

Cyclotron Institute Texas A&M University
Nuclear Data Evaluation Center

Data-Based Research Project:
Could revisiting the Principles of
a Level Scheme bring new Insight
into High-Spin Physics?

N. Nica

Content

I. Experimental Evidence

II. Level Scheme Re-Concept

III. Insight into High-Spin Physics

I. Experimental Evidence

Case study: ^{171}Yb nucleus high spin rotational bands

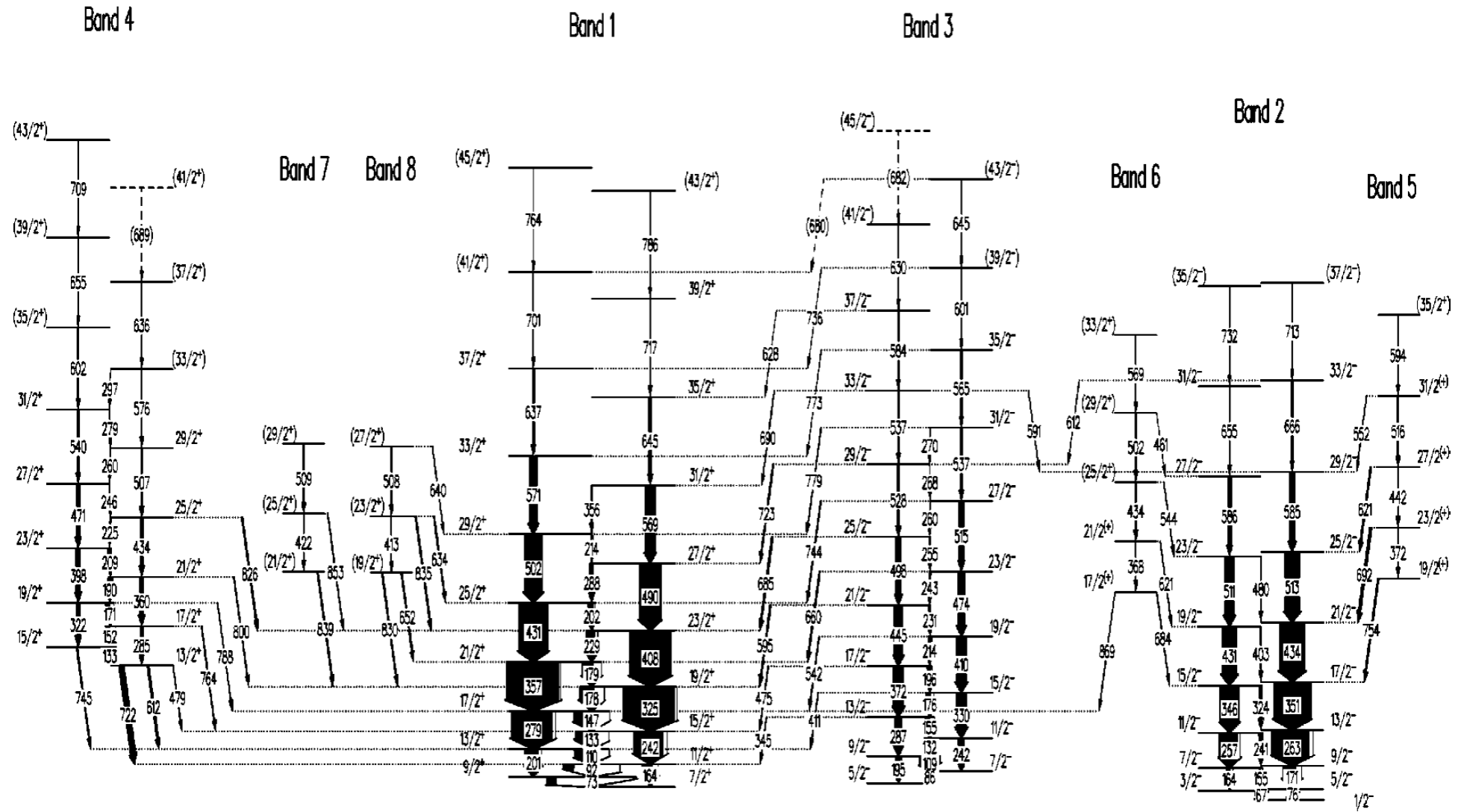


FIG. 5. Level scheme for ^{171}Yb .

D.E.Archer *et al*, Phys.Rev. C57, 2924 (1998)

How the bands can be described? $(\Delta E_\gamma^x, \Delta E_\gamma^y)$ Differential Coincidence Matrix

Bohr-Mottelson Collective Rotor

$$E(I) = \frac{\hbar^2}{2\mathfrak{I}} I(I+1), \quad c = \frac{\hbar^2}{2\mathfrak{I}}$$

$$E_\gamma = E(I) - E(I-2) = \frac{\hbar^2}{2\mathfrak{I}} (4I-2) = 2c(2I-1)$$

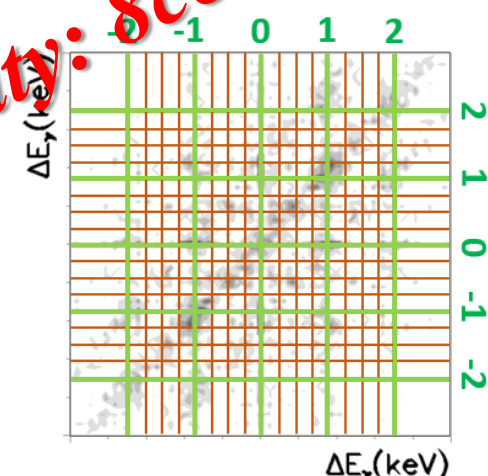
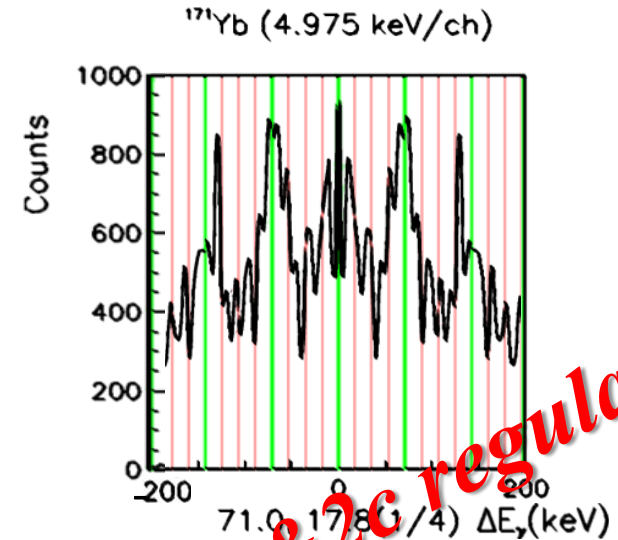
$$\Delta E_\gamma = E_\gamma(I) - E_\gamma(I-2) = 8 \frac{\hbar^2}{2\mathfrak{I}} = 8c$$

New Parametrization (average behavior)

$$E_\gamma = 2c(2I + k - 1), \quad k \text{ integer}$$

- $2c$ Moment of Inertia, Real
- $(2I+k-1)$ Angular Momentum, Integer

Bitmap Distribution



Repeatability: $8c \& 2c$ regular grid!

How the bands can be described?

Bohr-Mottelson Collective Rotor

$$E(I) = \frac{\hbar^2}{2\mathfrak{I}} I(I+1), \quad c = \frac{\hbar^2}{2\mathfrak{I}}$$

$$E_\gamma = E(I) - E(I-2) = \frac{\hbar^2}{2\mathfrak{I}} (4I-2) = 2c(2I-1)$$

$$\Delta E_\gamma = E_\gamma(I) - E_\gamma(I-2) = 8 \frac{\hbar^2}{2\mathfrak{I}} = 8c$$

New Parametrization (average behavior)

$$E_\gamma = 2c(2I + k - 1), \quad k \text{ integer}$$

- $2c$ Moment of Inertia, Real
- $(2I+k-1)$ Angular Momentum, Integer

¹⁷¹Yb Rotational Bands

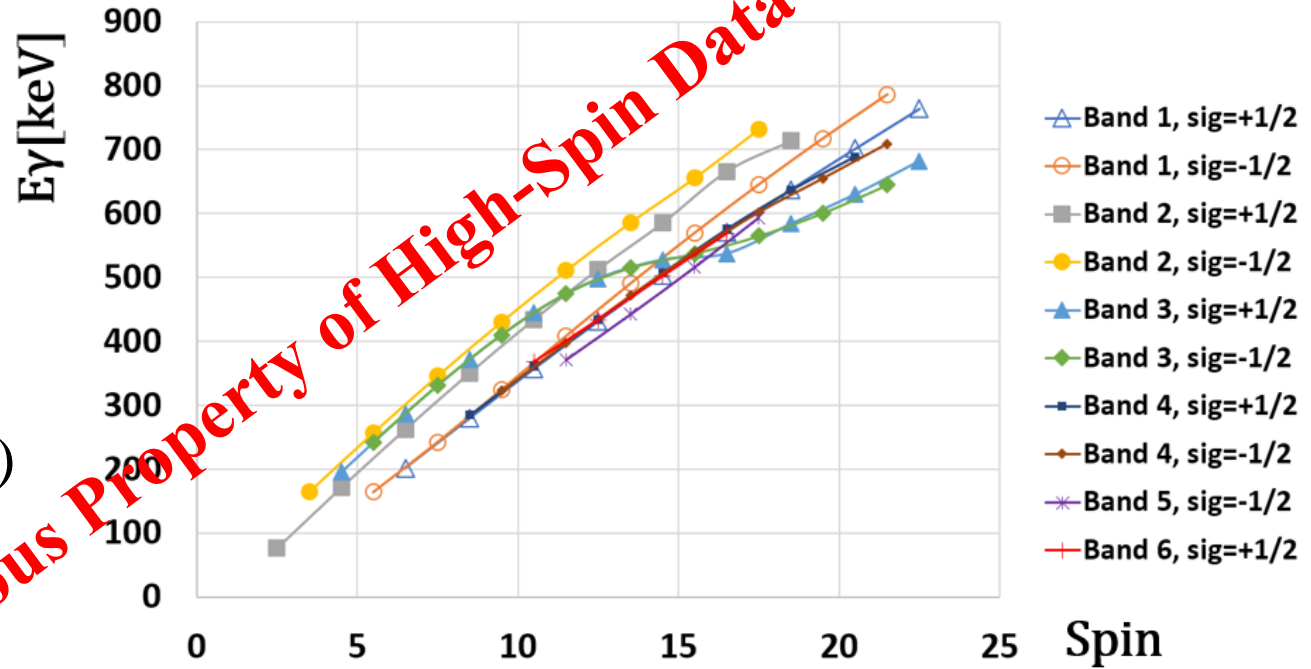


Fig. (E_γ 's versus spins)

-Quasi-linear beam almost parallel and equidistant

-Average behavior: $2c(2I+k-1)$, k integer

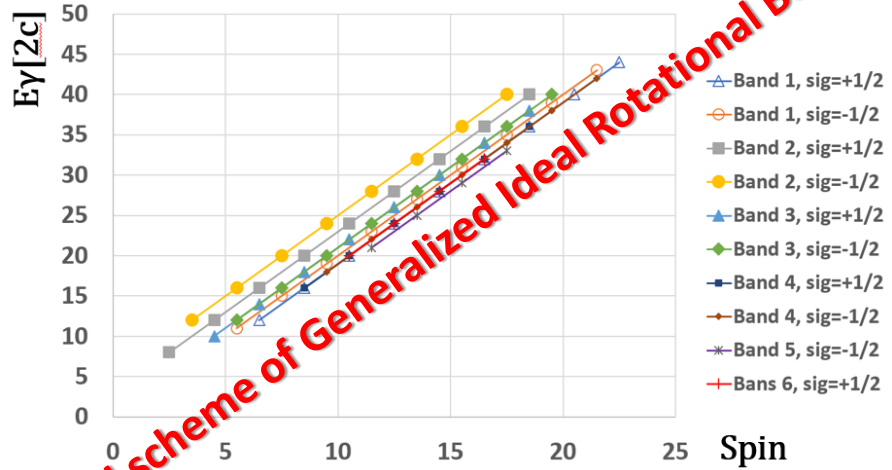
-Determine from fit over all bands' γ rays:

$$2c, k\text{'s, from } \sum (E_\gamma(I)/2c - (2I+k-1))^2 = \min$$

- Same slope ($2c$) for all k -bands \Rightarrow same $\mathcal{J}_{\text{eff}}^{(2)} = \frac{\hbar^2}{2c}$

Case study: ^{171}Yb nucleus high spin rotational bands

^{171}Yb Generalized Ideal Rotational Bands



^{171}Yb band fits using $E_\gamma=2c(2I+k-1)$ parametrization, $\Sigma(E_\gamma(I)/2c-(2I+k-1))^2 = \min$

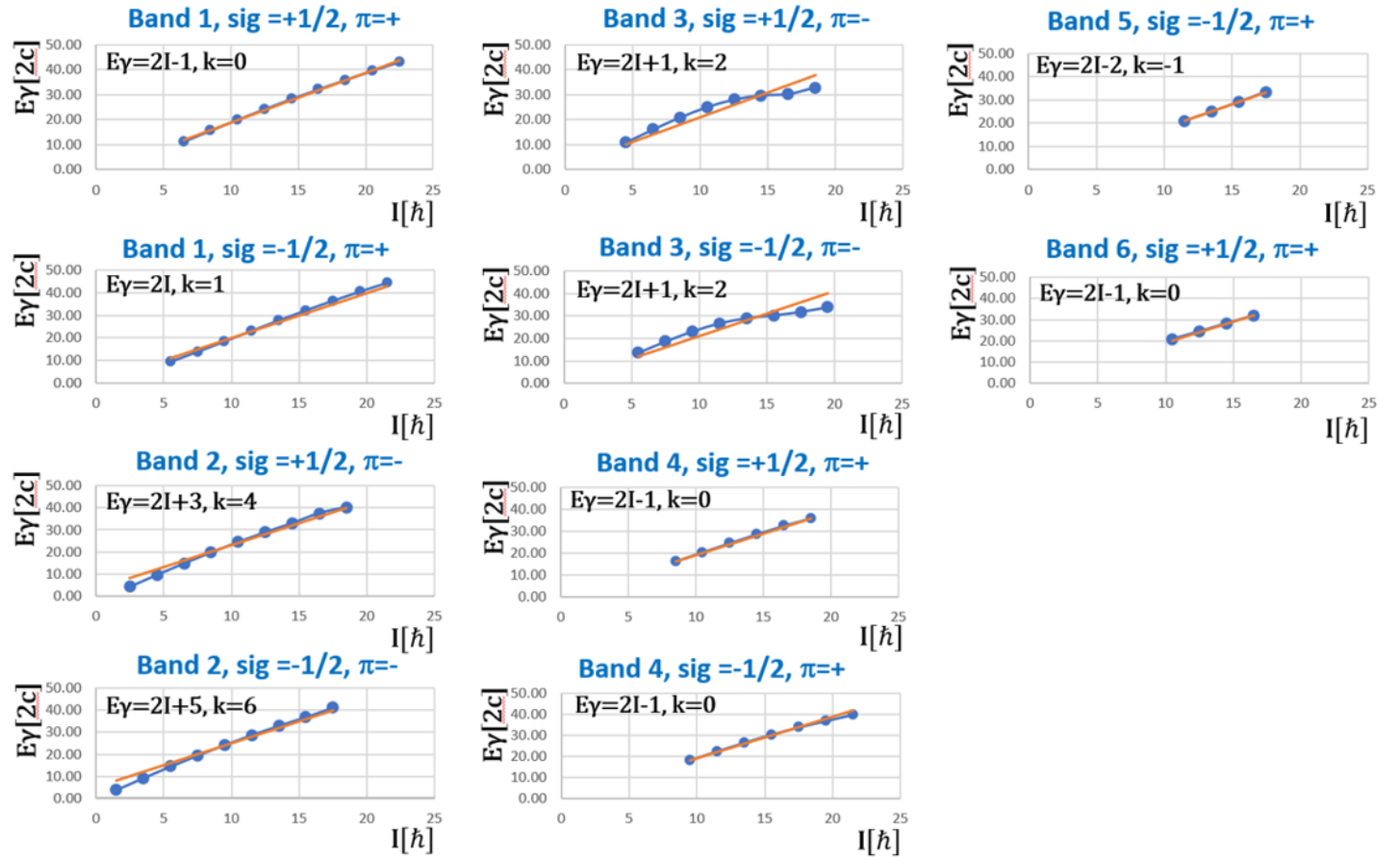


Fig. (^{171}Yb bands fit)

$$2c = 17.75 \text{ keV}$$

$$J_{eff}^{(2)} = 56.34 \text{ h}^2/\text{MeV}$$

Fig. (Generalized Ideal Rotation Bands)

^{171}Yb Rotational Bands

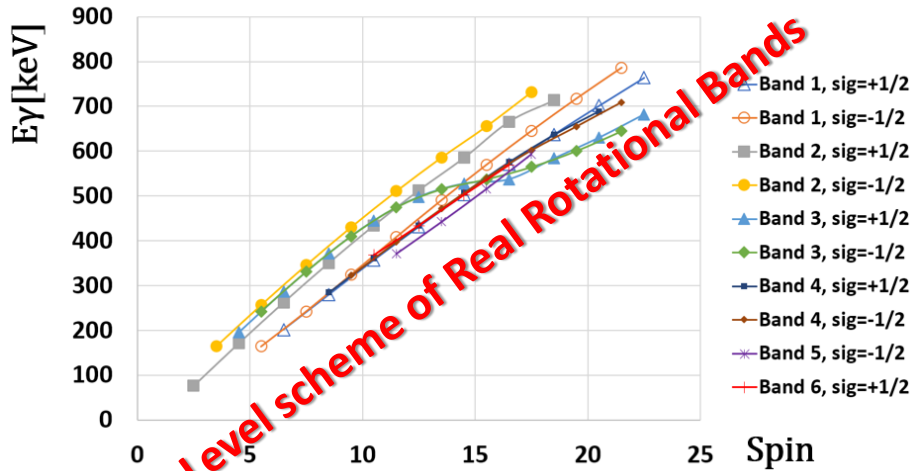


Fig. (E_γ 's versus spins)

II. Level Scheme Re-Concept

What we got for the average description of ^{171}Yb bands?

k -Generalized Ideal Rotor bands:

For $k=0$, Bohr-Mottelson Ideal Rotor bands: described by the $2cI(I+1)$ rule for even and odd spins

For $k \neq 0$, k -Generalized Ideal Rotor bands: have the same $J_{eff}^{(2)}$ (same $2c$) but are no longer described by the $2cI(I+1)$ rule.

Q: How to place the k -generalized ideal rotor bands in the level scheme?

A: By adding $2I+1$ "stairs" of $2c$ levels to the $k=0$ band!

One gets a "parabolic 2D building" on which:

- $k=0$ bands are vertical paths
- $k \neq 0$ bands are tilted paths
- In general, the energy levels can be indexed by three integer numbers, (I, m, n) , where:
 - I is the nuclear spin,
 - m is the position of the "stair" level relative to the spin "floor",
 - n the energy of the level, which is a natural number in units of $2c$.
- Triple coordinates suggest 3D level scheme!

^{171}Yb Generalized Ideal Rotational Bands

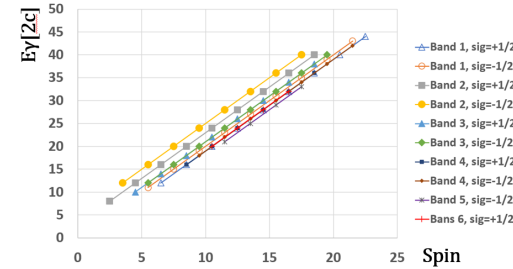


Fig. (Generalized Ideal Rotation Bands)

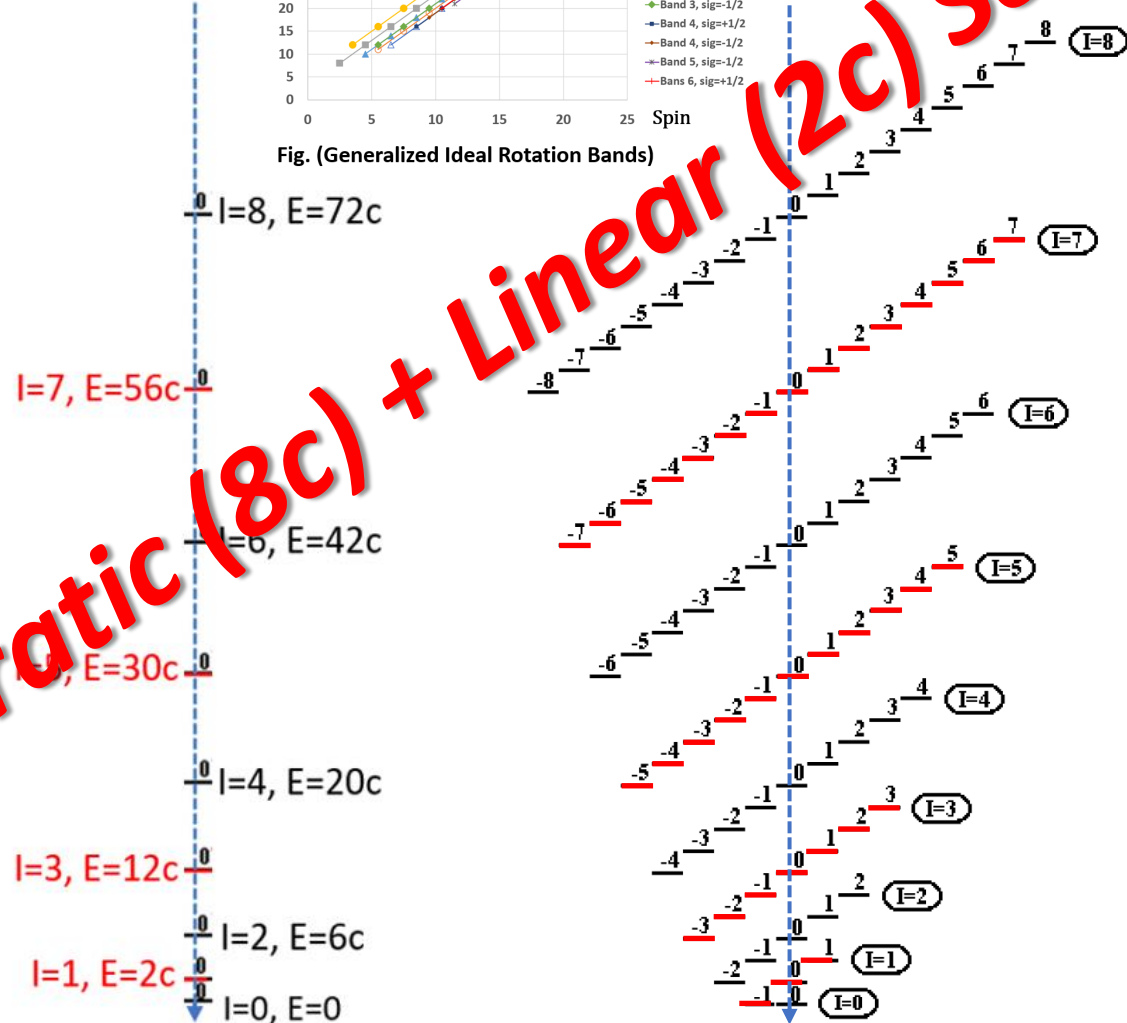


Fig. (Ideal rotor)

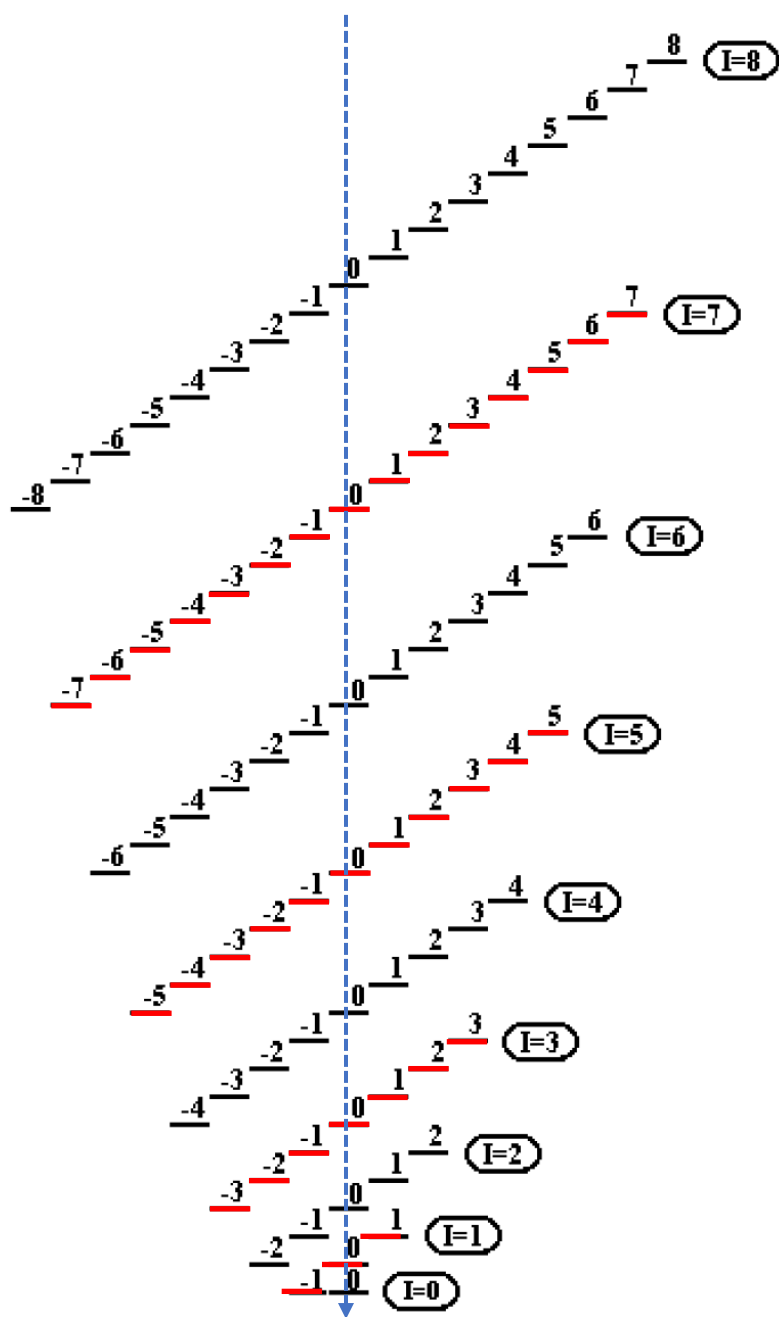
Fig. (Opened generalized ideal rotor)

Quadratic (8c) + Linear (2c) Scales

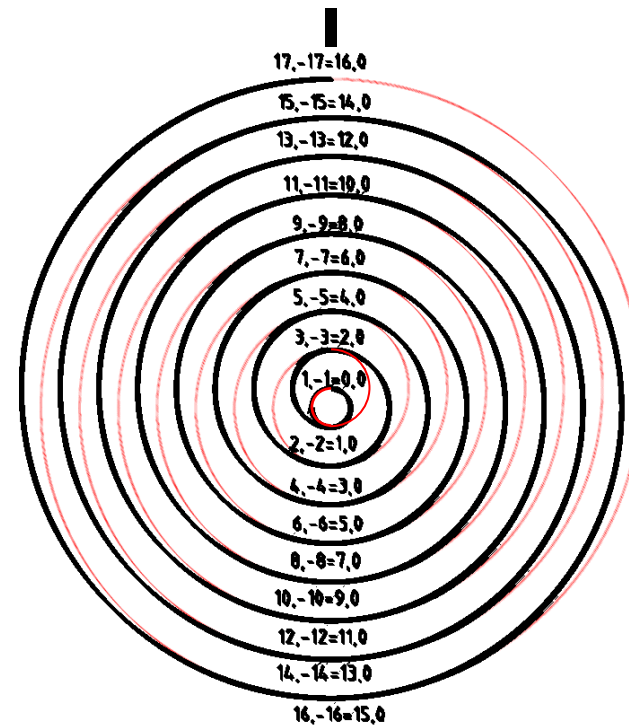
Parabolic Level Scheme for k bands

$$E_{\gamma}^k(I) = 2c (2I + k - 1), \quad k = \pm 1, \pm 2, \dots, I$$

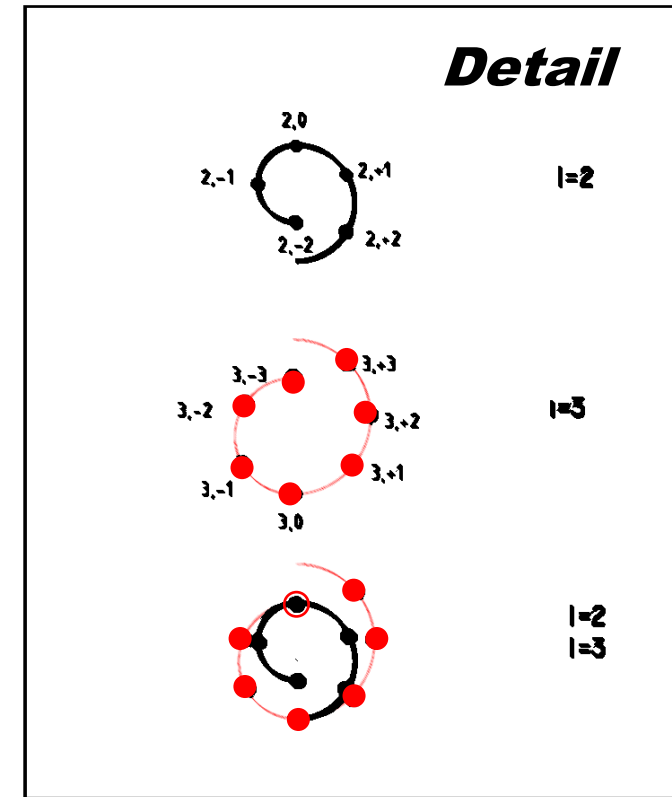
DOUBLE HELIX



2D view



3D view (from above)



Double-Helix for even-A and odd-A Nuclei

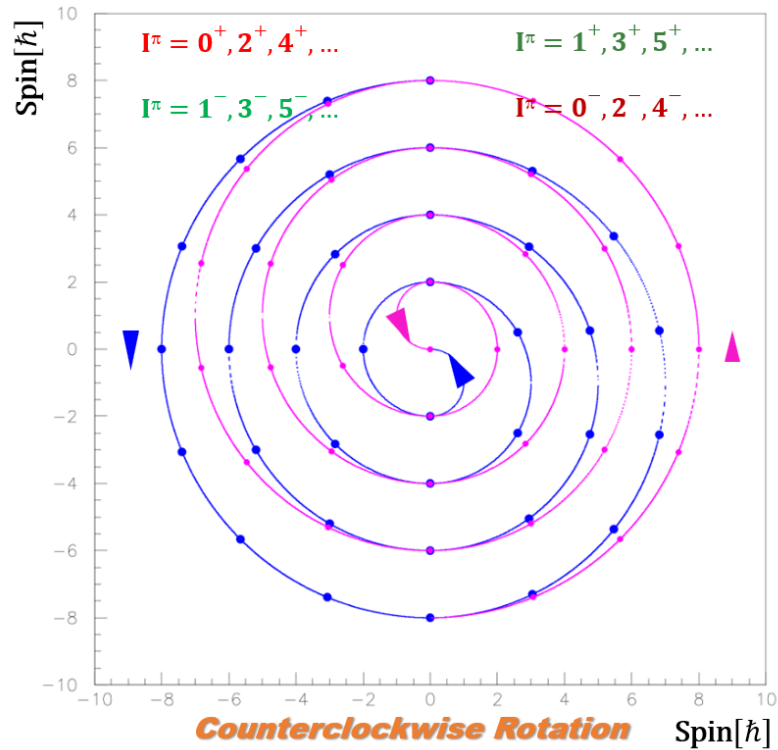


Fig. (Set of 2 helixes for integer spins)

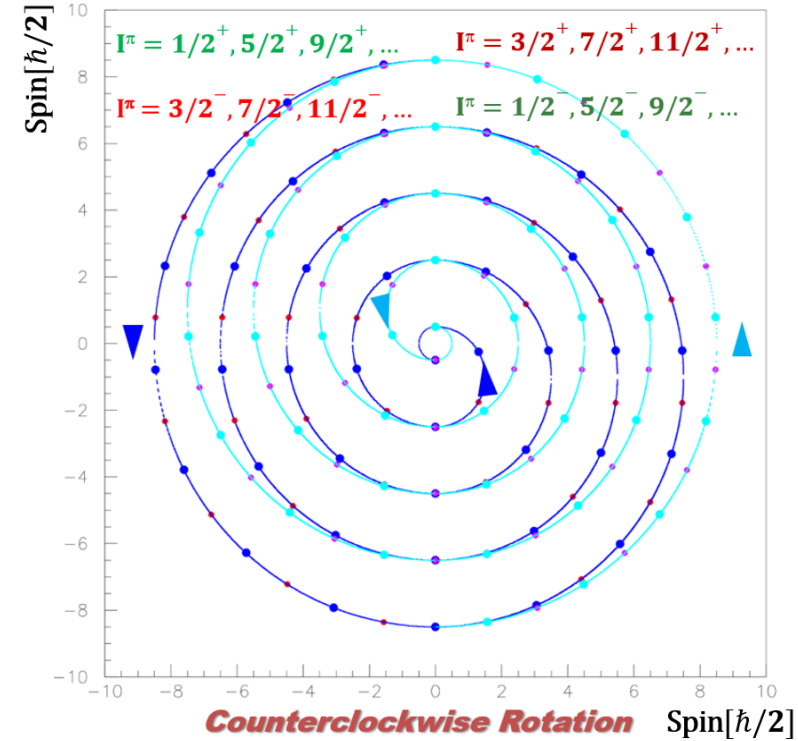


Fig. (Set of 2 helixes for integer half-spins)

Double-Helix Level Scheme of ^{171}Yb nucleus

Decomposition of experimental E_γ 's of all bands

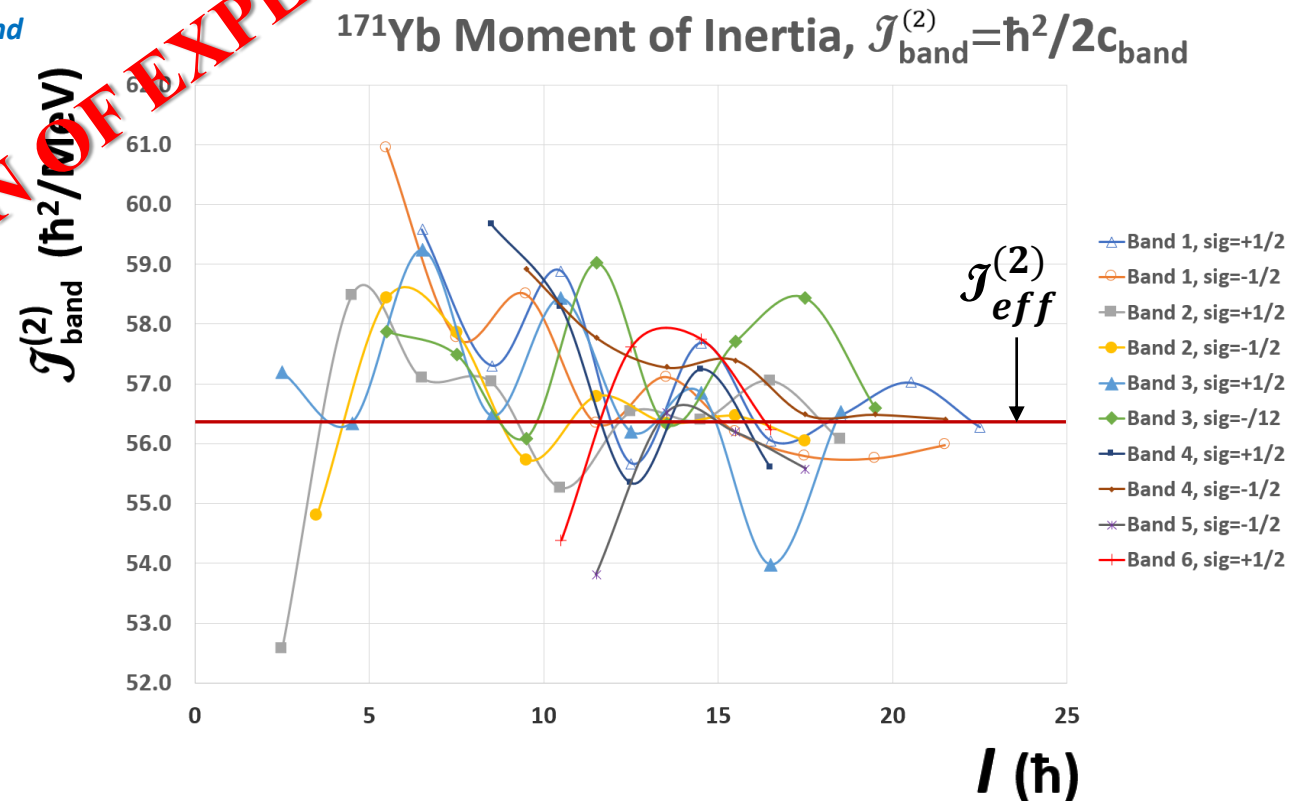
$$E_\gamma = 2c(2I+k-1 + k'+fn)$$

$$E_\gamma = 2c_{\text{band}} \times (2I+k+k'-1), \text{ with } 2c_{\text{band}} = 2c[1+fn/(2I+k+k'-1)],$$

with $2c_{\text{band}}$ real and $(2I+k+k'-1)$ integer

- $(2I+k+k'-1)$ generalized angular momentum
- One gets *Bands Moment of Inertia*, $\mathcal{J}_{\text{band}}^{(2)} = \hbar^2/2c_{\text{band}}$

COMPLETE DESCRIPTION OF EXPERIMENTAL BANDS



171Yb nucleus Double Helix Level Scheme

Part I: Level scheme of Generalized Ideal Rotational Bands

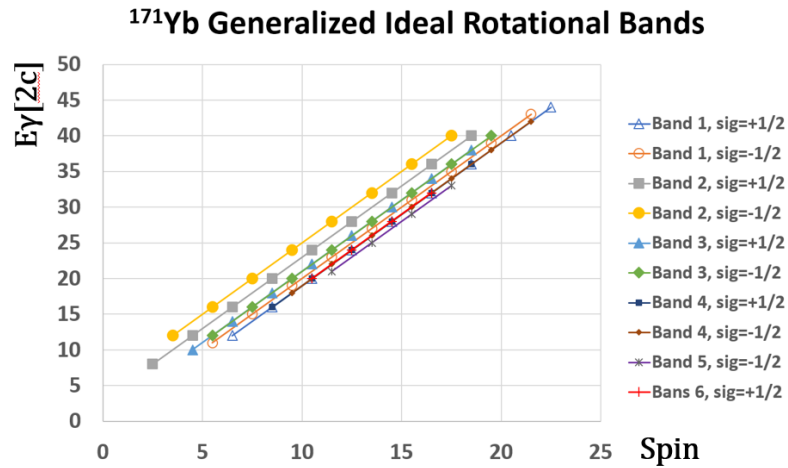


Fig. (Generalized Ideal Rotation Bands)

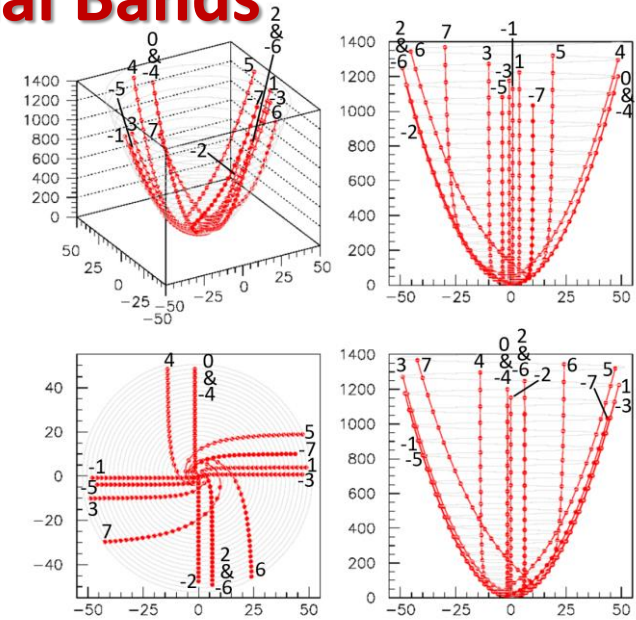
Average Ideal Rotational Description

Parametrization:

$$E_{\gamma} = 2c(2I + k - 1)$$

$2c$ *Real*, $(2I + k - 1)$ *Integer*

$2c, k$ from *least-squares fit*



Part II: Level scheme of Real Rotational Bands

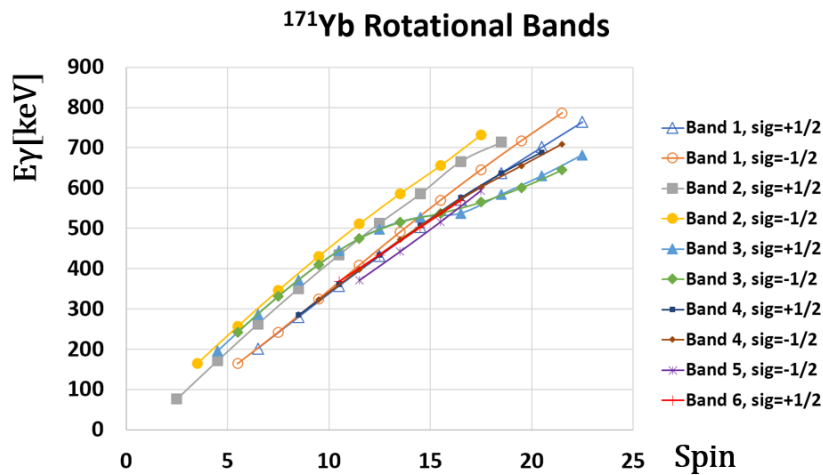


Fig. (Eγ's versus spins)

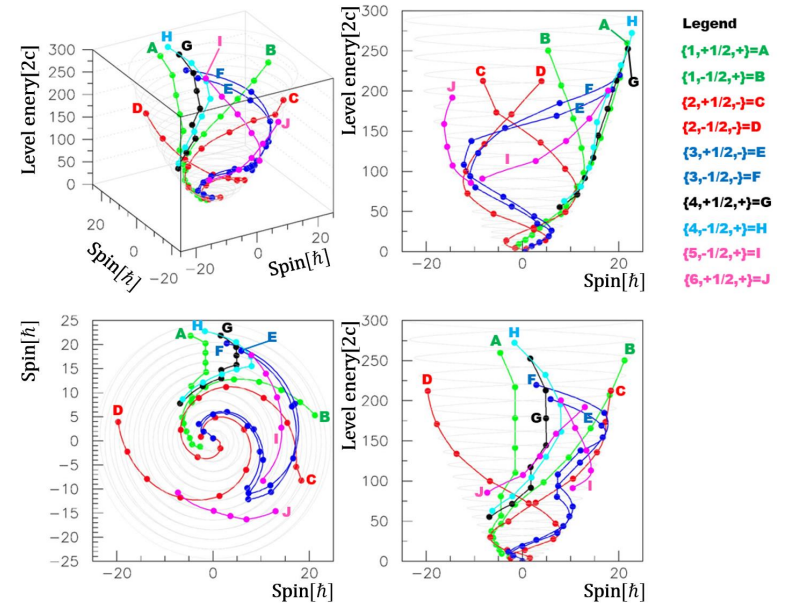
Real Bands Rotational Description

Decomposition:

$$E_{\gamma} = 2c_{band}(2I + k + k' - 1)$$

$2c_{band}$ *Real*, $(2I + k + k' - 1)$ *Integer*

$$2c_{band} = 2c[1 + fn / (2I + k + k' - 1)]$$



III. Insight into High-Spin Physics

Elementary Helix Loop with γ transition

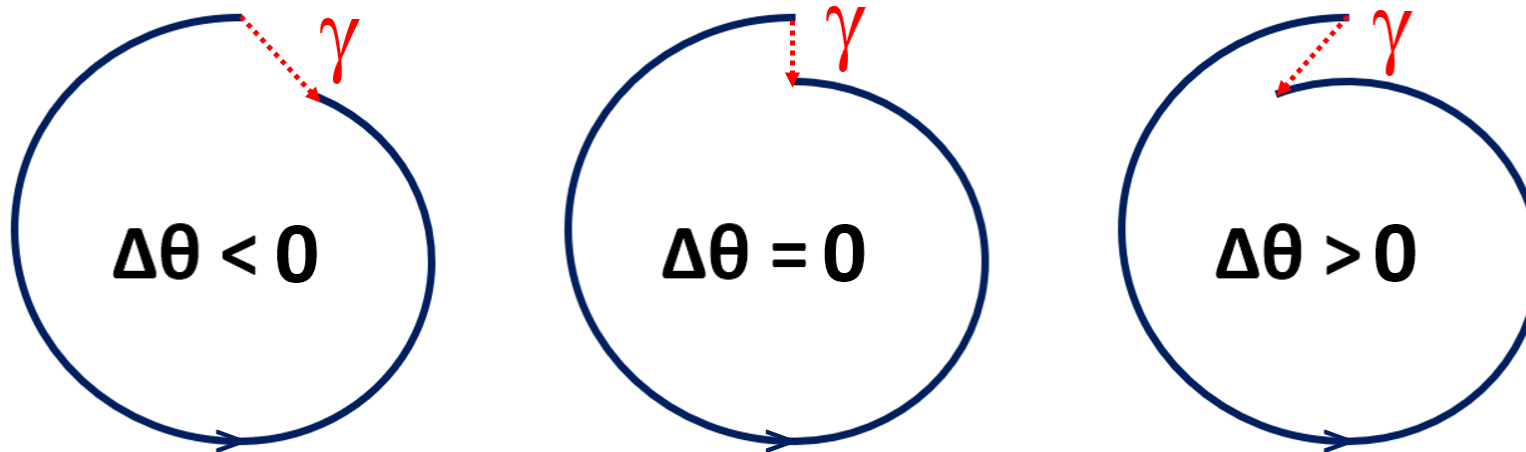
$$E\gamma = 2c_{band}(2I + k + k' - 1):$$

$\Delta\theta = 0, 2\pi$ Elementary Helix Loop Rotation *due to* $2I$ Macroscopic Collective Motion

- γ -decay paths: along vertical diameter

$\Delta\theta \neq 0$ Band's Apparent Rotation on the Helix *due to* $k + k'$ Microscopic S.P. Motion

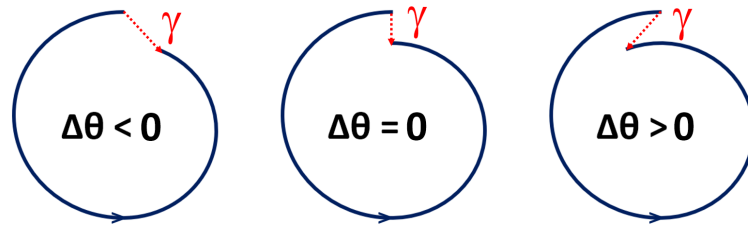
- γ -decay paths: $\Delta\theta < 0$ clockwise precession
- γ -decay paths: $\Delta\theta > 0$ counterclockwise precession



Phase angle of levels on helicoid: $\theta(I, m) = \sum_{I, m} (I + \frac{m}{I})\pi$

Phase shift between two consecutive band levels: $\Delta\theta(I) = \theta(I) - \theta(I-2) - 2\pi$

Fig. ($\Delta\theta$ apparent band rotation on the helicoid)



^{171}Yb Phase Shifts

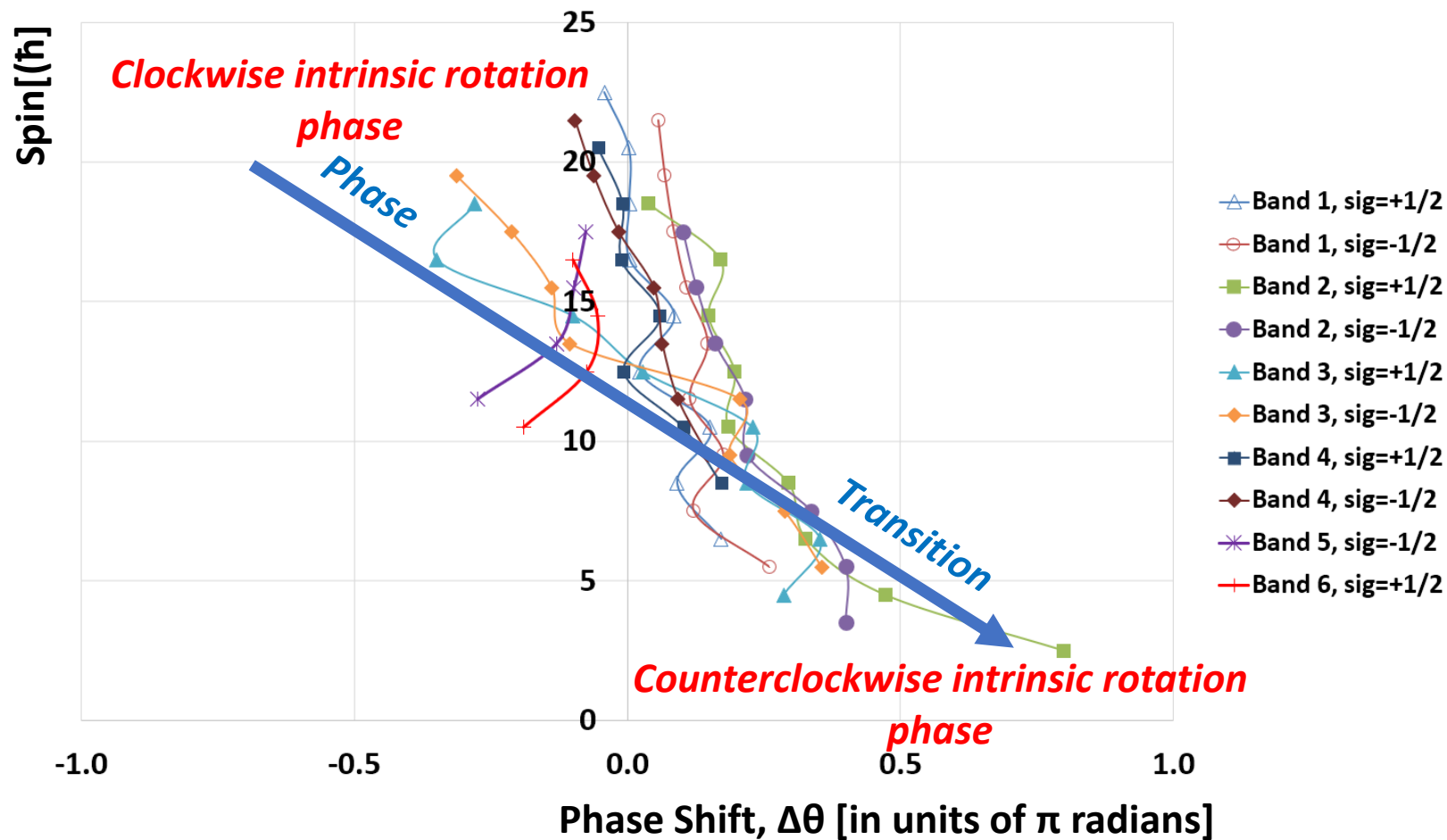


Fig. (^{171}Yb Phase Shifts)

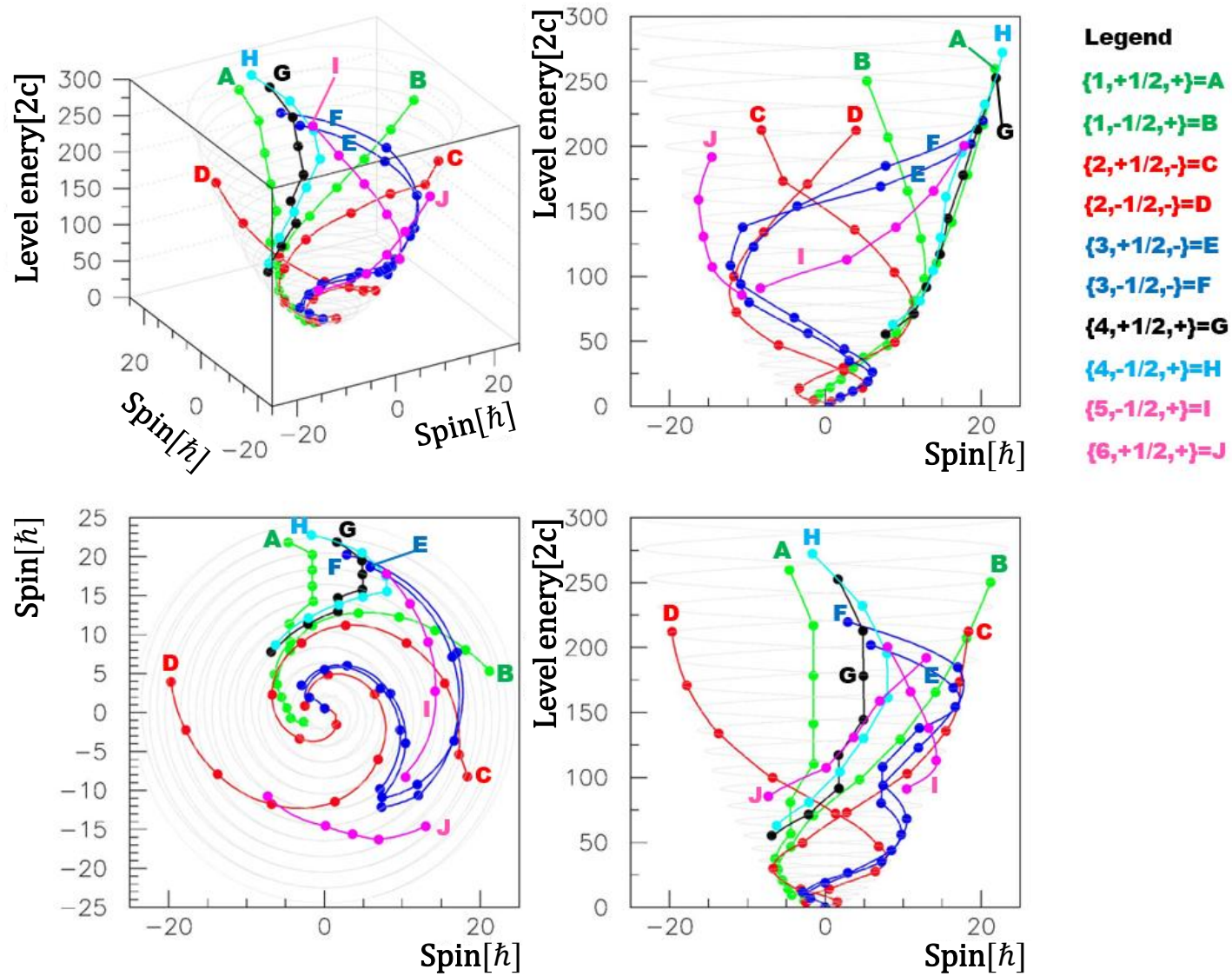


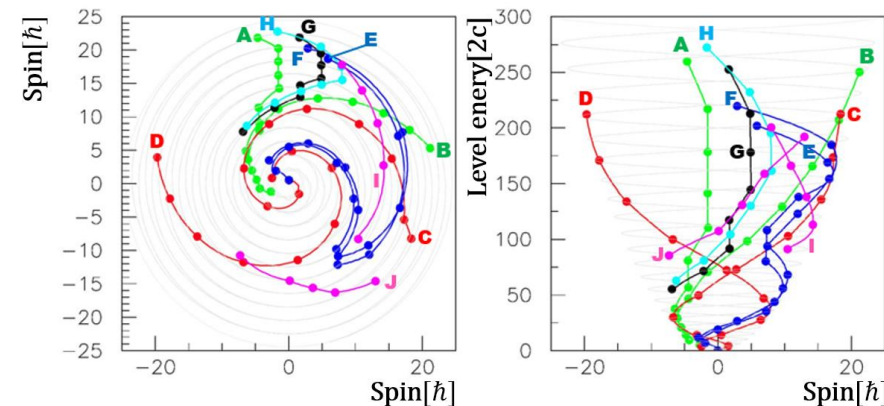
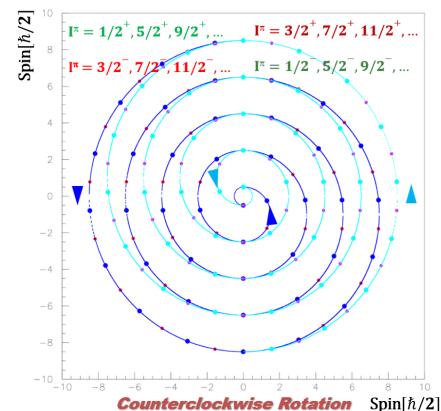
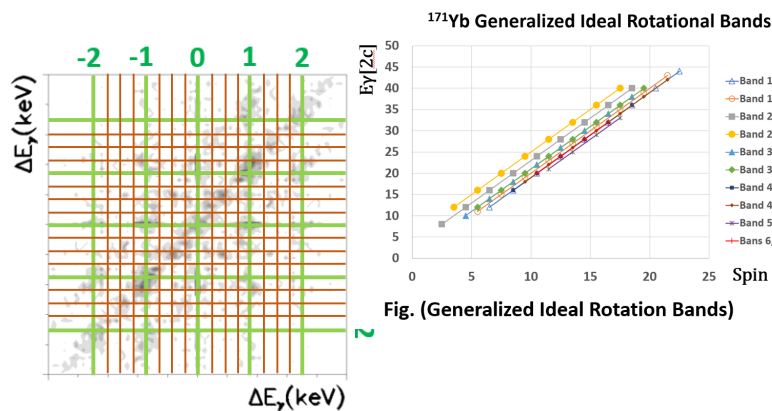
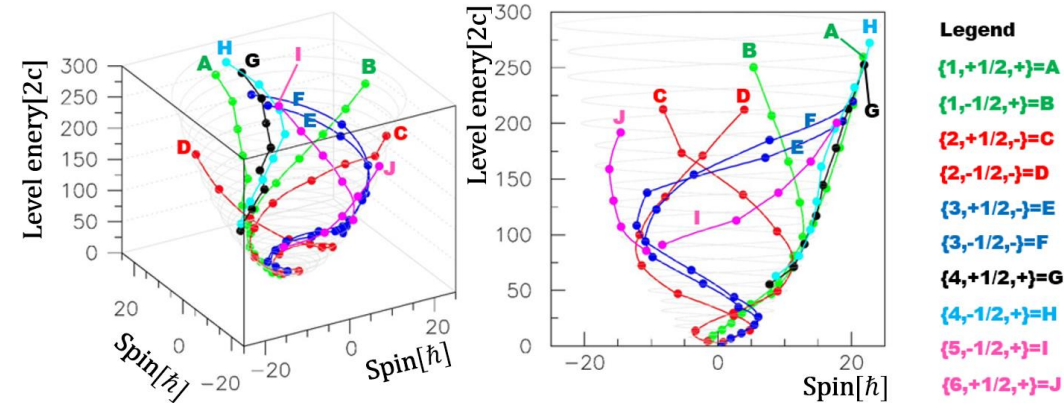
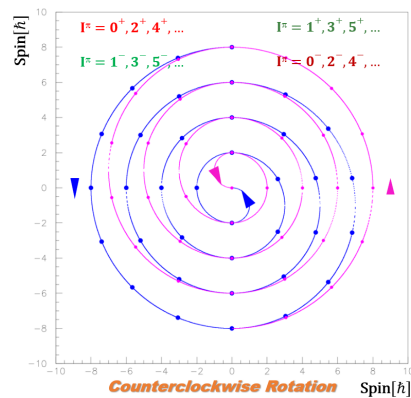
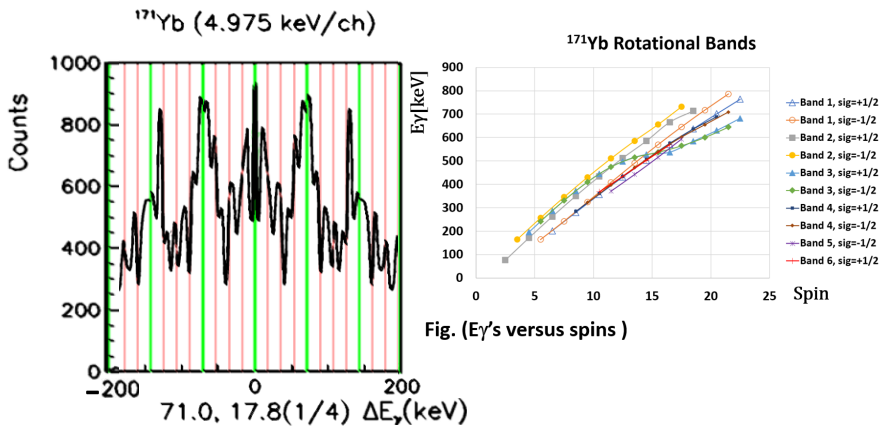
Fig. (Double helix of ^{171}Yb nucleus)

Double Helix Level Scheme Summary

I Repeatability

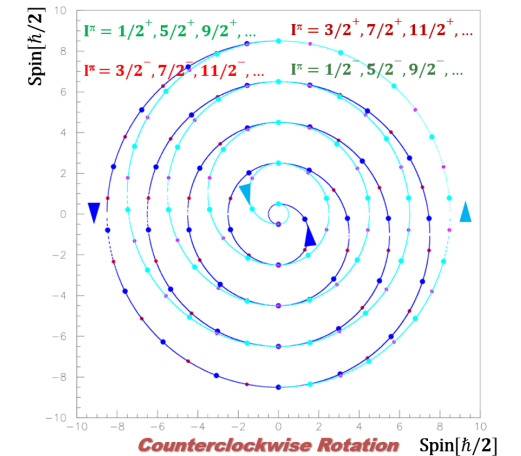
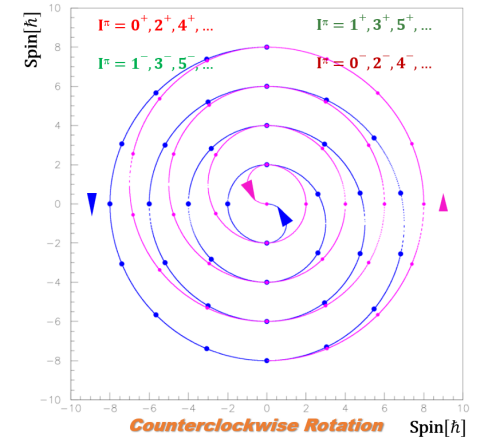
II Double Helix

III Double Helix Level Scheme



Double Helix Level Scheme Insight: Investment in Nuclear Structure Building

- *Double Helix is the geometrical place of the discrete set of spin states available for the rotational motion of the nucleus, which defines a Semiclassical Meta-Trajectory*
- *On average, one can assume that Nuclear Matter itself follows the Semiclassical Meta-Trajectory on Double Helix, with the actual levels selected by the rotational bands' paths*
- *Semi-classically, through Repeatability Nuclear Matter's Double Helix Motion can be seen as a Vortex Motion*
- *This can indicate vorticity in the liquid drop and relax the irrotational flow hypothesis on Bohr-Mottelson model*



DOUBLE HELICOID LEVELS SCHEME (DHLS)

