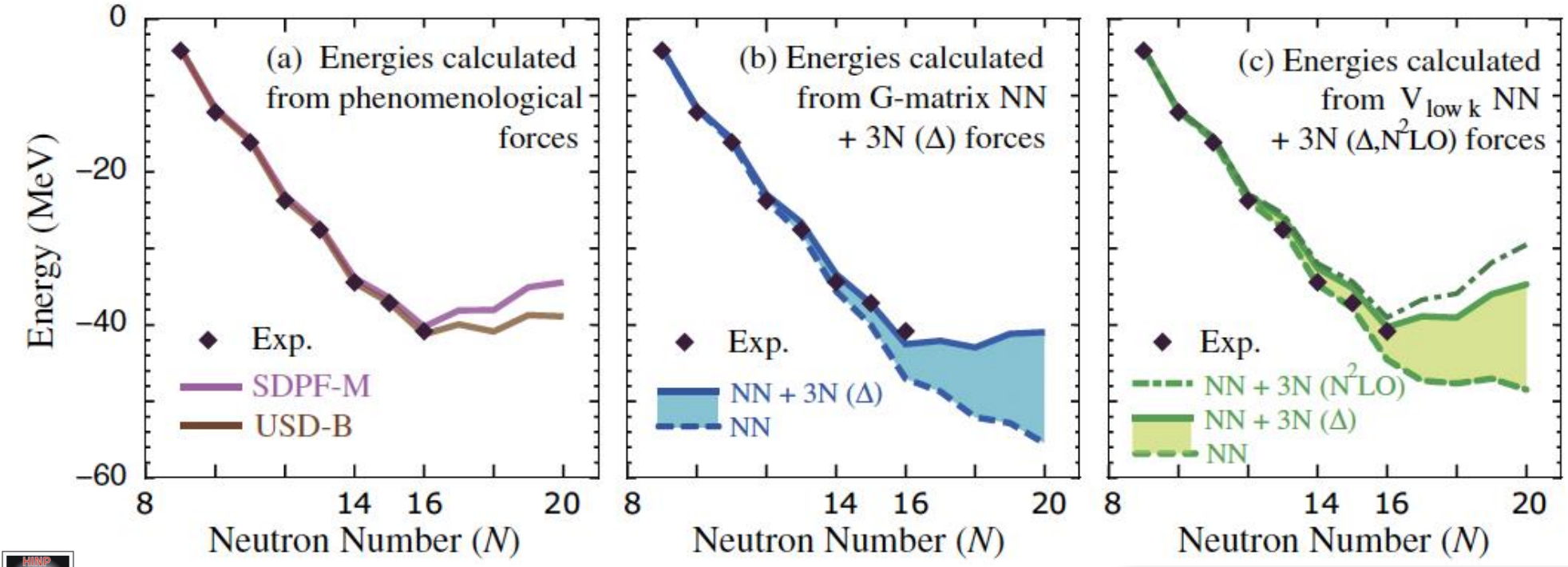


$$V_{ab,cd,J}^{(3\mathrm{NF})} = \sum_{h,J'} \frac{\hat{J'}^2}{\hat{J}^2} {}_A\langle (a,b),h;JJ'\,|V_{3N}|\,(c,d),h;JJ'\rangle_A$$

First study explicitly including the 3NF in the derivation of the effective SM Hamiltonian

With an *NN* force-only, g.s. energies do not stop to decrease putting the dripline at N = 20, while 3N contributions correct the g.s behavior bringing a significant raise from N = 16 to 18 **then provide the correct location of the drip line ↔ increase the $d_{5/2}$ - $d_{3/2}$ SO splitting**

Ground-state energies of oxygen isotopes measured from ^{16}O



T. Otsuka et al, PRL 105, 032501 (2010)

Outline of calculations

_NN_ and 3N forces within the ChPT framework at N^{3}LO and at NNLO, respectively

[D. R. Entem, R. Machleidt, PRC 66 (2002) 014002; and c_D c_E from P. Navrátil et al, PRL 99 (2007)042501]

H_{eff} within the framework of the MBPT ($\hat{Q}$-box folded diagram expansion)

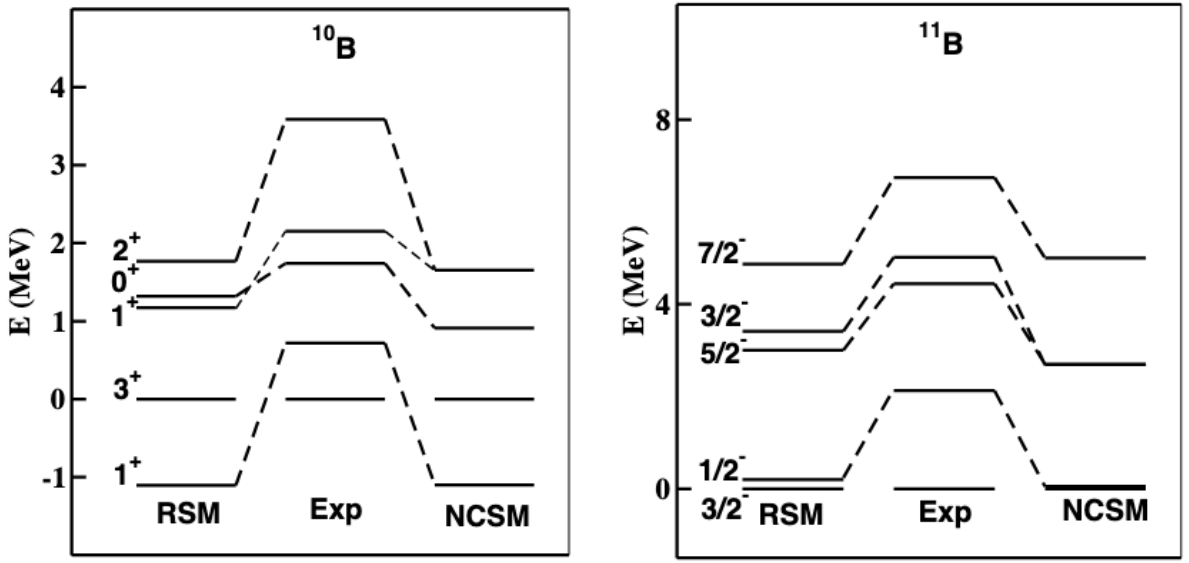
by including in the $\hat{Q}$-box one- and two-body Goldstone diagrams at third order in the _NN_ potential and at first order in the 3N one

+ density-dependent two-body contributions to account for induced 3N forces which arise in systems with more than 2-valence nucleons

[T. Fukui et al, PRC 98 (2018) 044305; Y. Z. Ma et al, PRC 100 (2019) 034324]

p-shell nuclei: B isotopes

Benchmark calculations: comparison of SM and ab initio NCSM results

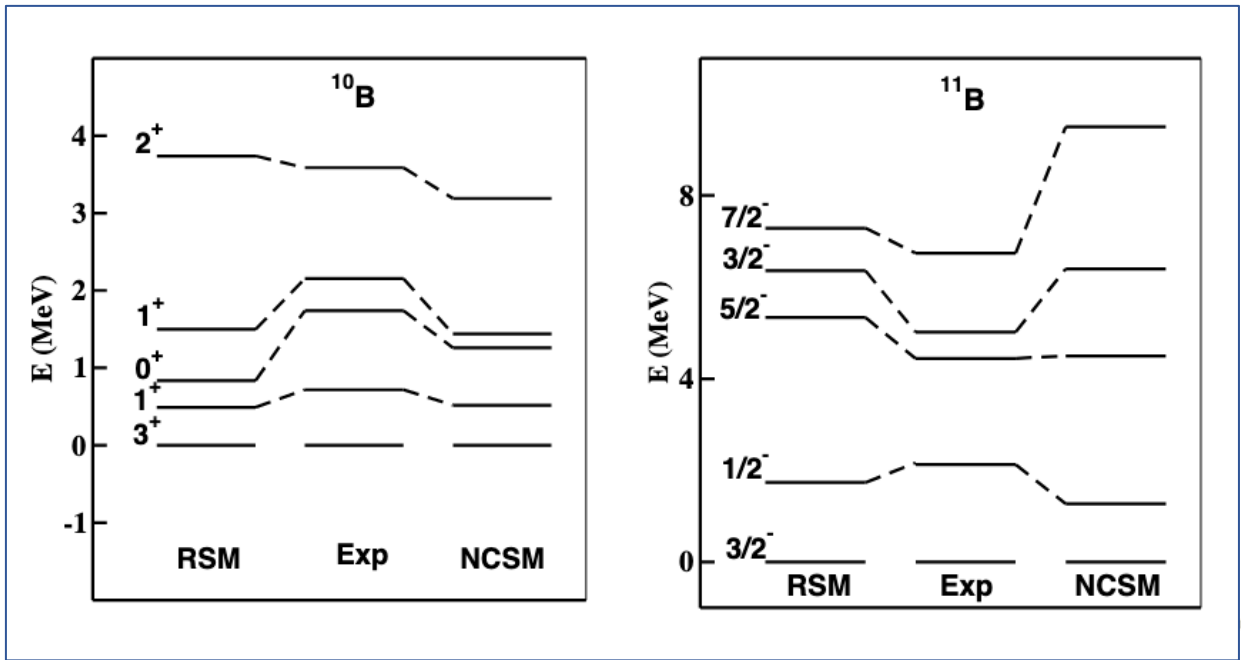


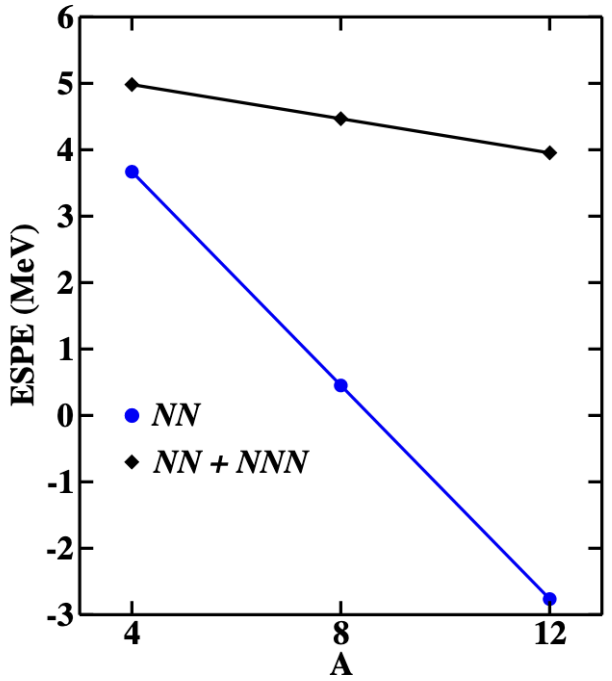
NN force

L. Coraggio, et al, Ann. Phys. (NY) 327 (2012) 2125

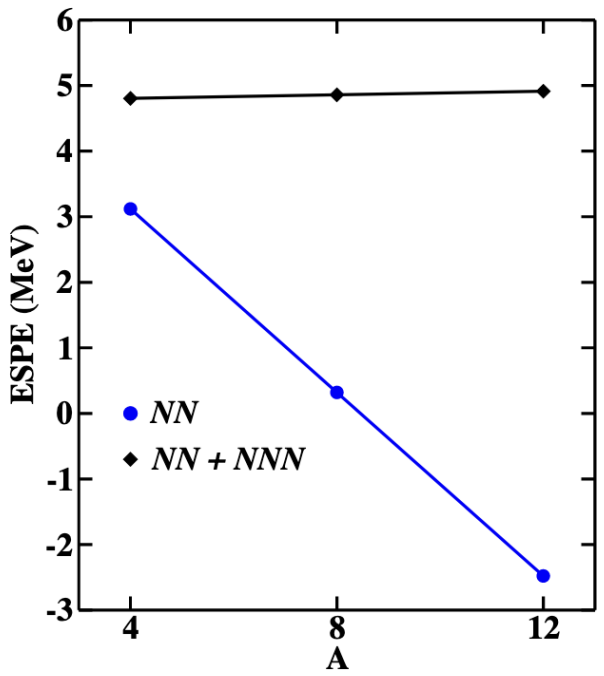
NN+3N forces

T. Fukui et al, PRC 98 (2018) 044305





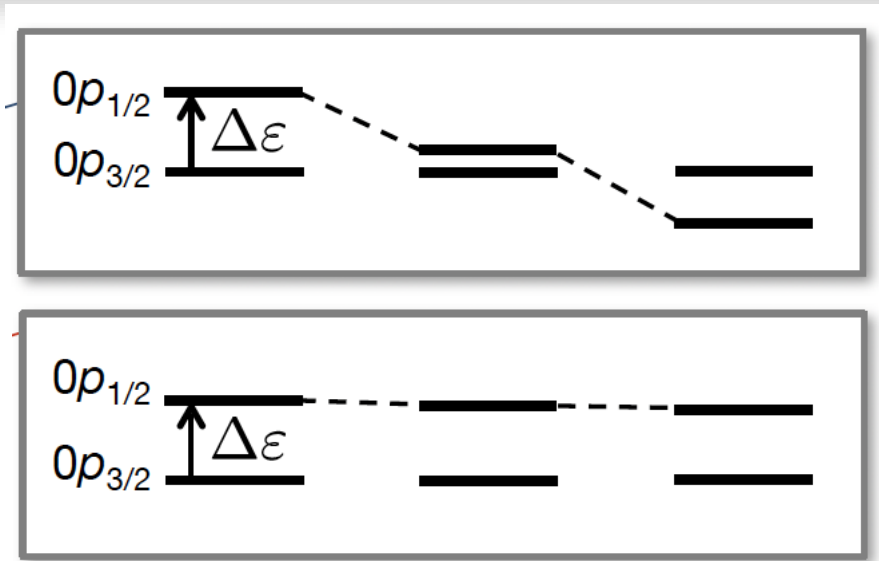
proton $0p_{1/2}$ ESPE relative to $0p_{3/2}$



neutron $0p_{1/2}$ ESPE relative to $0p_{3/2}$

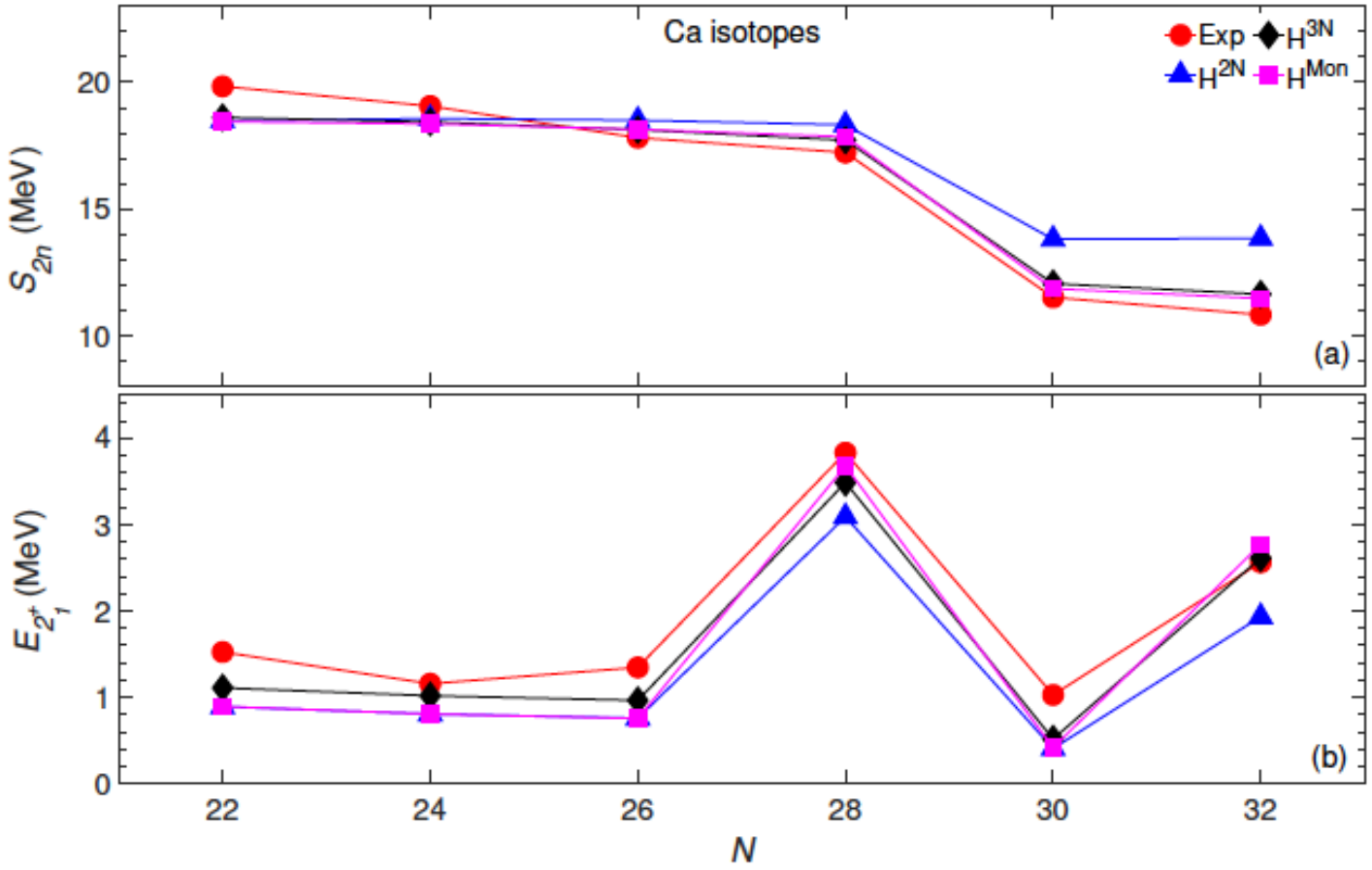
Relative ESPE rapidly drops down when considering only the N^{3}LO *NN* potential, even becoming negative around A=8, while relative ESPE is almost constant (4 ~ 5 MeV) when the 3N potential is taken into account → which reflects on the spectra of ^{10}B and ^{11}B

3NF is essential to make explicit the spin-orbit splitting of the 0p orbitals and ensure better closure properties



Two-neutron separation energies and 2_1^+ excitation energies for calcium isotopes from N = 20 to 32

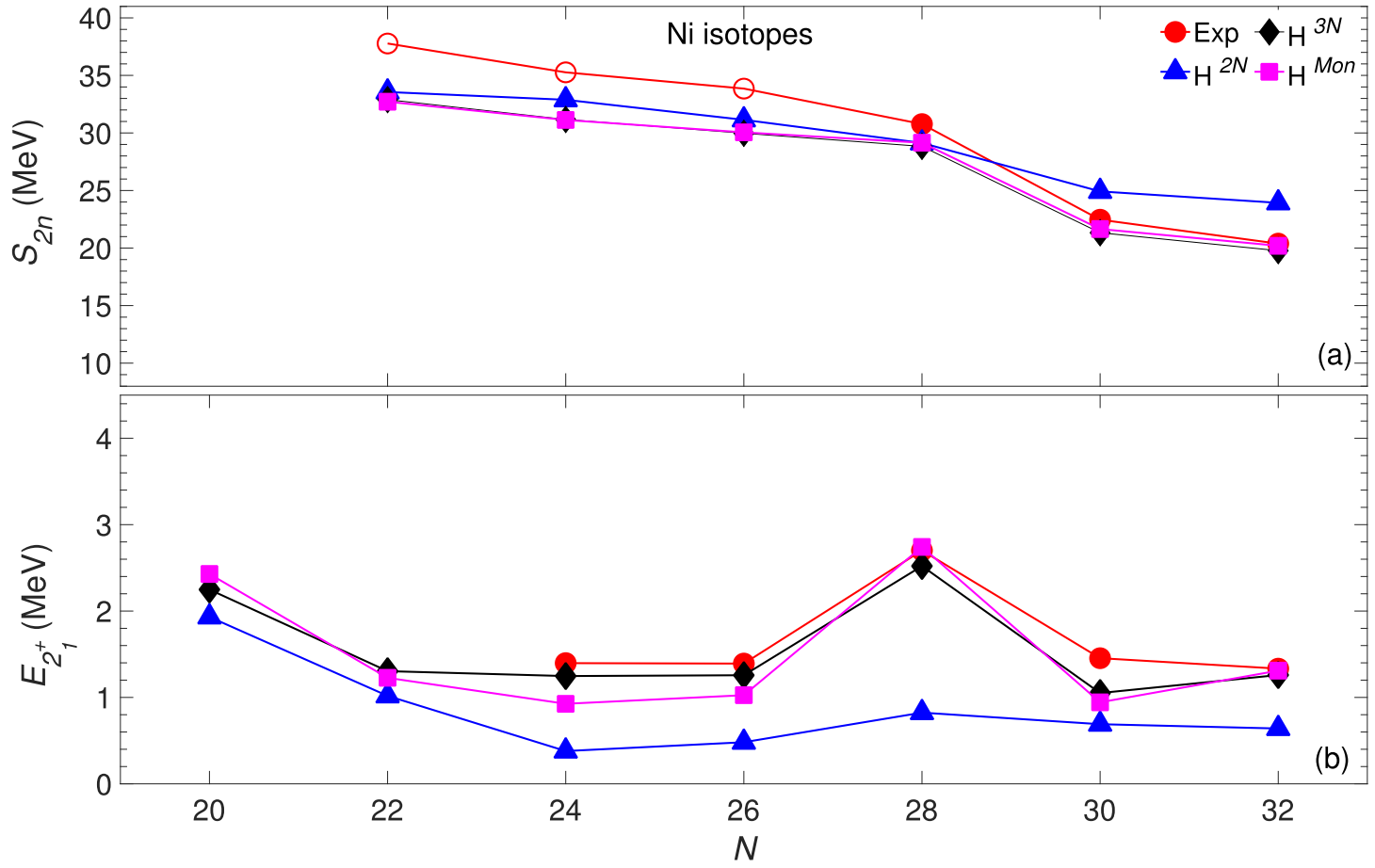
fp valence space



H^{2N}	only induced 3N forces
H^{3N}	genuine + induced 3N forces
H^{Mon}	monopole component of H^{3N} + multipole component of H^{2N}

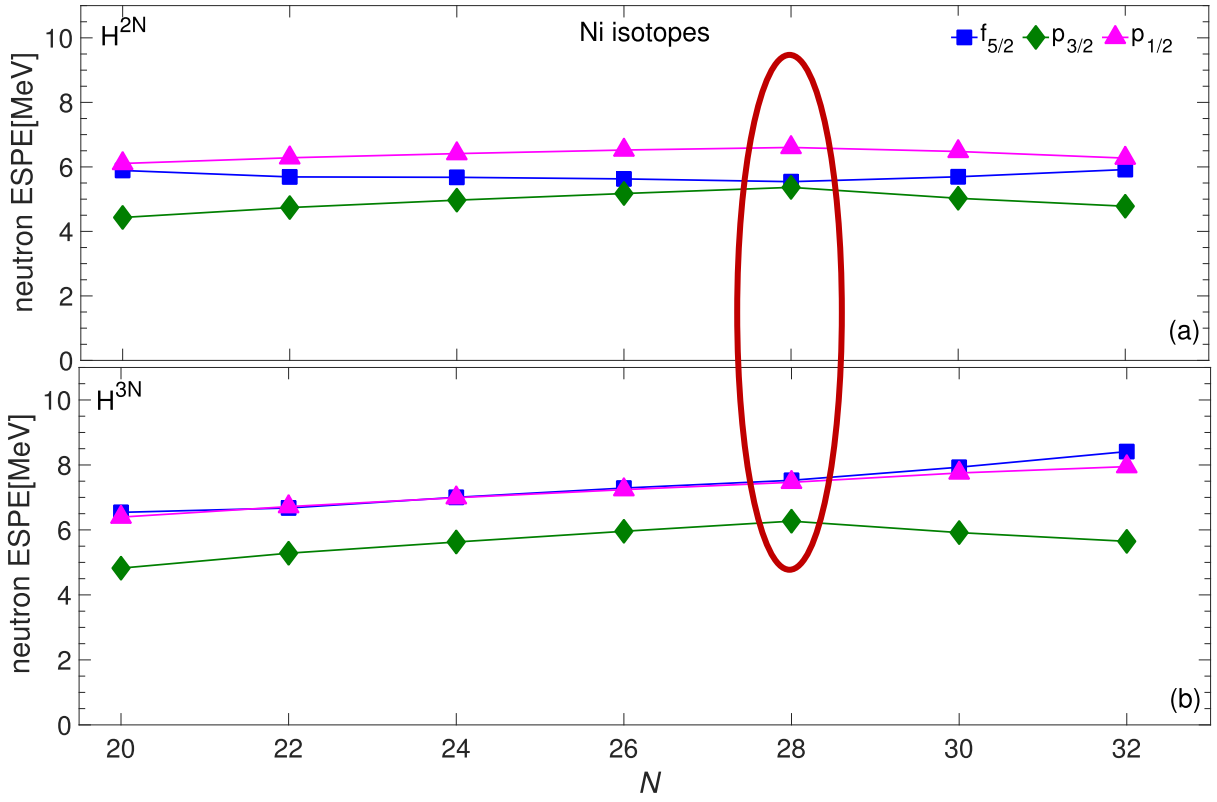
Two-neutron separation energies and 2_1^+ excitation energies for nickel isotopes from N = 20 to 32

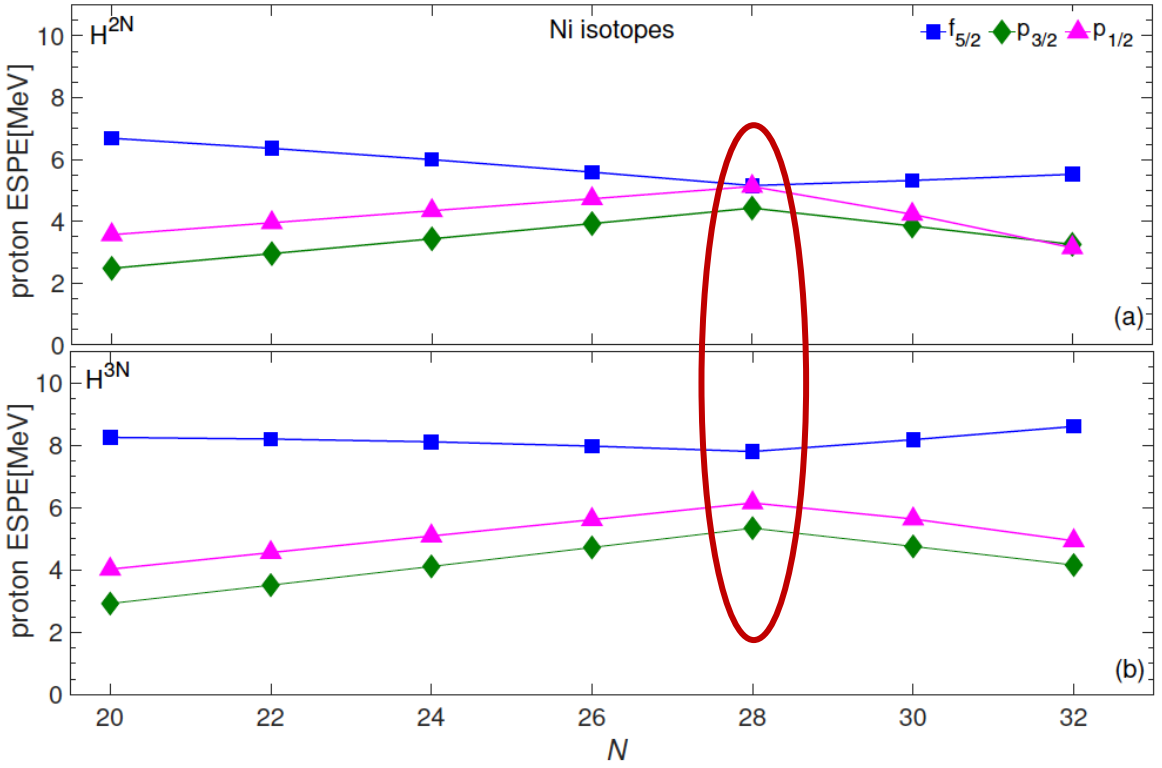
fp valence space



H^{2N}	only induced 3N forces
H^{3N}	genuine + induced 3N forces
H^{Mon}	monopole component of H^{3N} + multipole component of H^{2N}

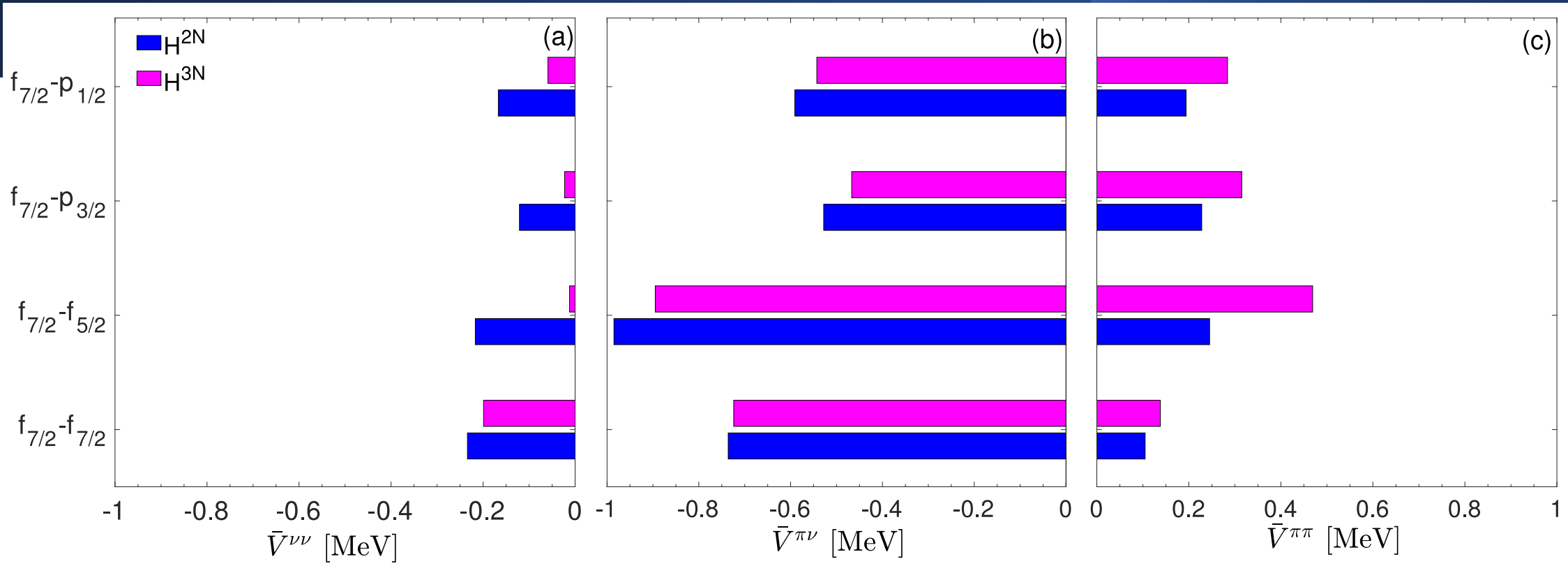
Neutron and proton ESPE for nickel isotopes with and without 3N force





H^{2N}	only induced 3N forces
H^{3N}	genuine + induced 3N forces

Monopole matrix elements $\bar{V}^{\tau\tau'}_{f_{7/2b}}$ of the effective interactions with and without 3N force



- 3NF provides a repulsive contribution to all matrix elements, which makes the nn and pn matrix elements less attractive and pp ones more repulsive.
- The size of the contributions depends on the involved orbitals, ranging from few 10 to ~ 200 keV → substantial changes in the spacings between the ESPE's in correspondence of a sizable occupation of a specific orbital
- Changes produced by the 3N force are larger for non diagonal with respect to diagonal matrix elements, which leads to a larger gap between the $f_{7/2}$ and the remaining orbitals

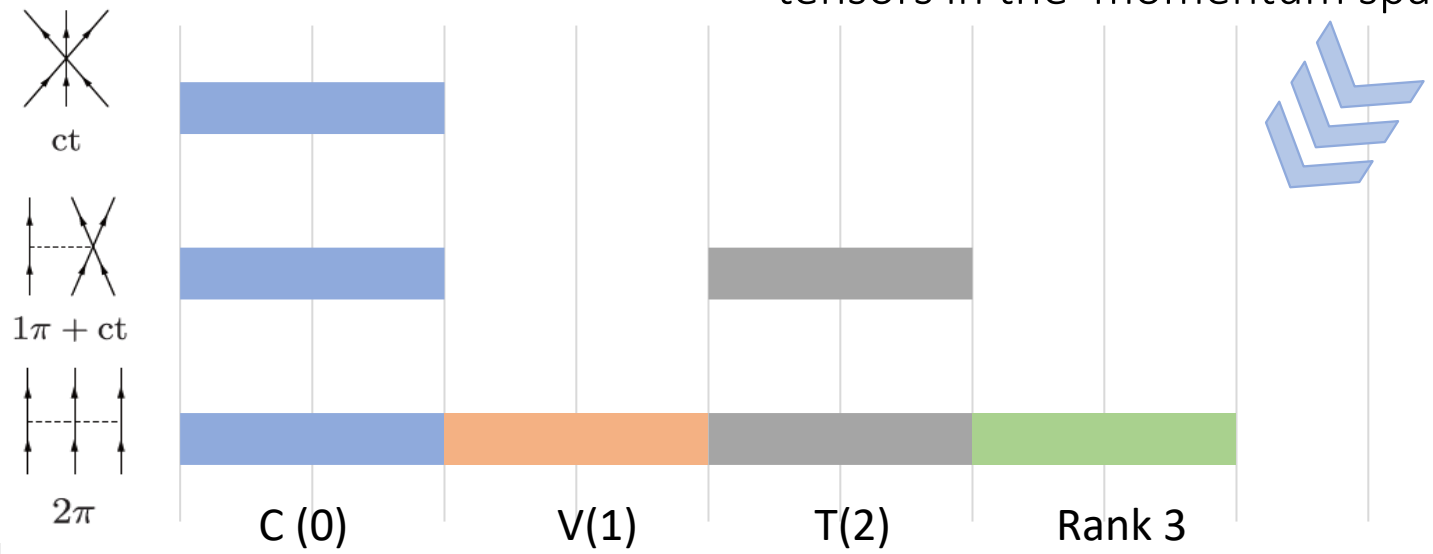
What is the specific mechanism behind the increase in the SO splitting produced by the chiral 3NF? Are there specific components of the 3NF leading to such an increasing?

Tensor decomposition of the chiral 3NF at N^{2}LO

3NF can be schematically written as

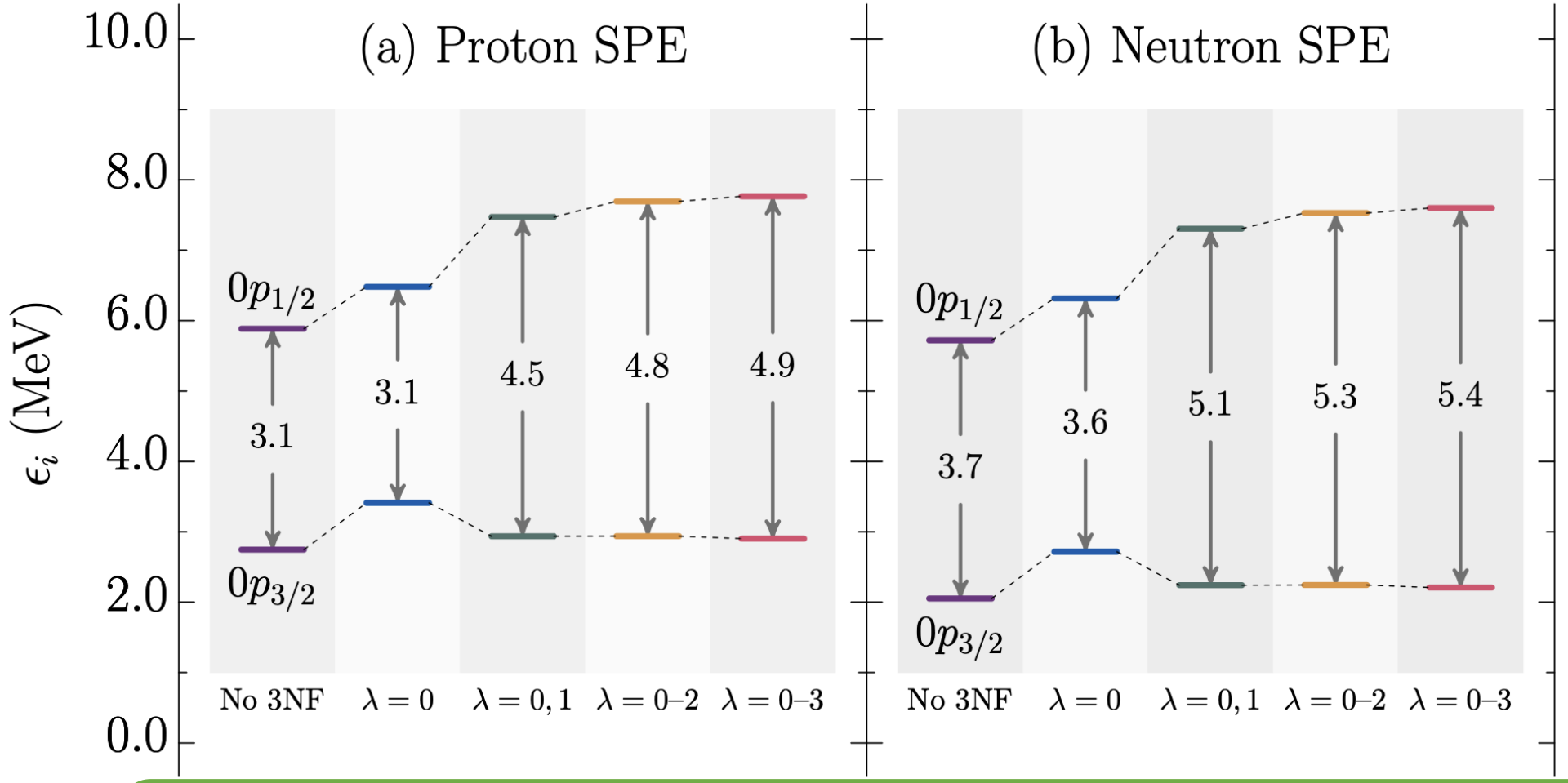
$$v_{3N}^{\alpha} \Longrightarrow T^{\alpha} \sum_{\lambda} S_{\lambda}^{\alpha} \cdot Q_{\lambda}^{\alpha} \qquad \alpha \in \{ct, 1\pi + ct, 2\pi\}$$

by coupling the spin tensor operators S_{λ}^{α} with the corresponding rank tensors in the momentum space Q_{λ}^{α}. T^{α} is the isospin part.



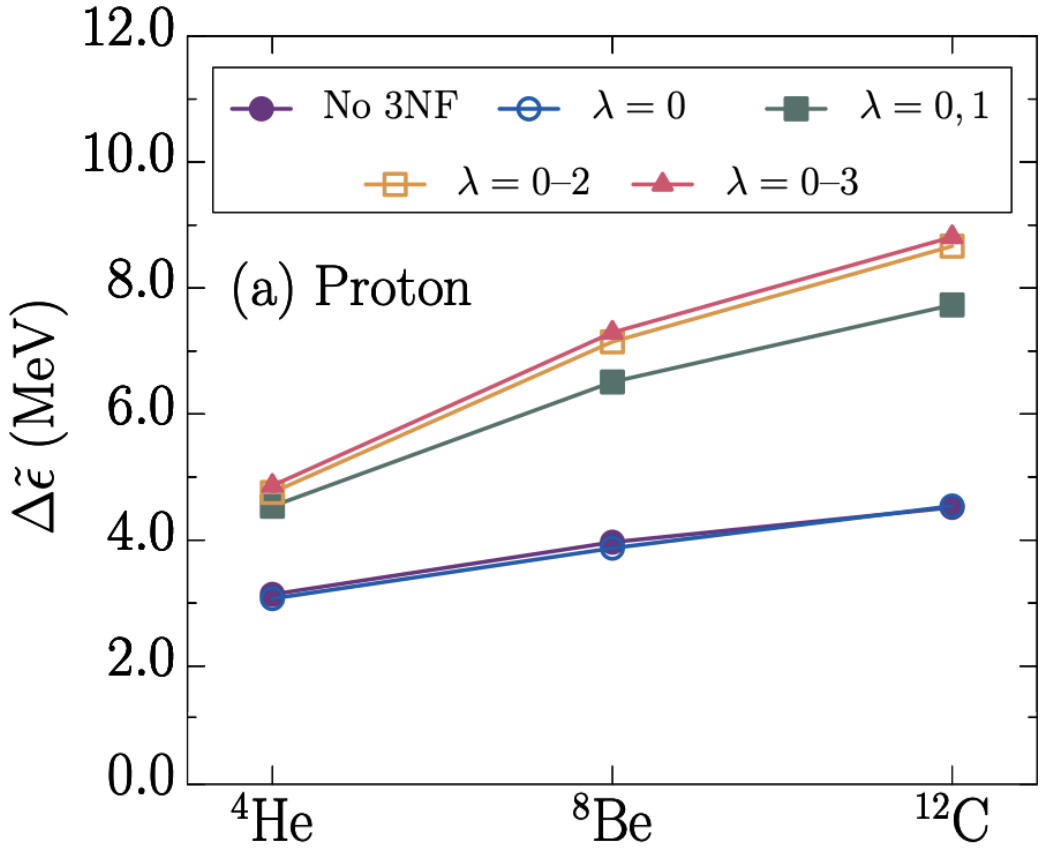
Formalism developed by Tokuro Fukui

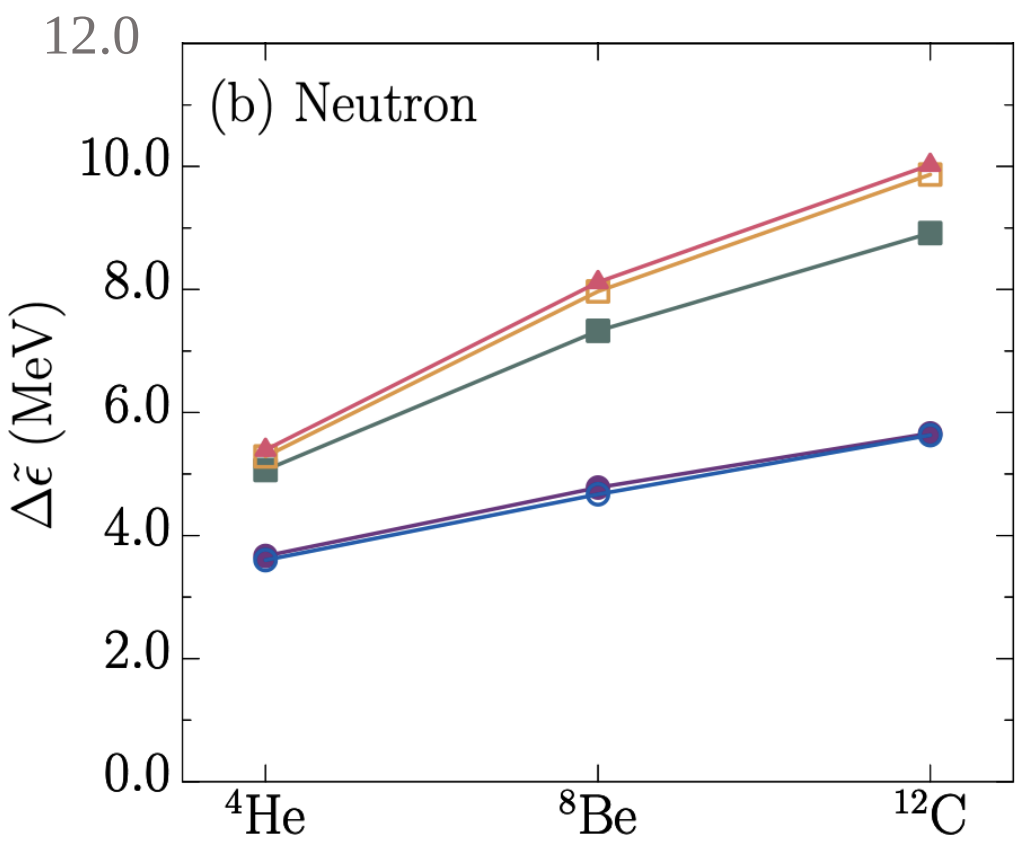
SPEs of the 0p3/2 and 0p1/2 orbitals



The SO splitting induced by the chiral 3NF is primarily governed by its rank-1 component arising from the 2 pion-exchange term

Gap of the ESPEs between the $0p_{3/2}$ and $0p_{1/2}$ orbitals





Similarly to the SPEs, the primal impact comes from the rank-1 3NF of the 2-exchange process, accounting for almost enhancements by the whole 3NF. The effect of the rank-2 3NF (about 20%) is smaller yet appreciable.

Summary

- Genuine 3NFs provide a repulsive contribution, which, although partially counterbalanced by induced 3N forces is essential in determining the location of the neutron dripline and the evolution of the shell structure.

- 3NFs affect essentially the monopole component, then confirming the need to introduce monopole adjustments when using effective SM Hamiltonians derived from NN potentials.

- The size of the contributions depends on the involved orbitals, ranging from few 10 to ~ 200 keV → substantial changes in the spacings between the ESPE's in correspondence of a sizable occupation of a specific orbital

- A crucial role in the SO splitting is played by the vector (rank-1) component arising from the the 1π+ct exchange term of the chiral 3NF

Collaborators

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Thanks for your attention