Nuclear shape / phase transitions in the N = 40, 60, 90 regions

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This work is dedicated to the memory of Adam Prášek

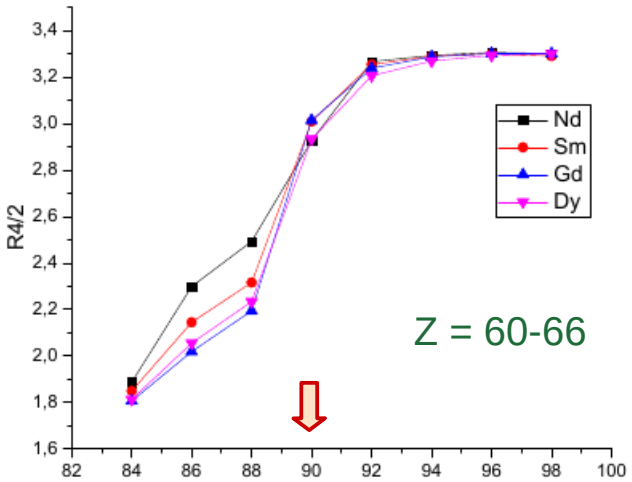
Outline

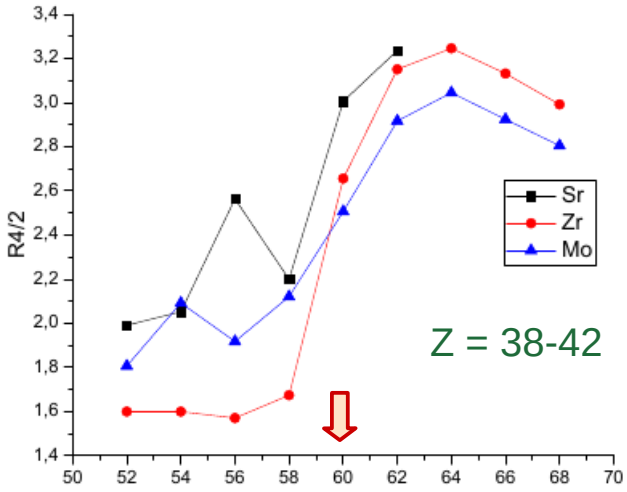
- Manifestations of structural change
- Shell model interpretation
- Microscopic self-consistent calculations
- Algebraic Collective Model calculations

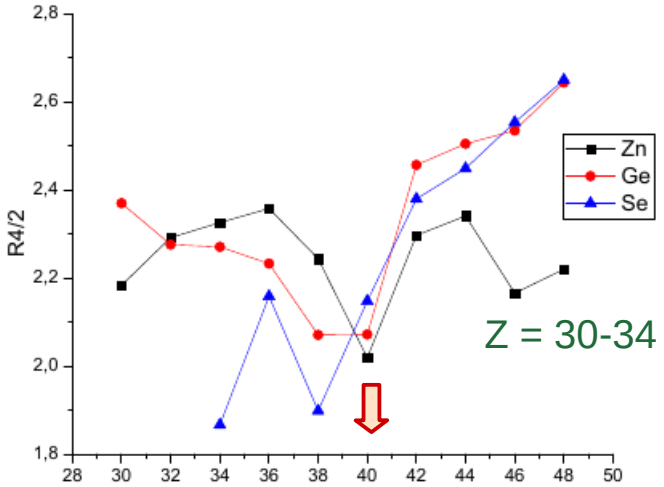
Energy ratios $R_{4/2}$

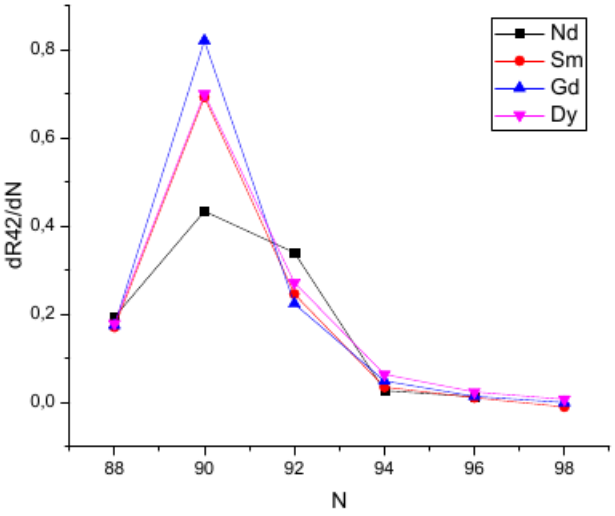
$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

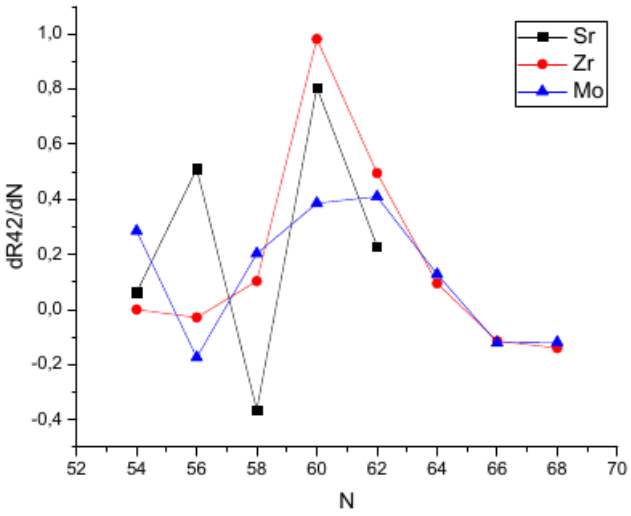
$$\frac{dR_{4/2}}{dN}(N) = R_{4/2}(N) - R_{4/2}(N-2)$$

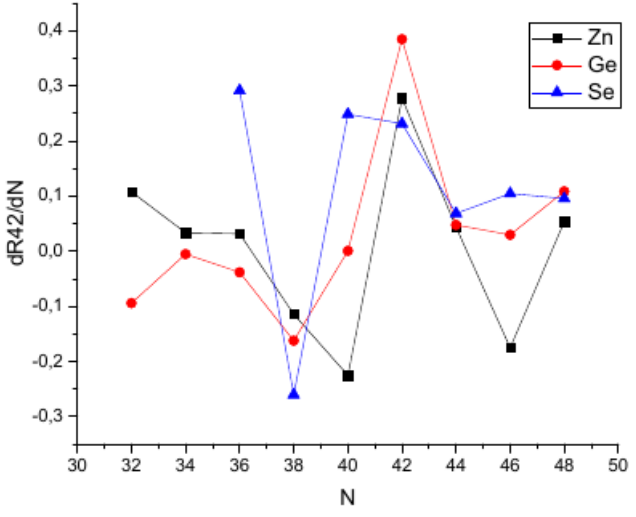








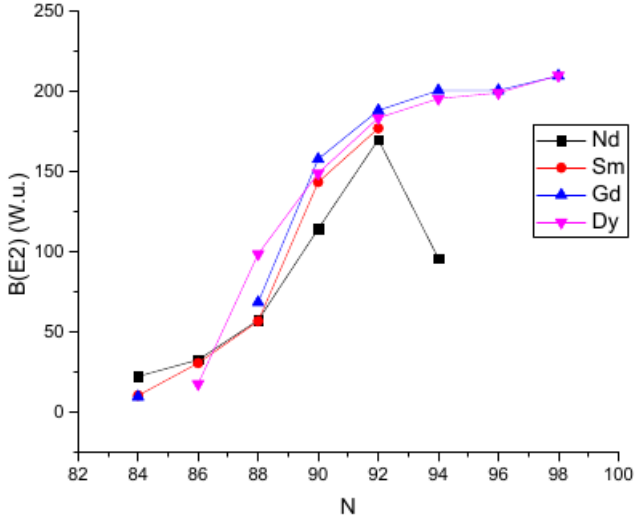


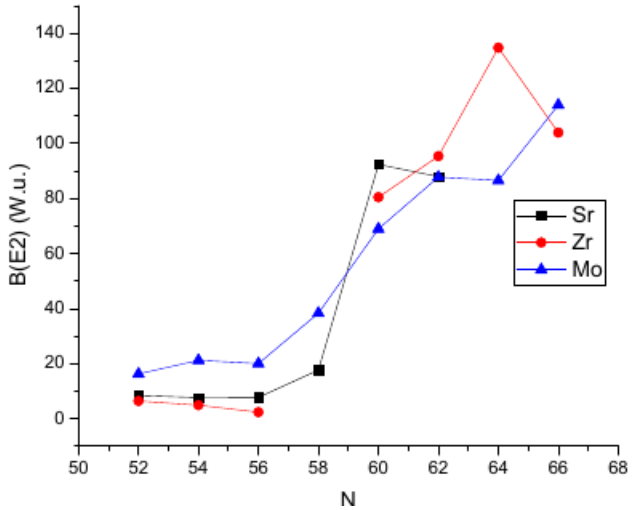


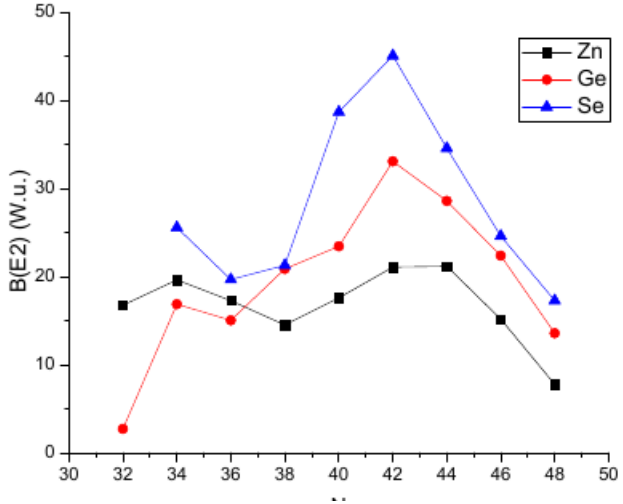
B(E2) transition rates

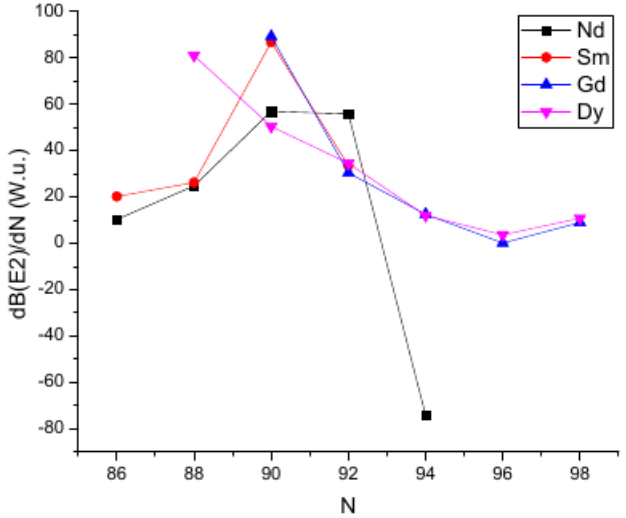
$$B(E2; 2_1^+ \rightarrow 0_1^+)$$

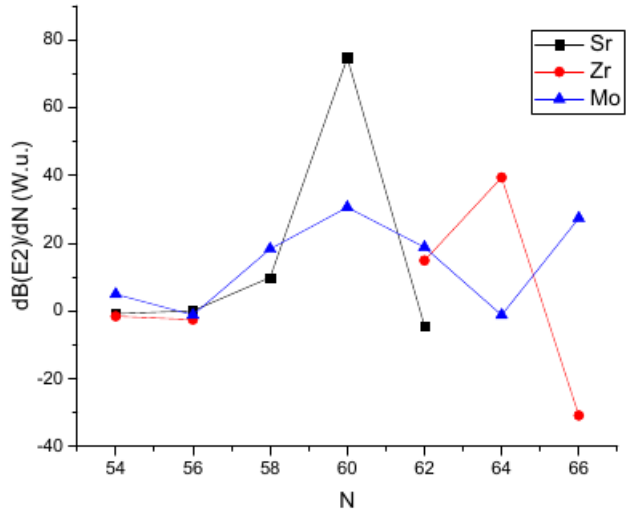
$$\frac{dB(E2; 2_1^+ \rightarrow 0_1^+)}{dN}(N) = B(E2; 2_1^+ \rightarrow 0_1^+)(N) - B(E2; 2_1^+ \rightarrow 0_1^+)(N-2)$$

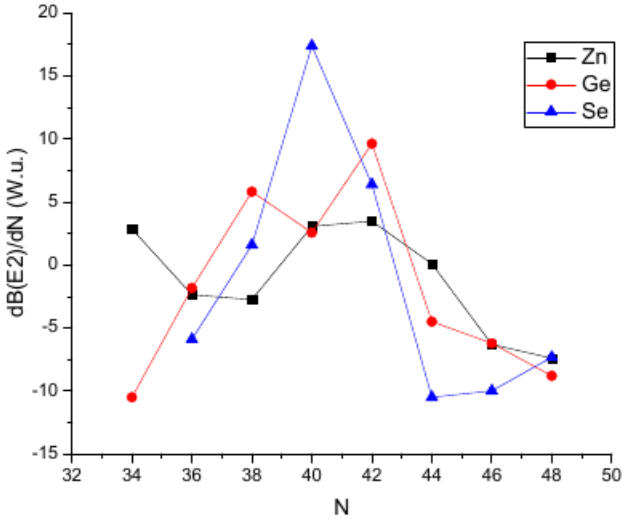






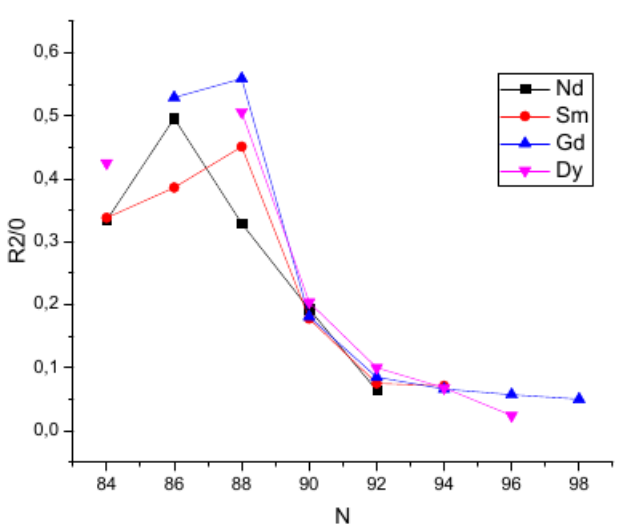


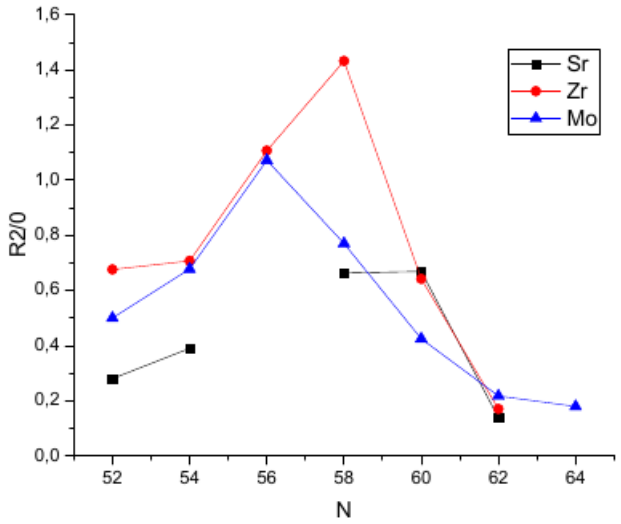


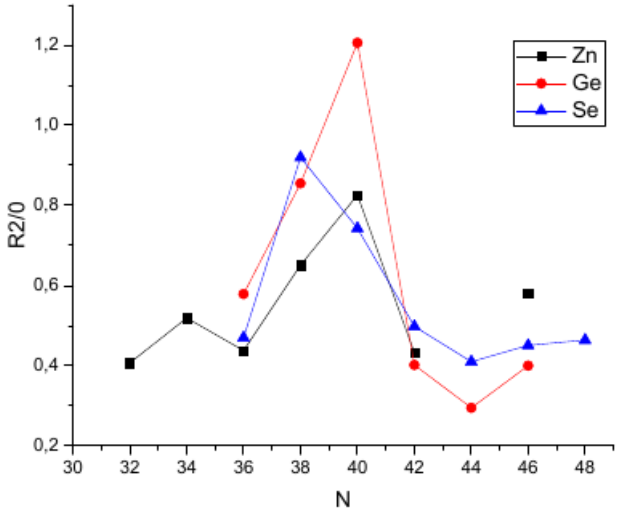


Energy ratios $R_{2/0}$

$$R_{2/0} = \frac{E(2_1^+)}{E(0_2^+)}$$



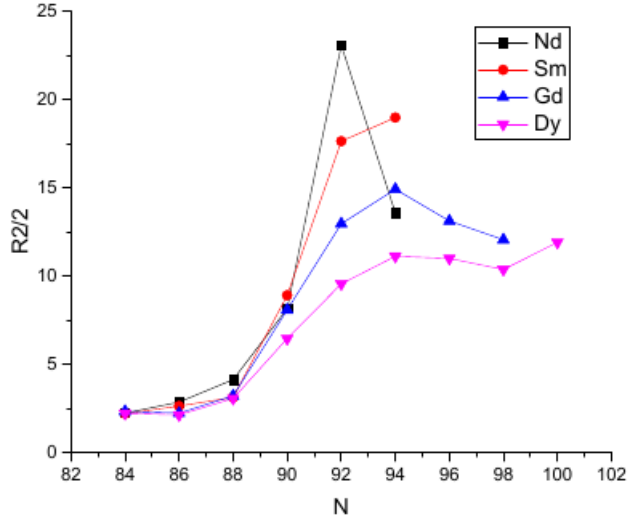


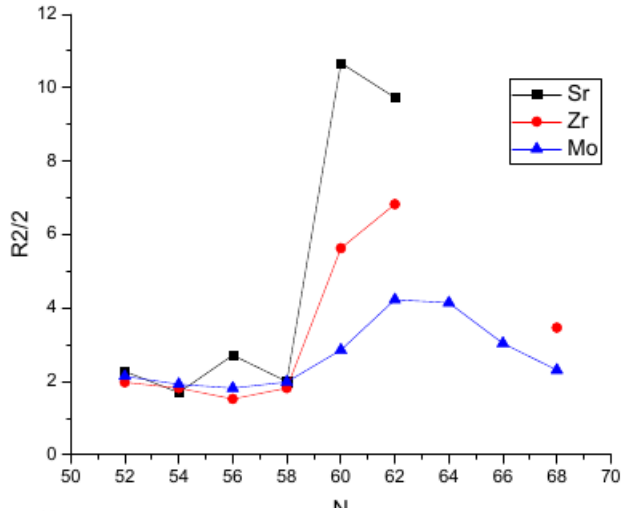


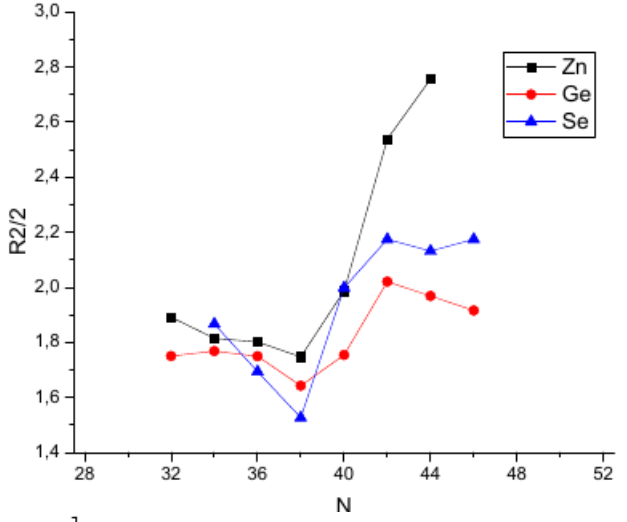
Energy ratios $R_{2/2}$

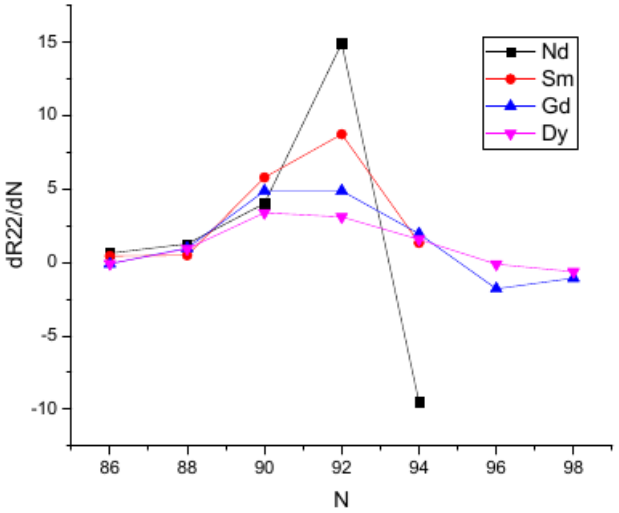
$$R_{2/2} = \frac{E(2^+_\gamma)}{E(2^+_1)}$$

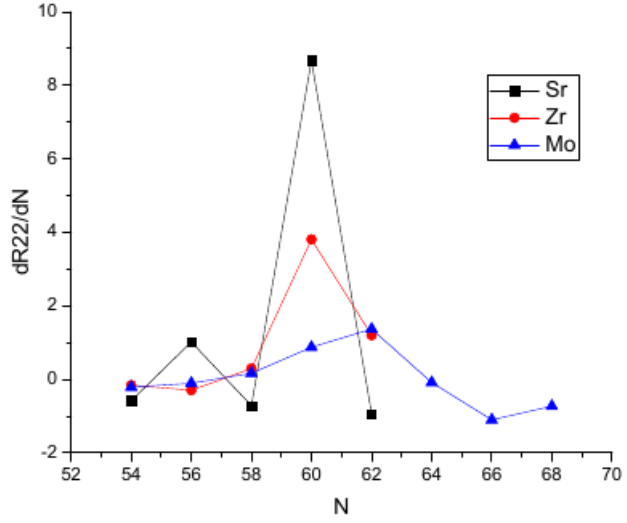
$$\frac{dR_{2/2}}{dN}(N) = R_{2/2}(N) - R_{2/2}(N-2)$$

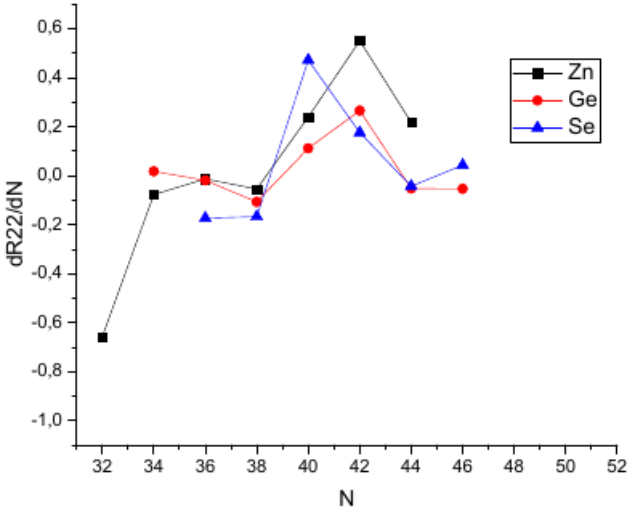












A mechanism for the onset of deformation

interactions between valence protons and neutrons

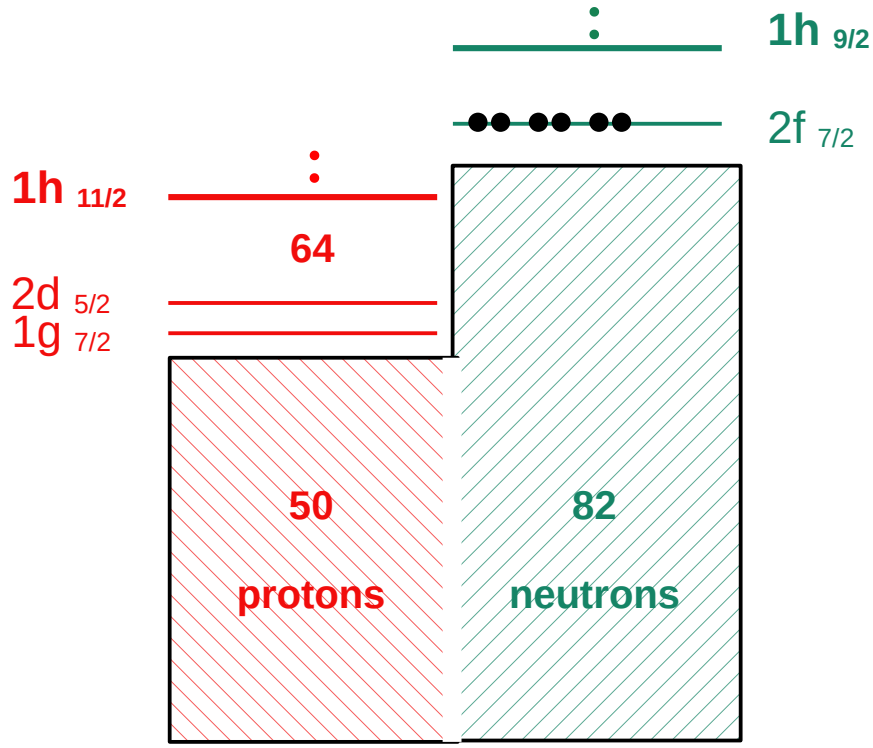
midshell => increased collectivity => low-lying 2^+_1

subshell structure (gaps) depend on the number of protons and neutrons present
=> *effective* shells

e.g. for Z ≈ 60, N ≈ 90
when N < 90 *effective* proton shell: Z = 50 - 64

=> midshell at Z ≈ 56

=> as the neutron $h_{9/2}$ begins to fill



monopole p-n interaction between spin-orbit partner orbitals $h_{11/2}$ proton and $h_{9/2}$ neutron lowers the $h_{11/2}$ proton level, eliminating the Z = 64 gap

P. Federman and S. Pittel, Phys. Lett. B 69, 385 (1977)
R. F. Casten, Nuclear Structure from a Simple Perspective (Oxford University Press, Oxford, 1990)
T. Otsuka, Physics 4, 258 (2022)

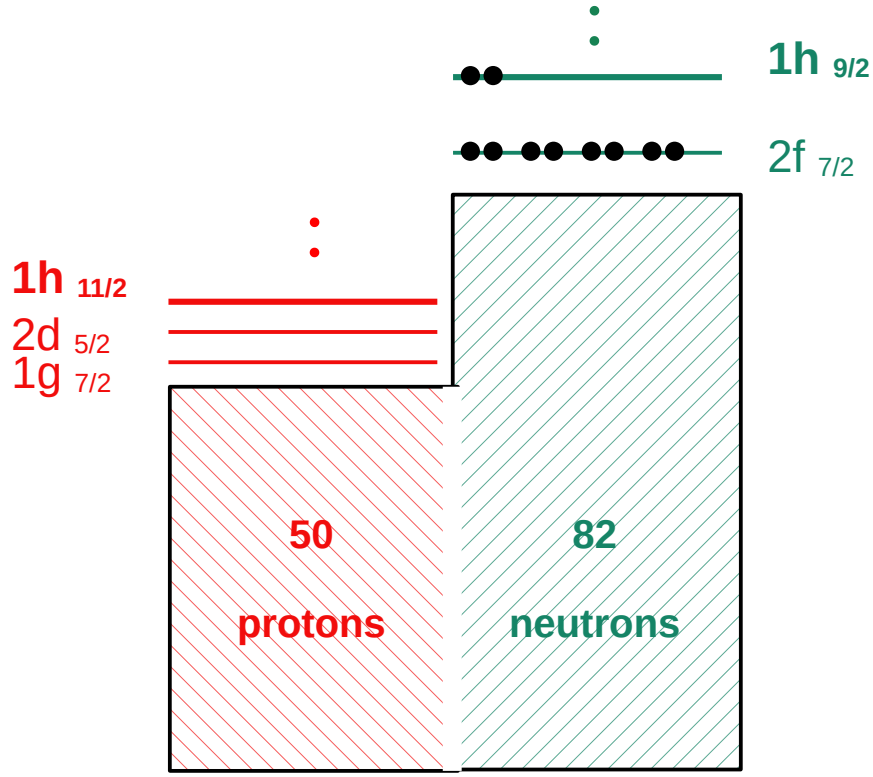
A mechanism for the onset of deformation

when N ≥ 90 *effective* proton shell: Z = 50 - 82

=> new midshell at Z ≈ 64, 66

=> new position for the lowest-lying 2^+_1

=> affects nuclei with Z = 60-66 (Nd, Sm, Gd, Dy)



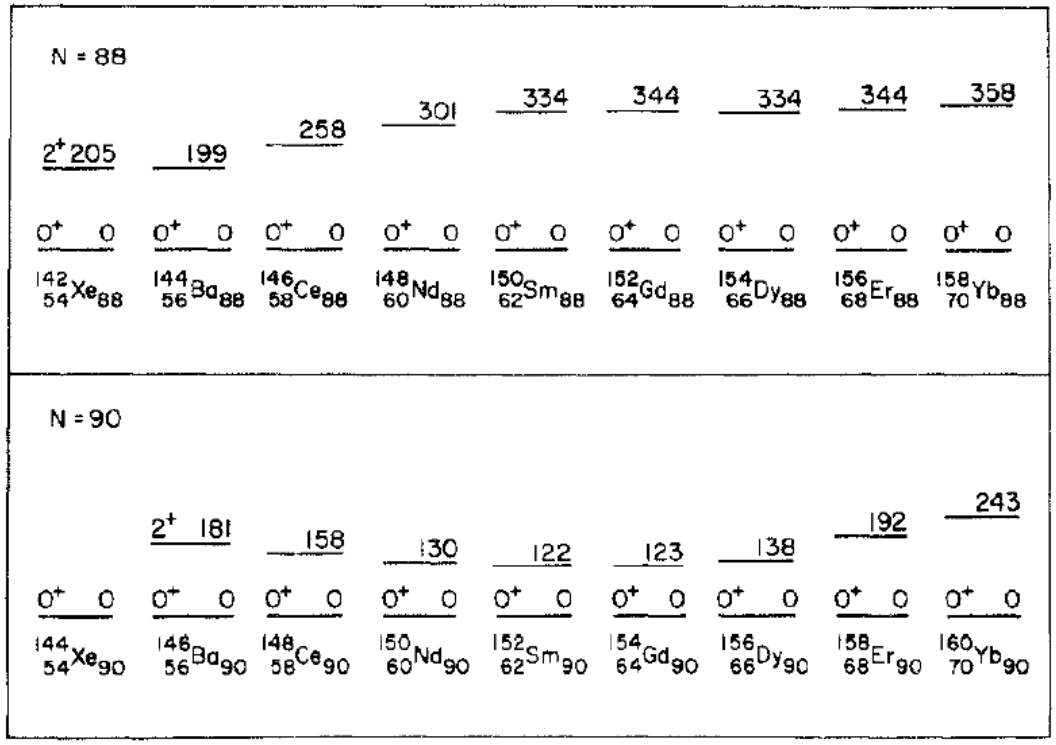


fig. from R. F. Casten *op.cit.*

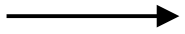
valence nucleons also important in proxy-SU(3) scheme →
talks by N. Minkov and D. Bonatsos

Microscopic calculations: SHF + BCS

Hartree-Fock
equation

+

Skyrme
effective interaction



mean-field

$$\hat{h}\psi_\alpha = \varepsilon_\alpha \psi_\alpha$$

+

BCS

$$(\varepsilon_\alpha - \epsilon_{F,q_\alpha})(u_\alpha^2 - v_\alpha^2) = \Delta w_\alpha u_\alpha v_\alpha$$

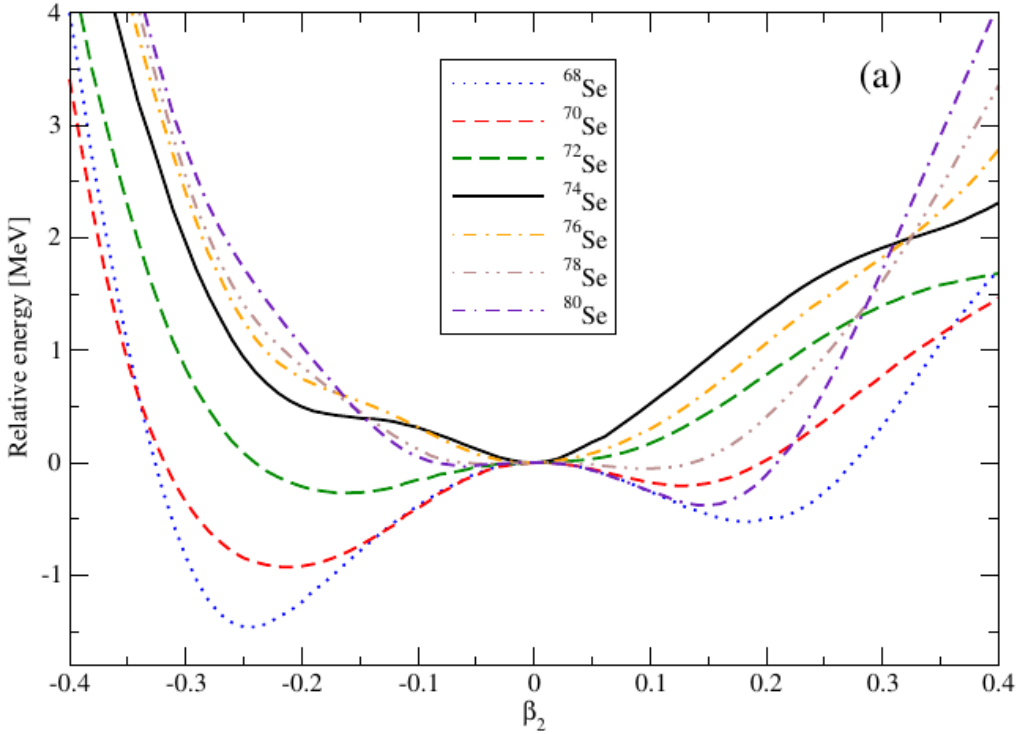
SkyAx code:
initial single particle states = Nilsson orbitals

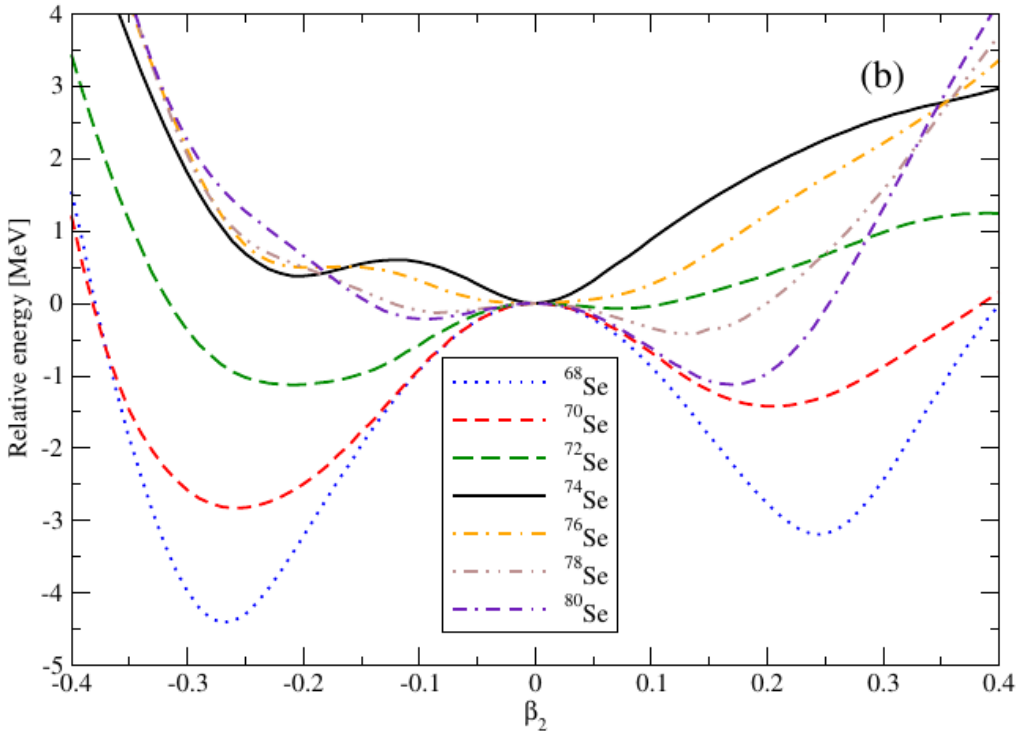
P.-G. Reinhard, B. Schuetrumpf, and J. A. Maruhn, Comp. Phys. Comm. 258, 107603 (2021).

15 Skyrme parametrizations: {SV-bas, SV-tls, SV-mas07,... }

constrained calculations: (quadrupole) moment is fixed => Potential Energy Curves (PEC)

PEC for Se isotopes (N=34-46)



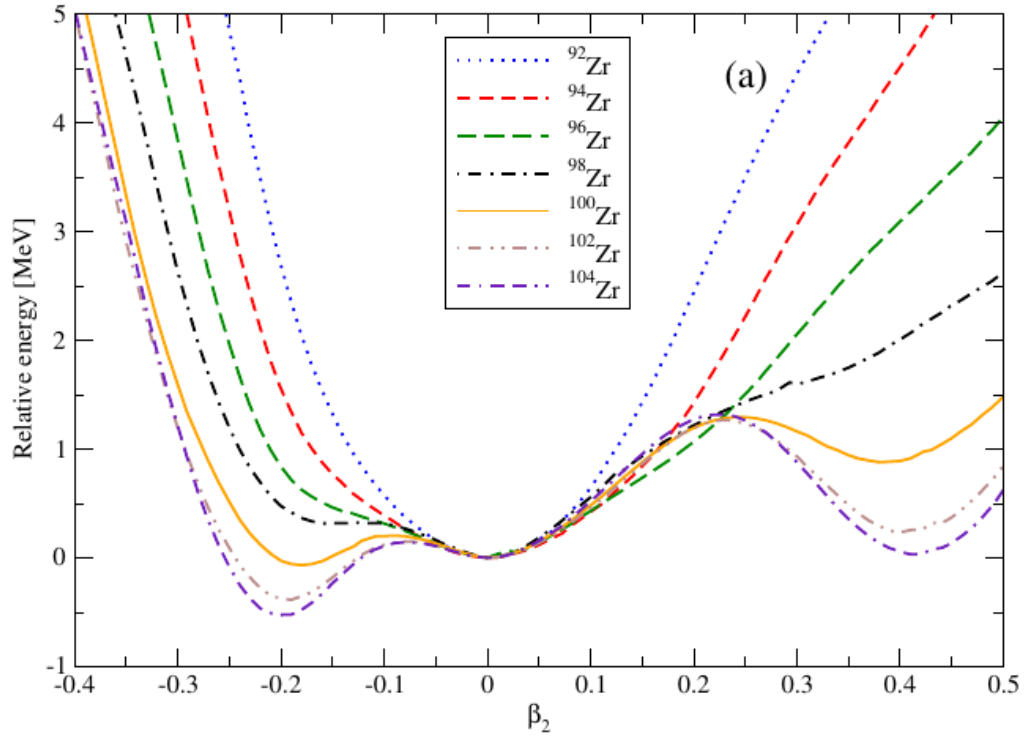


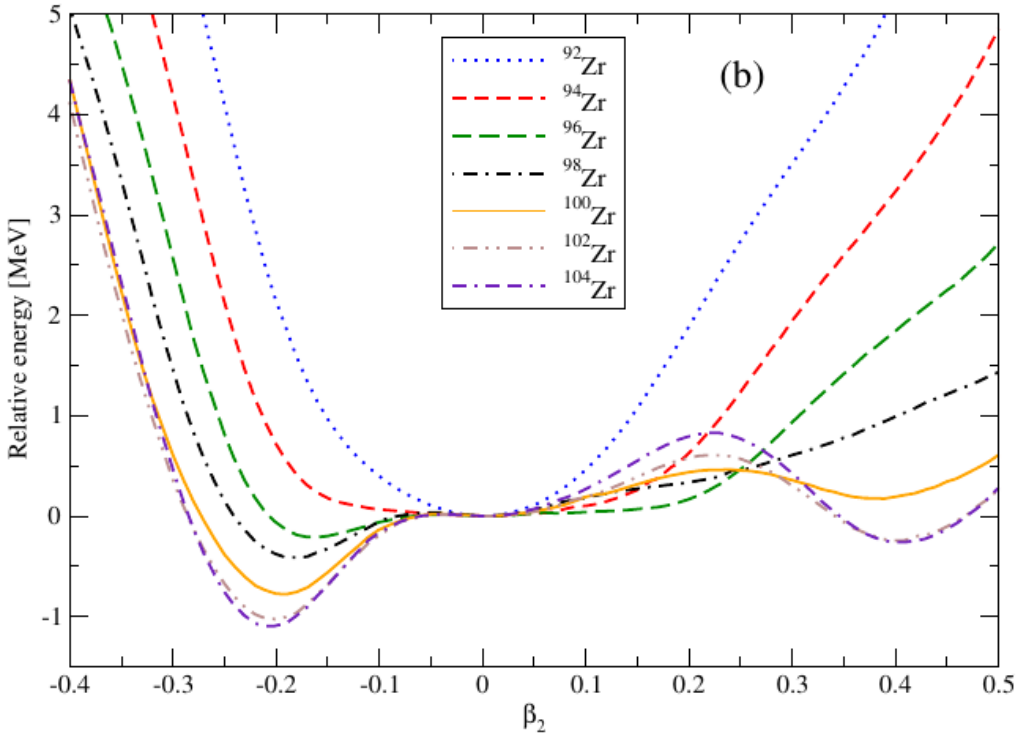
SV-bas

N = 40 → ^{74}Se

SV-mas07

PEC for Zr isotopes (N=52-64)



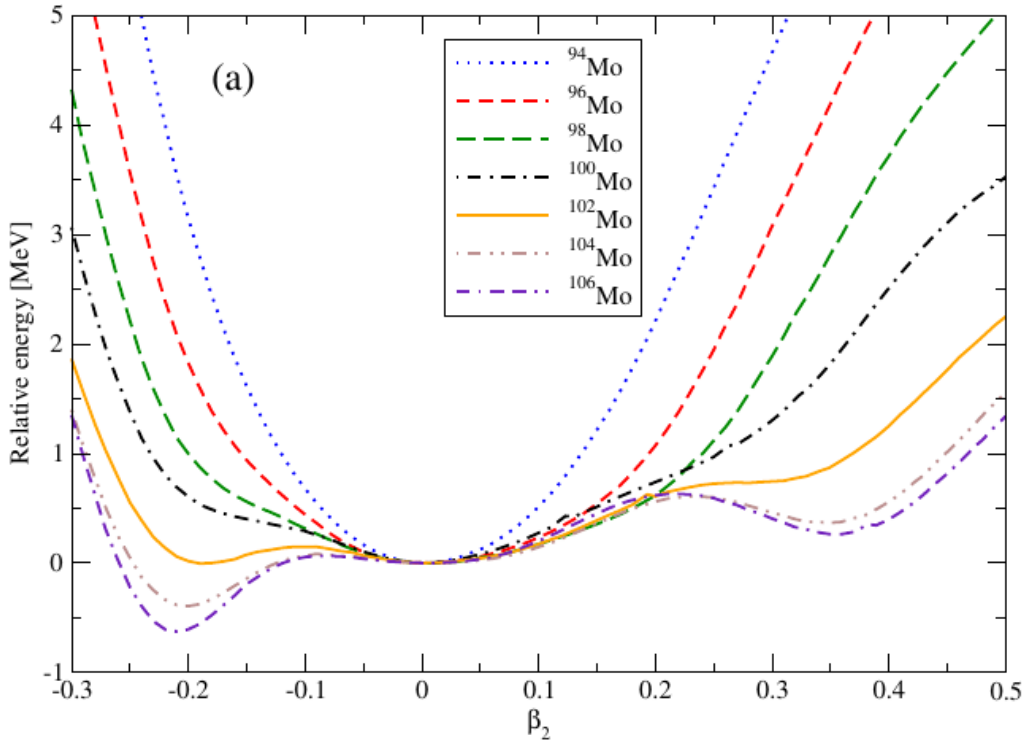


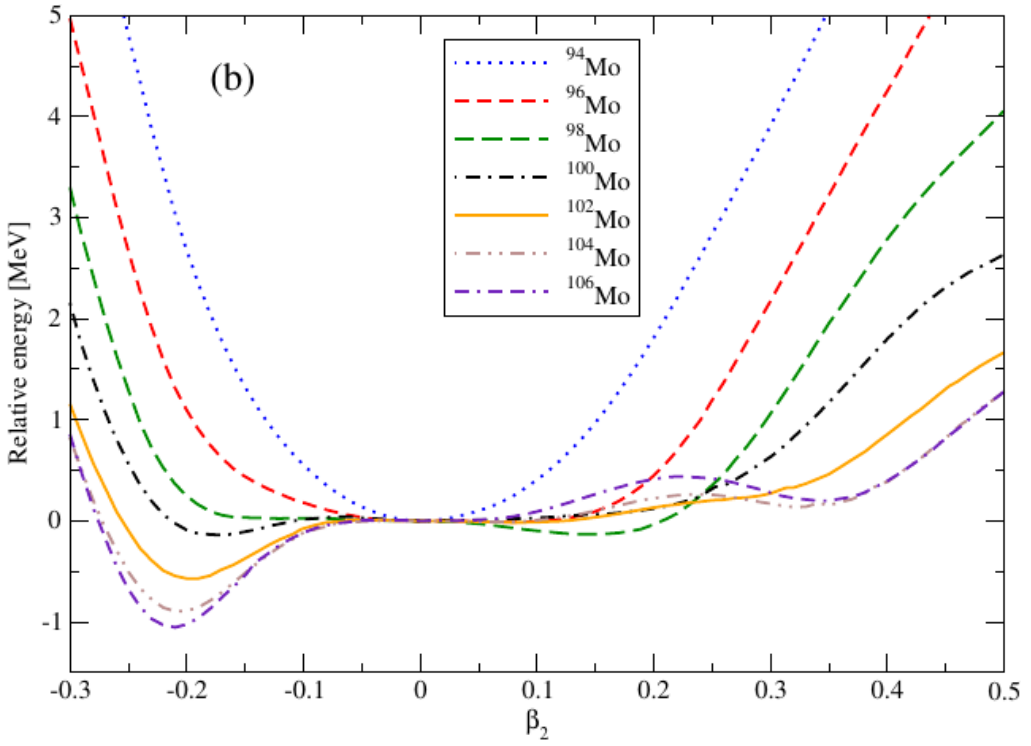
SV-bas

N = 60 → ^{100}Zr

SV-mas07

PECs for Mo isotopes (N=52-64)



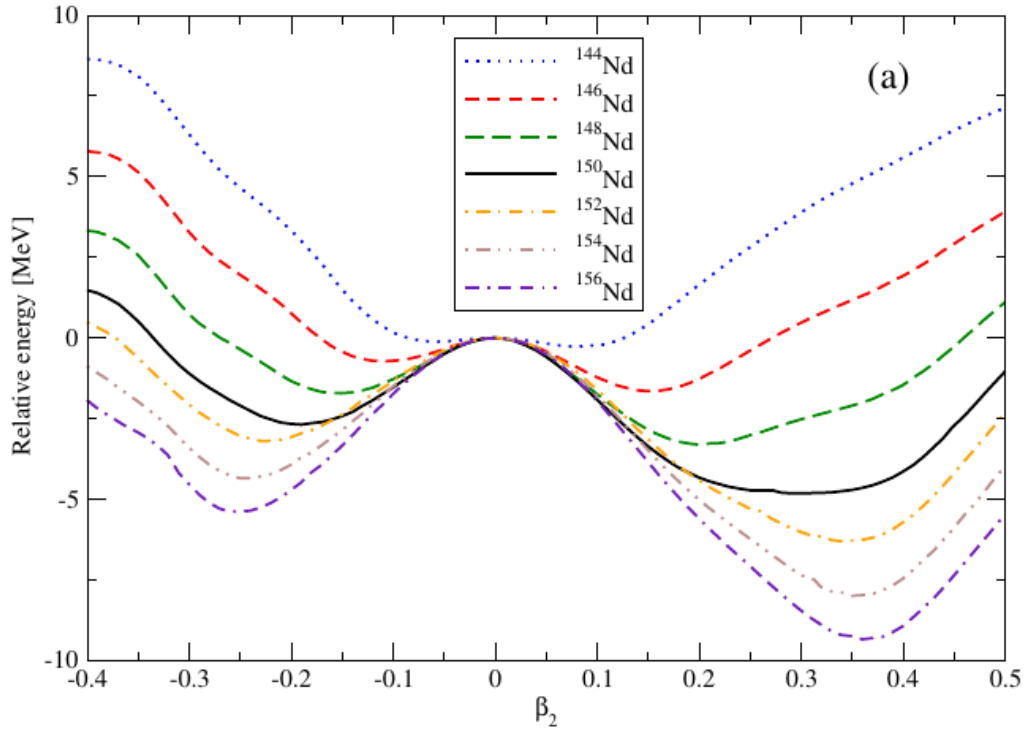


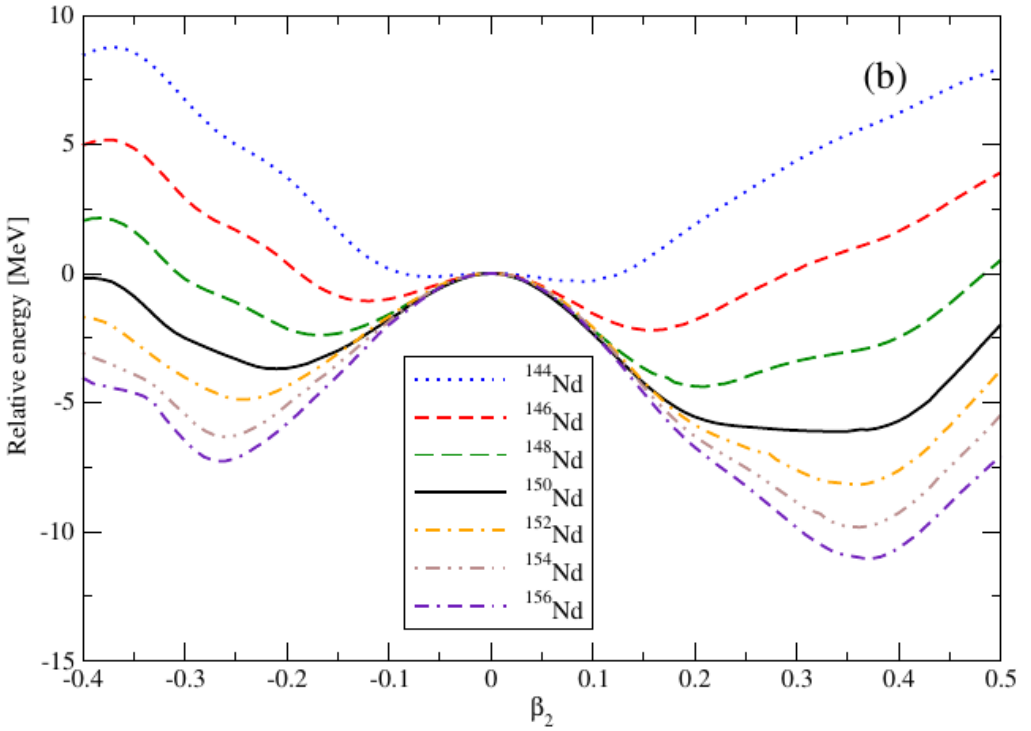
SV-bas

N = 60 → ^{102}Mo

SV-mas07

PECs for Nd isotopes (N=84-96)





SV-bas

N = 90 → ^{150}Nd

SV-mas07

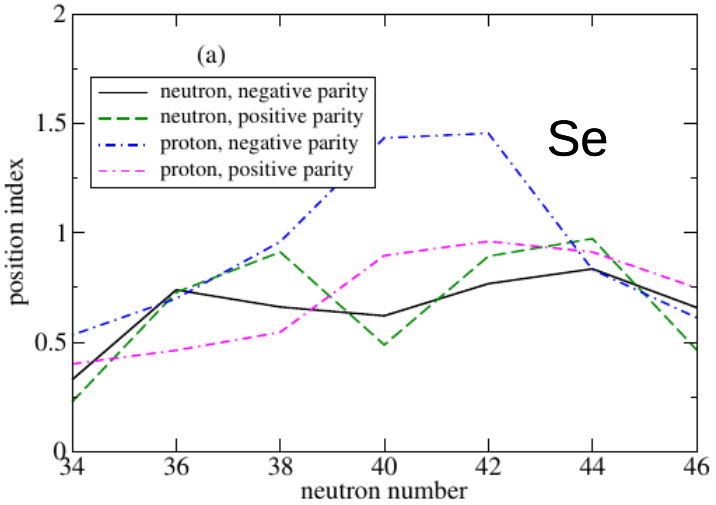
Position Index

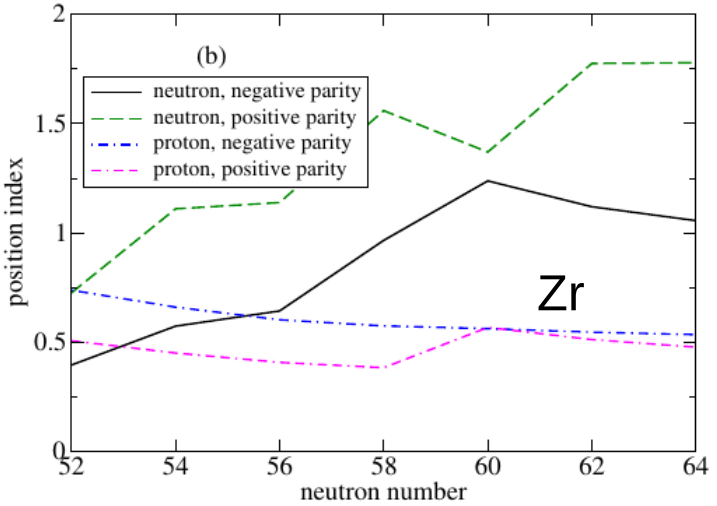
$$I(q,\pi) = \sum_{i_{(q,\pi)}} \left(0.5 - \left|v_i^2 - 0.5\right|\right)$$

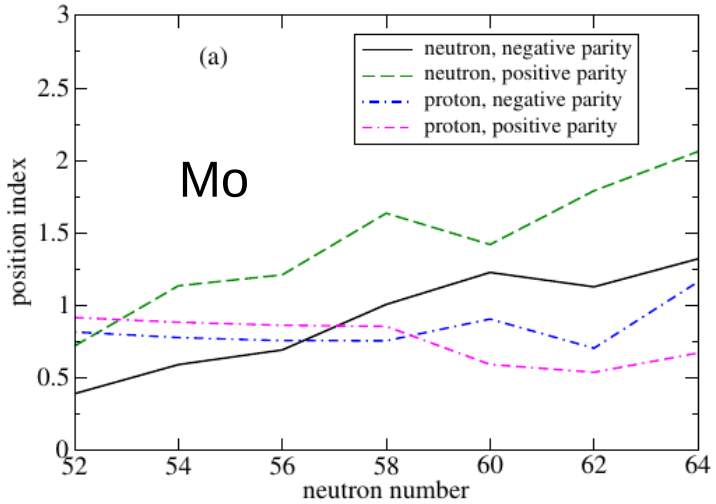
v_i^2: occupation probability of state i

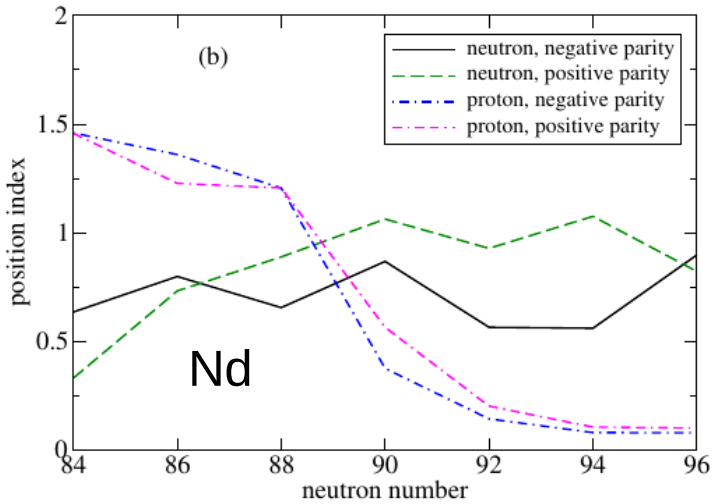
$q = n, p$ and $\pi = \pm$

what kind of levels are closest to the Fermi surface









Algebraic Collective Model (ACM)

A computationally tractable version of the collective model of **Bohr** and **Mottelson**

$$\hat{H} = x_1\nabla^2 + x_2 + x_3\beta^2 + x_4\beta^4 + \frac{x_5}{\beta^2}$$
$$+ x_6\beta\cos 3\gamma + x_7\beta^3\cos 3\gamma + x_8\beta^5\cos 3\gamma + \frac{x_9}{\beta}\cos 3\gamma$$
$$+ x_{10}\cos^2 3\gamma + x_{11}\beta^2\cos^2 3\gamma + x_{12}\beta^4\cos^2 3\gamma + \frac{x_{13}}{\beta^2}\cos^2 3\gamma$$
$$+ \frac{x_{14}}{\hbar^2}[\hat{\pi}\otimes\hat{q}\otimes\hat{\pi}]_0$$

$$\nabla^2 = \frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2}\hat{\Lambda}$$

x_1-x_{14} parameters: fitted to data

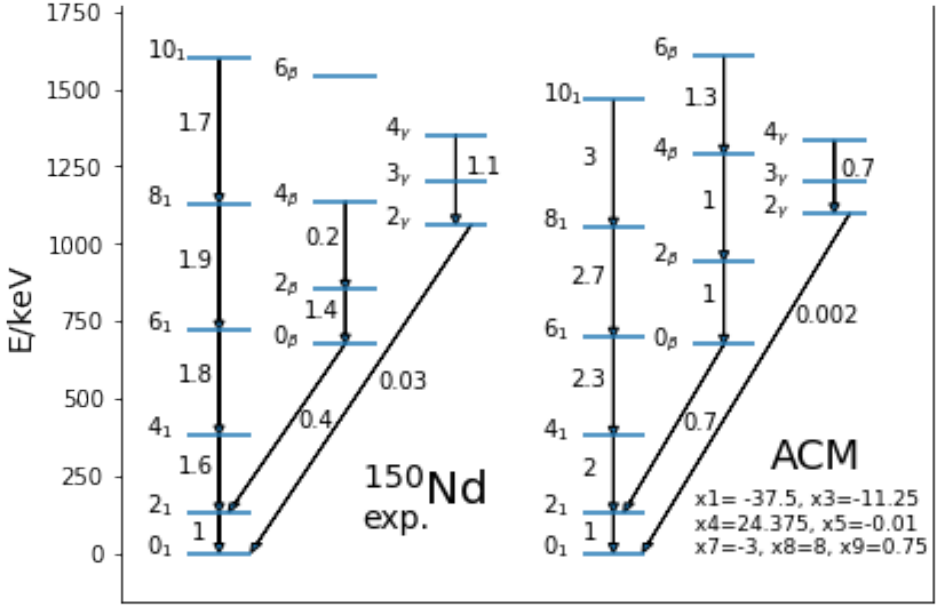
basis w.f.: $\mathrm{SU(1,1)}\times\mathrm{SO(5)} \supset \mathrm{U(1)}\times\mathrm{SO(3)} \supset \mathrm{SO(2)}$

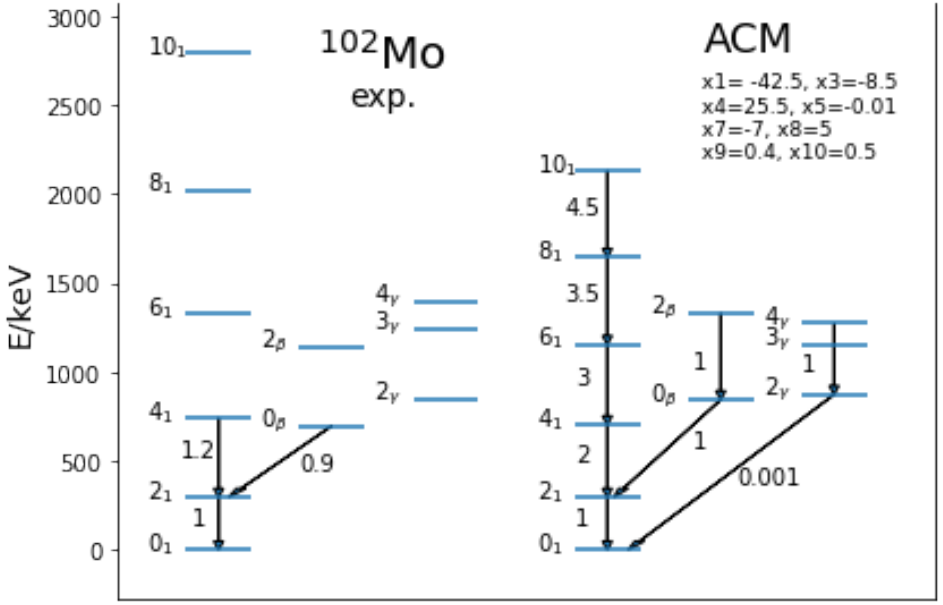
- SU(1,1): radial β-w.f.
- SO(5): angular 5-dim sph. har.

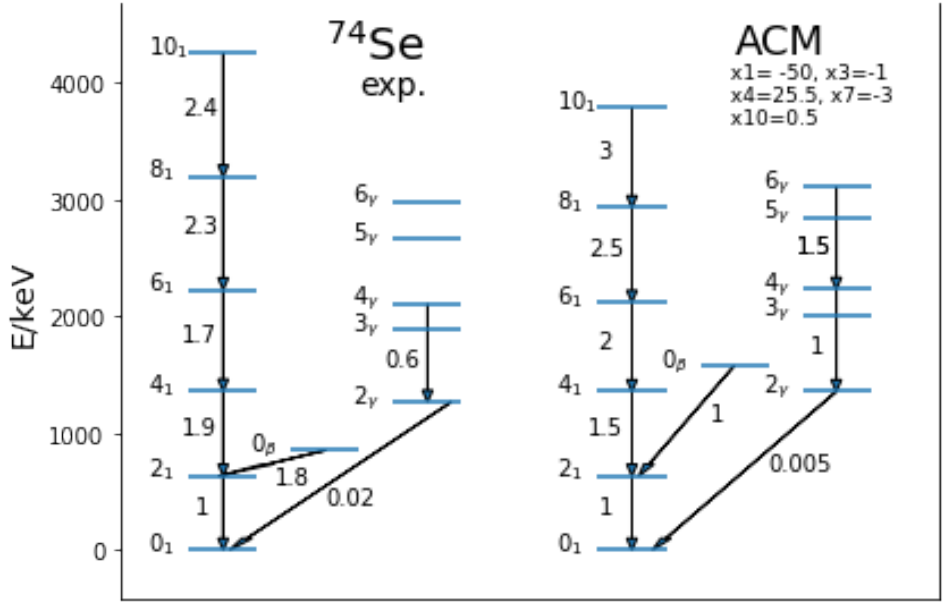
=> matrix elements computed analytically

D. J. Rowe, T. A. Welsh, and M. A. Caprio, Phys. Rev. C 79, 054304 (2009).
D. J. Rowe and J. L. Wood, Fundamentals of nuclear models (World Scientific, Singapore, 2010)
T. A. Welsh and D. J. Rowe, Comput. Phys. Commun. 200, 220 (2016)

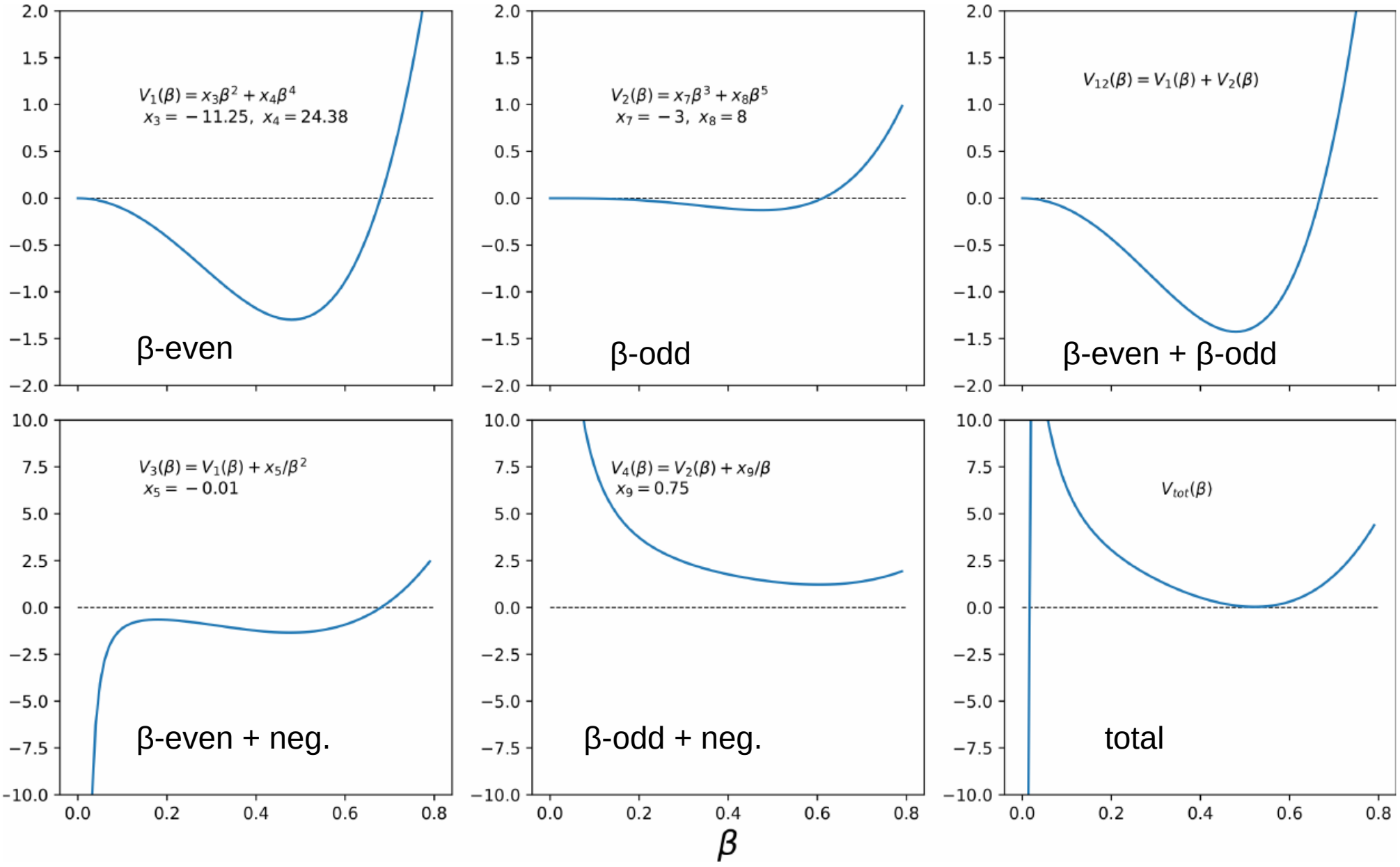
ACM calculations: spectra and B(E2)s





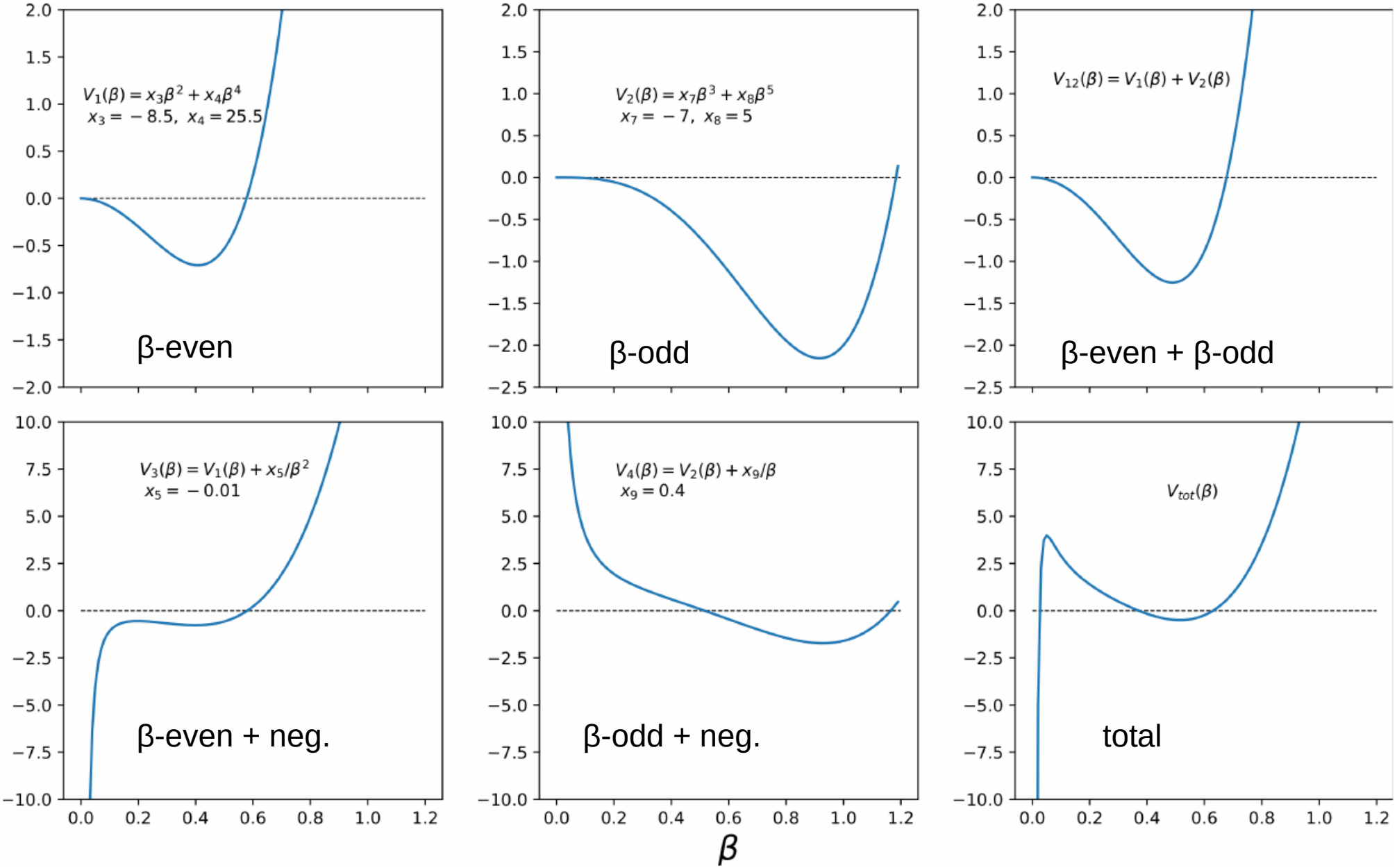


ACM Potential Energy Curves



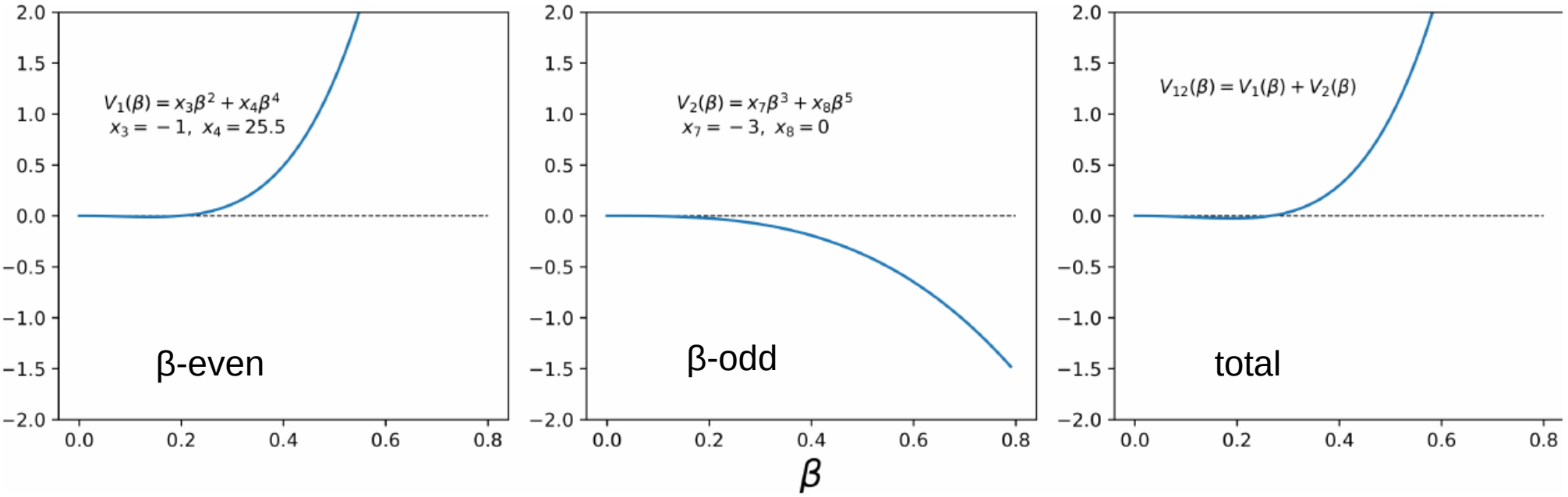
^{150}Nd

ACM Potential Energy Curves



^{102}Mo

ACM Potential Energy Curves



^{74}Se

In general, potentials resulting from ACM calculations show similarities with ones used in the Bohr Hamiltonian in the context of critical point symmetries, e.g.

- displaced infinite square well,
- infinite square well with a sloped wall,
- Davidson
- Kratzer

Summary

- Certain experimental quantities, such as energy ratios: R4/2,… and B(E2) transition rates serve as benchmarks for nuclear structure
- Valence **p-n** interactions driving force for structural change
- Microscopic calculations in the N = 40, 60, 90 regions show signs of a *first*-order phase transition with coexisting minima in the PECs
- Potentials resulting from the ACM calculations show similarities with ones used in the context of the Bohr Hamiltonian (critical point symmetries)

IOP PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **34** (2007) R285–R320 doi:10.1088/0954-3899/34/7/R01

TOPICAL REVIEW

Quantum phase transitions and structural evolution in nuclei

R F Casten and E A McCutchan

Wright Nuclear Structure Laboratory, Yale University, New Haven, CT 06520, USA

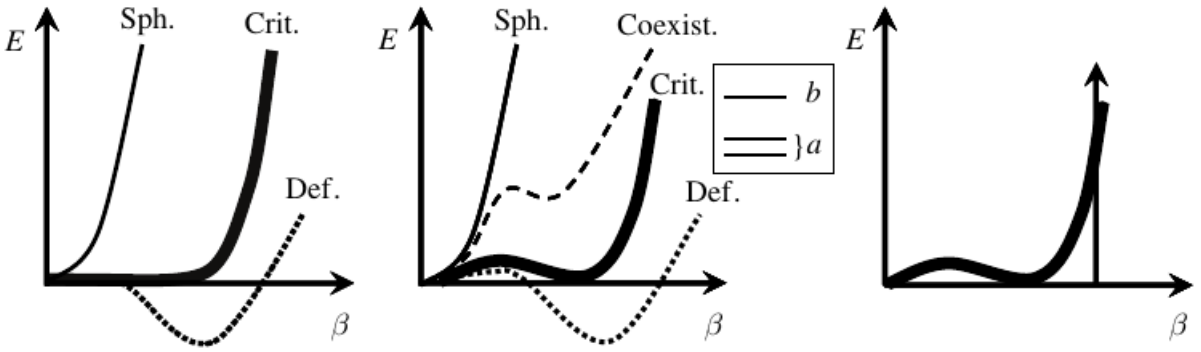
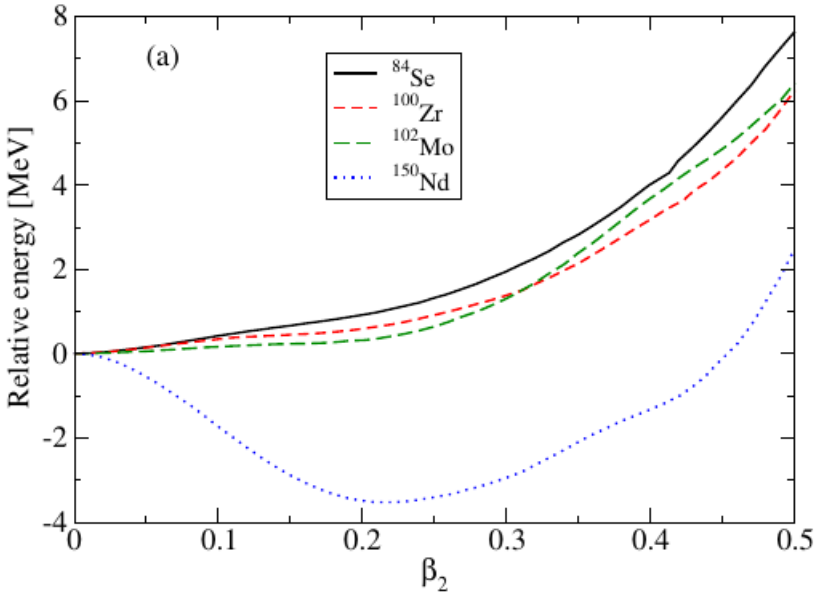
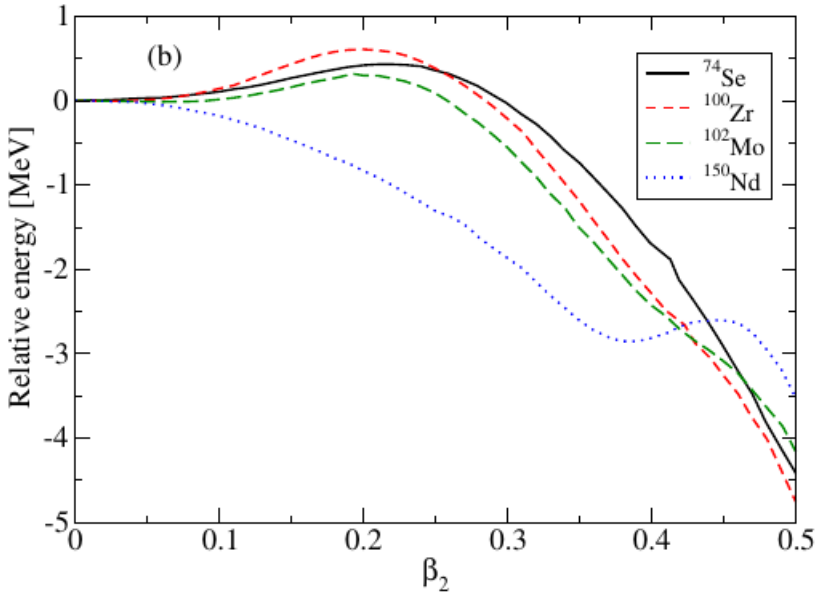


Figure 7. Energy surfaces for nuclei with successively larger numbers of valence nucleons plotted against the quadrupole deformation β (left) for a second-order phase transition and (middle) for a first-order phase transition. Right: the curve 'Crit.' repeated along with the square well ansatz that embodies the essential features of X(5) and E(5). The inset to the middle figure represents a set of shell model orbits a and b.


(a)
^{84}Se
^{100}Zr
^{102}Mo
^{150}Nd
Relative energy [MeV]
β_2
8
6
4
2
0
-2
-4
0
0.1
0.2
0.3
0.4
0.5


(b)
^{74}Se
^{100}Zr
^{102}Mo
^{150}Nd
Relative energy [MeV]
β_2
1
0
-1
-2
-3
-4
-5
0
0.1
0.2
0.3
0.4
0.5