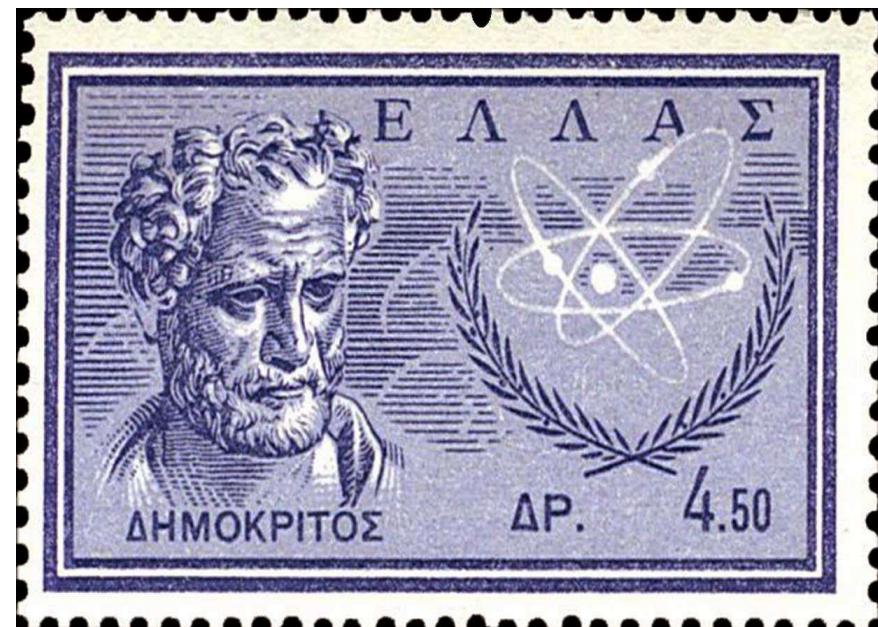


# Seven years of the proxy-SU(3) shell model symmetry

Dennis Bonatsos  
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# Nuclear Research Centre Demokritos 1961



# The people behind the work

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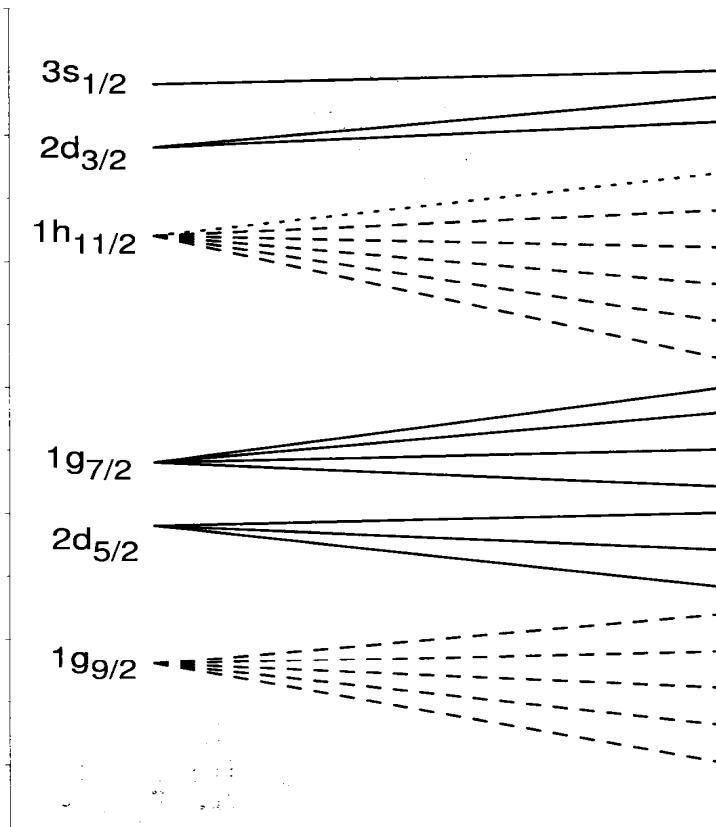
N. Minkov (INRNE Sofia)

Review: Symmetry 15 (2023) 29 (feature article)

# Proxy-SU(3) scheme

- Restoration of SU(3) symmetry of 3D-HO
- Approximation to the Nilsson model
- Connection to spherical shell model
- Highest weight irreps
- Prolate over oblate dominance
- Prolate to oblate shape transition
- Islands of shape coexistence
- Preponderance of triaxial shapes

# Restoration of 3D-HO SU(3) symmetry proxy-SU(3)      $1h11/2 \rightarrow 1g9/2$ PRC95 (2017) 064325



# Justification: Matrix of the Nilsson Hamiltonian for the 50-82 proton shell

PRC95 (2017) 064325

# Matrix of the proxy-SU(3) Hamiltonian for the sdg proton shell

PRC95 (2017) 064325

# Connection of proxy-SU(3) to the spherical shell model

A. Martinou et al. EPJA 56 (2020) 239

Elliott to shell model basis

$$[N_z \ N_x \ N_y \ M_s] = R [n \ L \ M \ M_s]$$

R: unitary transformation

Davies and Krieger, Can. J. Phys. 69 (1991) 62

$$[n \ L \ M \ M_s] = C | n \ L \ J \ M_j \rangle$$

C: Clebsch Gordan coefficients

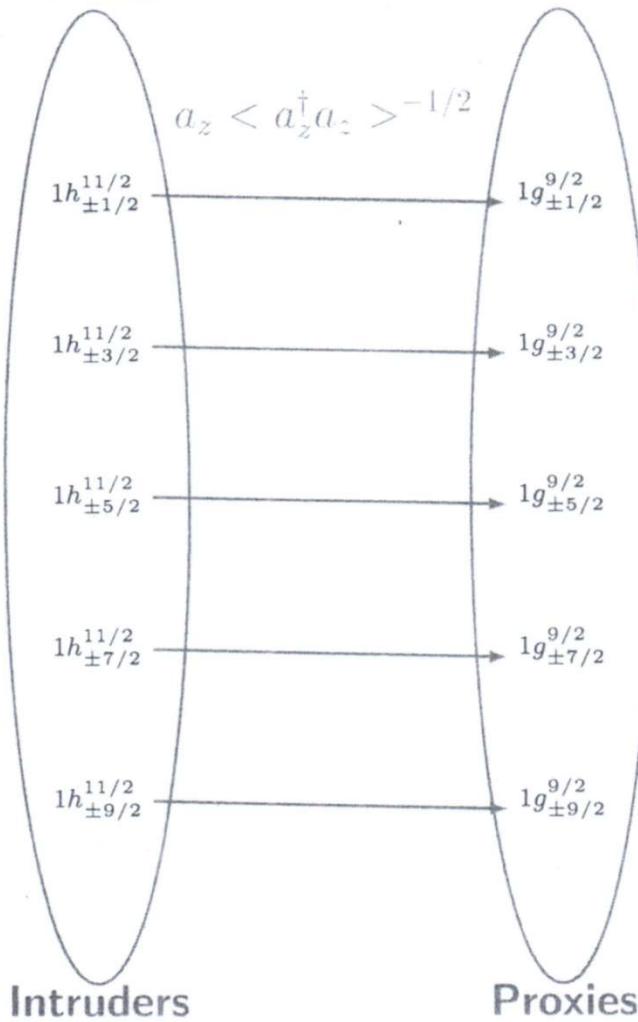
$$[N_z \ N_x \ N_y \ M_s] = R C | n \ L \ J \ M_j \rangle$$

Elliott                      shell model

# unitary transformation

$|0\ 1\ 1\ 0\rangle$  pairs     $|\Delta n\ \Delta L\ \Delta J\ \Delta M_j\rangle$

A. Martinou et al. EPJA 56 (2020) 239



# Dominance of highest weight (most symmetric) irreps

A. Martinou et al. EPJA 57 (2021) 83

- Pauli principle
- Total wave function = antisymmetric
- Total wave function= space x spin x isospin
- Experimentally known ( $S=0, T=1$ ) or ( $S=1, T=0$ )
- Spin x isospin = antisymmetric
- Spatial part = symmetric
- Short range of the nucleon-nucleon interaction

# prolate over oblate dominance

PRC 95 (2017) 064326

ANALYTIC PREDICTIONS FOR NUCLEAR SHAPES, ...

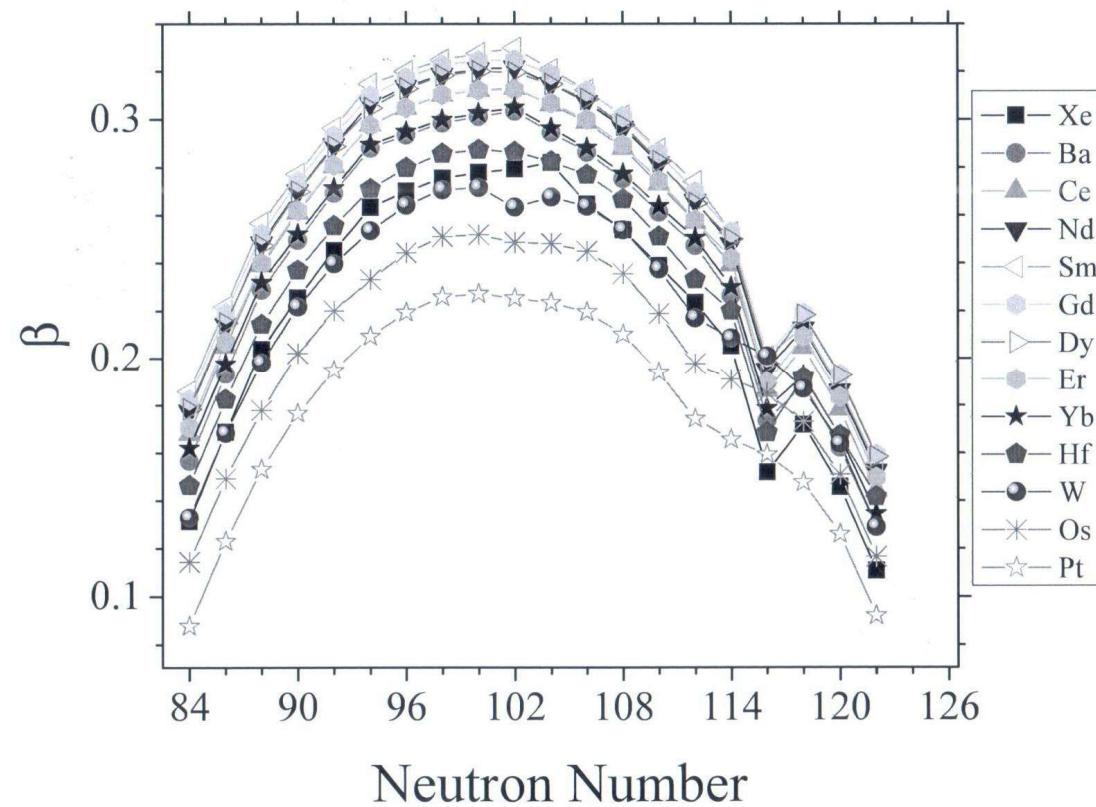
PHYSICAL REVIEW C 95, 064326 (2017)

TABLE II. Most leading SU(3) irreps [34,35] for nuclei with protons in the 50-82 shell and neutrons in the 82-126 shell. Boldface numbers indicate nuclei with  $R_{4/2} = E(4_1^+)/E(2_1^+) \geq 2.8$ , while \* denotes nuclei with  $2.8 > R_{4/2} \geq 2.5$ , and \*\* labels a few nuclei with  $R_{4/2}$  ratios slightly below 2.5, shown for comparison, while no irreps are shown for any other nuclei with  $R_{4/2} < 2.5$ . For the rest of the nuclei shown (using normal fonts and without any special signs attached) the  $R_{4/2}$  ratios are still unknown [46]. Irreps corresponding to oblate shapes are underlined.

<i>N</i>	<i>N<sub>val</sub></i>	<i>Z</i>	Ba	Ce	Nd	Sm	Gd	Dy	Er	Yb	Hf	W	Os	Pt
		<i>Z<sub>val</sub></i>	56	58	60	62	64	66	68	70	72	74	76	78
		irrep	6	8	10	12	14	16	18	20	22	24	26	28
			(18,0)	(18,4)	(20,4)	(24,0)	(20,6)	(18,8)	(18,6)	(20,0)	(12,8)	(6,12)	(2,12)	(0,8)
88	6	(24,0)	(42,0)*	(42,4)*	(44,4)*									
90	8	(26,4)	(44,4)	(44,8)	(46,8)	(50,4)	(46,10)	(44,12)	(44,10)*	(46,4)*	(38,12)*			
92	10	(30,4)	(48,4)	(48,8)	(50,8)	(54,4)	(50,10)	(48,12)	(48,10)	(50,4)	(42,12)*			
94	12	(36,0)	(54,0)	(54,4)	(56,4)	(60,0)	(56,6)	(54,8)	(54,6)	(56,0)	(48,8)	(42,12)	(38,12)*	
96	14	(34,6)	(52,6)	(52,10)	(54,10)	(58,6)	(54,12)	(52,14)	(52,12)	(54,6)	(46,14)	(40,18)	(36,18)*	
98	16	(34,8)	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	(52,16)	(52,14)	(54,8)	(46,16)	(40,20)	(36,20)*	
100	18	(36,6)	(54,6)	(54,10)	(56,10)	(60,6)	(56,12)	(54,14)	(54,12)	(56,6)	(48,14)	(42,18)	(38,18)	(36,14)*
102	20	(40,0)	(58,0)	(58,4)	(60,4)	(64,0)	(60,6)	(58,8)	(58,6)	(60,0)	(52,8)	(46,12)	(42,12)	(40,8)*
104	22	(34,8)	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	(52,16)	(52,14)	(54,8)	(46,16)	(40,20)	(36,20)	(34,16)*
106	24	(30,12)	(48,12)	(48,16)	(50,16)	(54,12)	(50,18)	(48,20)	(48,18)	(50,12)	(42,20)	(36,24)	(32,24)	(30,20)*
108	26	(28,12)	(46,12)	(46,16)	(48,16)	(52,12)	(48,18)	(46,20)	(46,18)	(48,12)	(40,20)	(34,24)	(30,24)	(28,20)*
110	28	(28,8)	(46,8)	(46,12)	(48,12)	(52,8)	(48,14)	(46,16)	(46,14)	(48,8)	(40,16)	(34,20)	(30,20)	(28,16)*
112	30	(30,0)	(48,0)	(48,4)	(50,4)	(54,0)	(50,6)	(48,8)	(48,6)	(50,0)	(42,8)	(36,12)	(32,12)	(30,8)**
114	32	(20,10)	(38,10)	(38,14)	(40,14)	(44,10)	(40,16)	(38,18)	(38,16)	(40,10)	(32,18)	(26,22)	(22,22)	(20,18)**
116	34	(12,16)	(30,6)	(30,10)	(32,10)	(36,6)	(32,12)	(30,14)	(30,12)	(32,6)	(24,14)	(18,28)*	(14,28)	(12,24) **
118	36	(6,18)	(24,18)	(24,22)	(26,22)	(30,18)	(26,24)	(24,16)	(24,24)	(26,18)	(18,26)	(12,30)	(8,30)*	(6,26) **
120	38	(2,16)	(20,16)	(20,20)	(22,20)	(26,16)	(22,22)	(20,24)	(20,22)	(22,16)	(14,24)	(8,28)	(4,28)*	(2,24) **

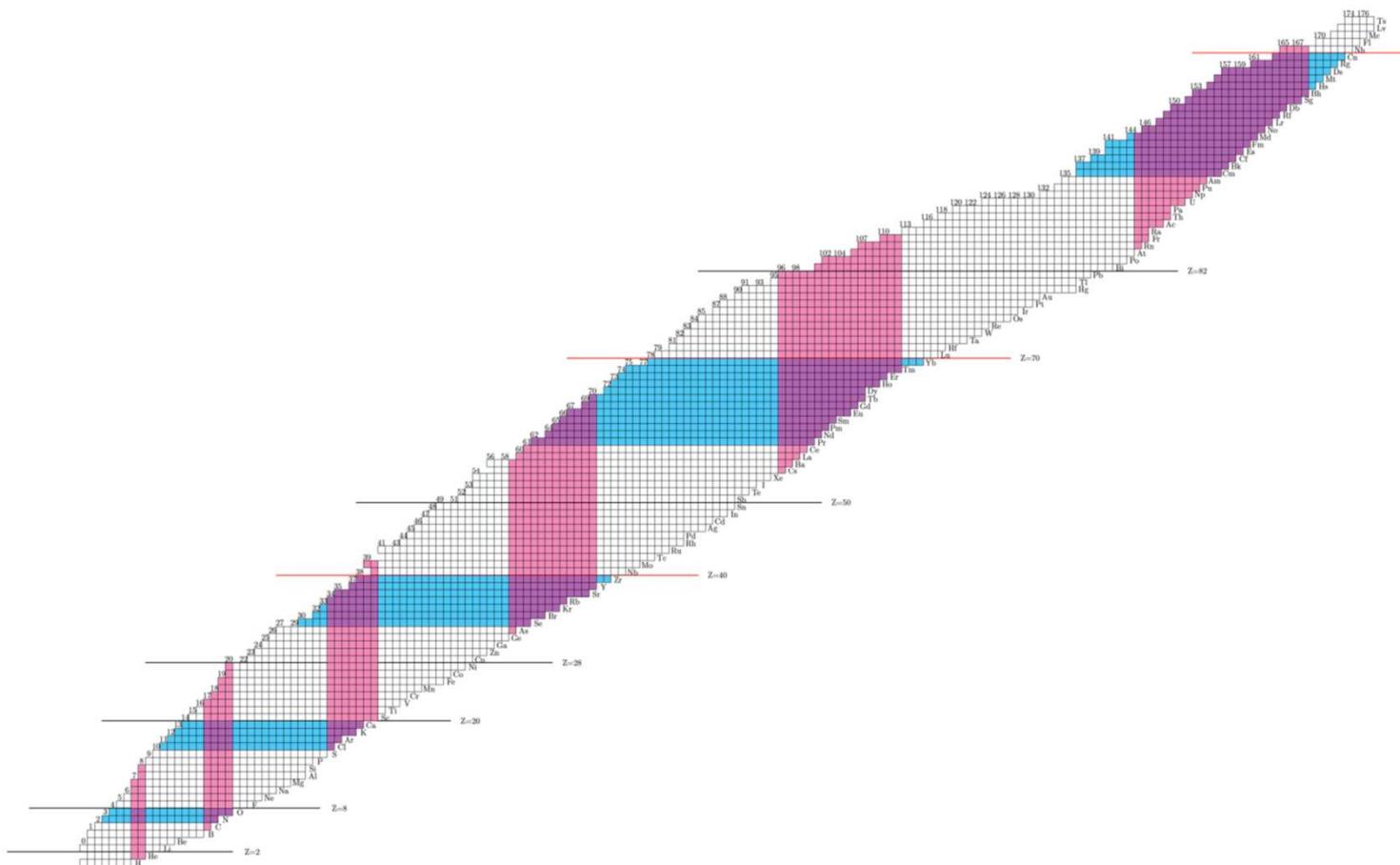
# Prolate to oblate transition

PRC 95 (2017) 064326



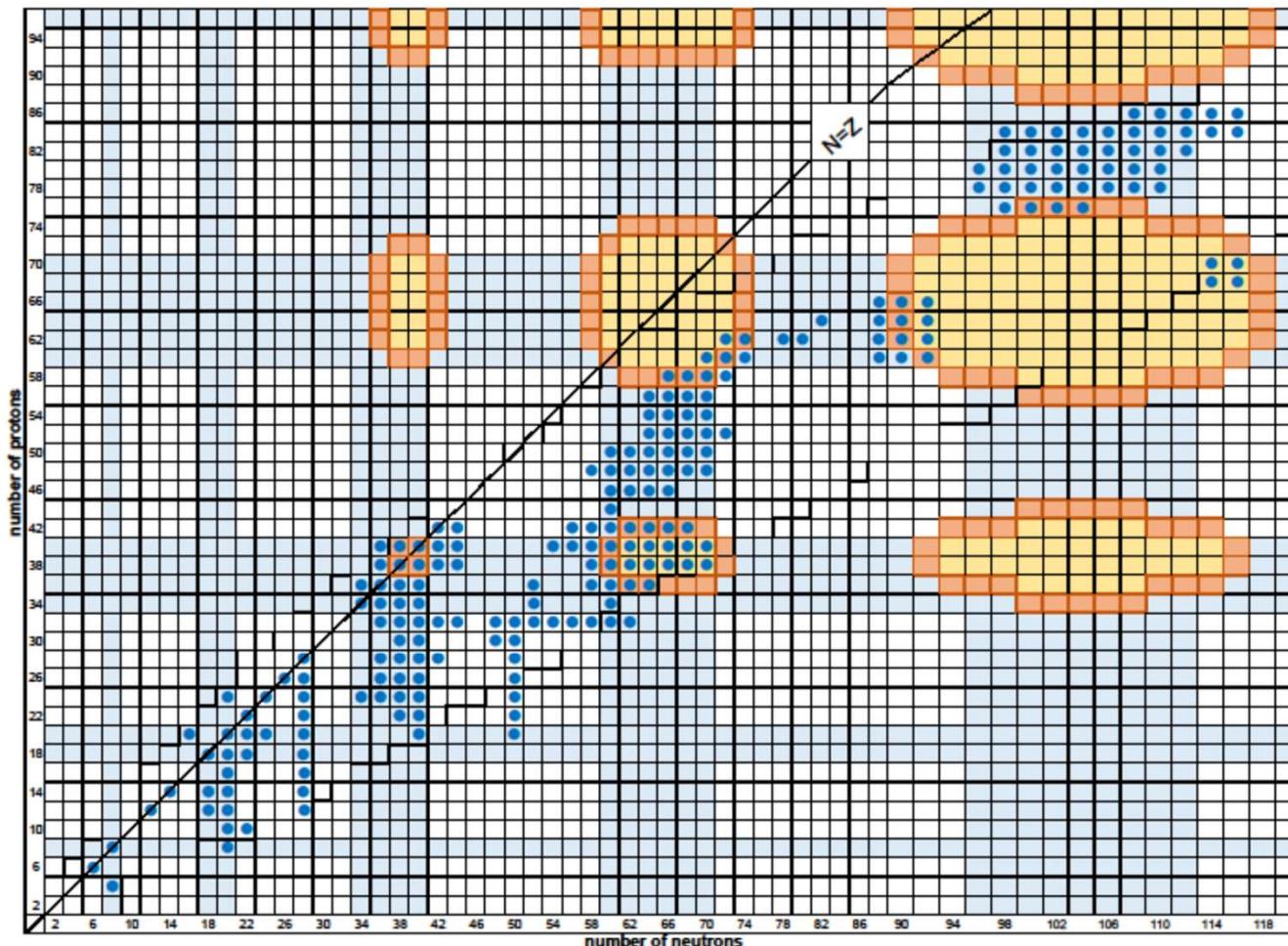
# proxy-SU(3) symmetry dual shell mechanism for shape coexistence

A. Martinou et al. EPJA 57 (2021) 84



# nuclei with shape coexistence

Atoms 11 (2023) 117 (cover story)

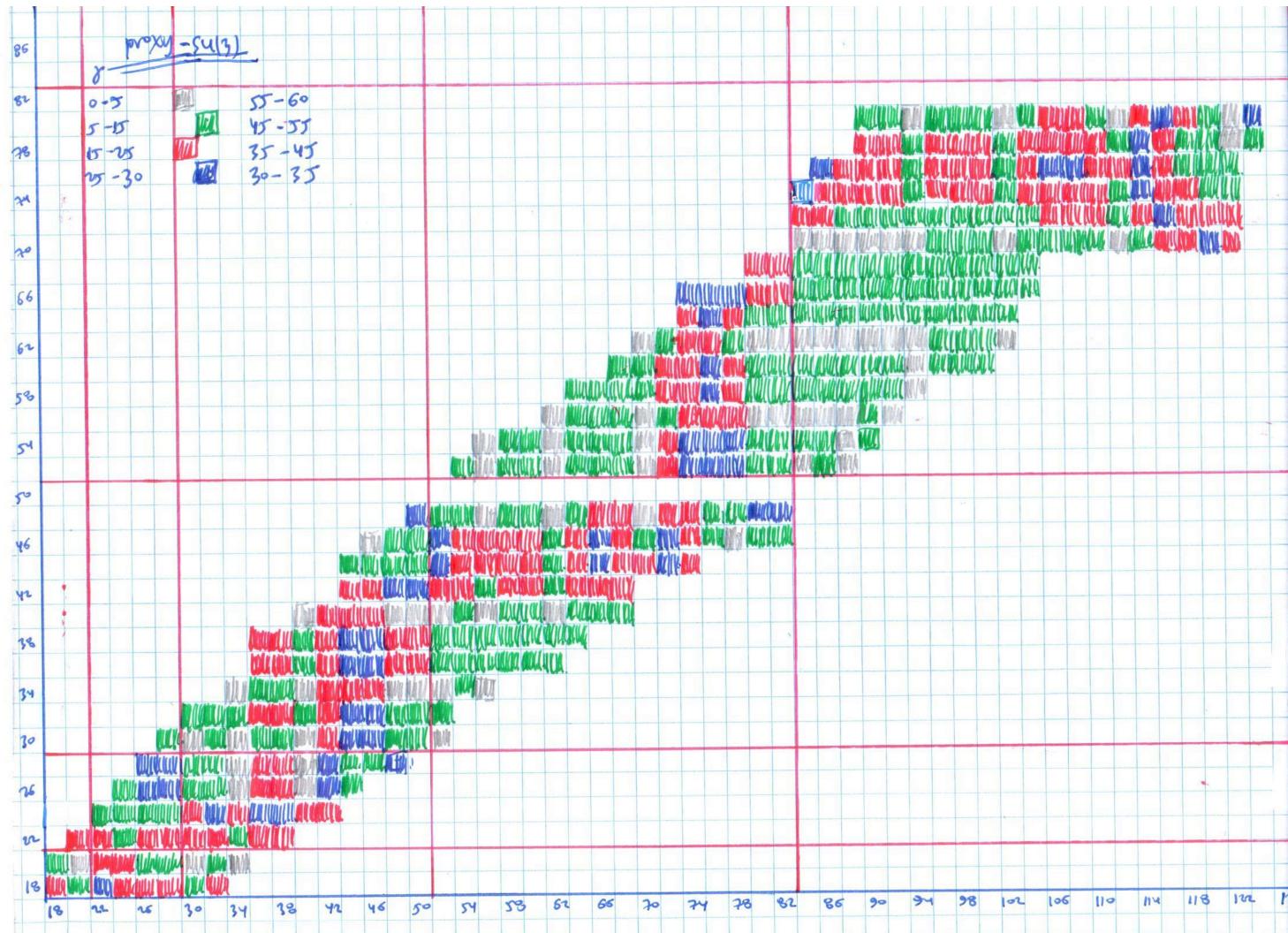




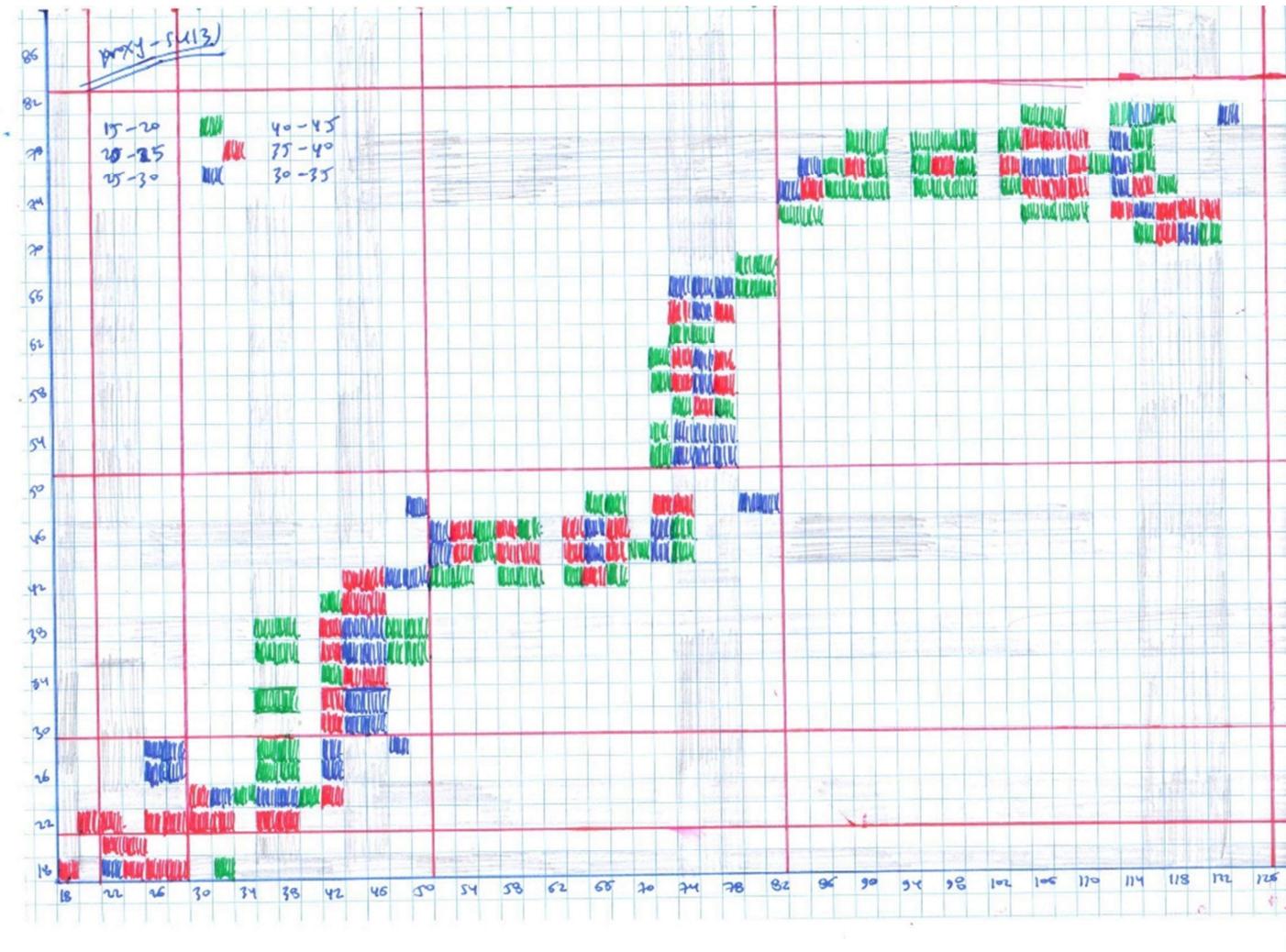
# Preponderance of triaxial shapes

- Monte Carlo shell model – tensor force  
Tsunoda and Otsuka PRC 103 (2021) L021303  
Otsuka et al. arXiv: 2303.11299 [nucl-th]
- Triaxial Projected Shell Model  
Rouoof et al. EPJA 60 (2024) 40
- Empirical (consistent modelling)  
Grosse and Junghans PLB 833 (2022) 137328

# Proxy-SU(3) predictions for triaxiality



# Proxy-SU(3) predictions for high triaxiality



EVERYONE ELSE  
WHO TRIED THIS  
HAS GOTTEN HURT.

SU(3)

HEAVY NUCLEI

EVERYONE ELSE  
DID IT THE  
WRONG WAY.