Single Folding Potential Calculations in ^{141}Pr

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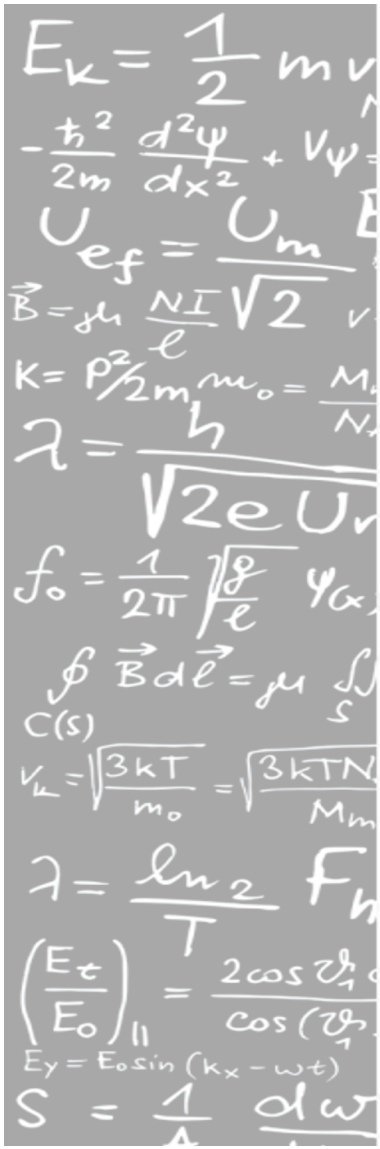
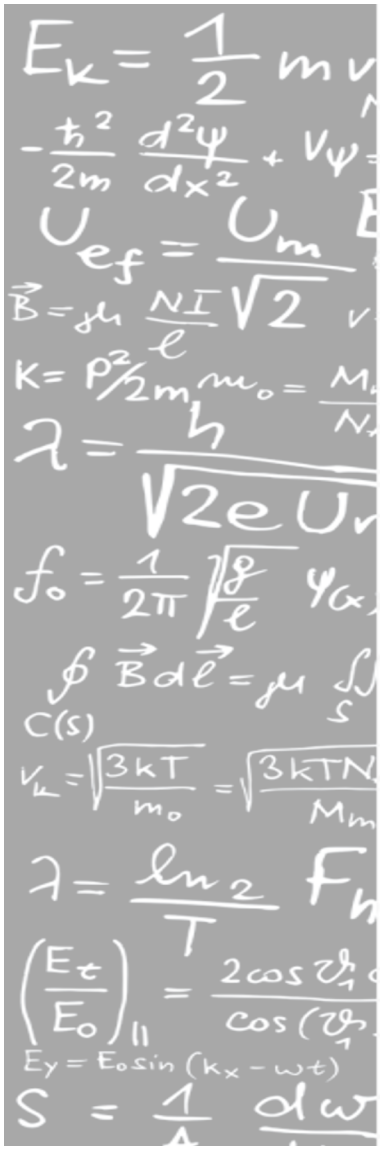


Introduction

The ground state properties of the nuclei, especially nucleon densities, are generally calculated using Skyrme and Gogny forces. The nucleon densities of Praseodymium-141 (^{141}Pr) have been calculated by using;

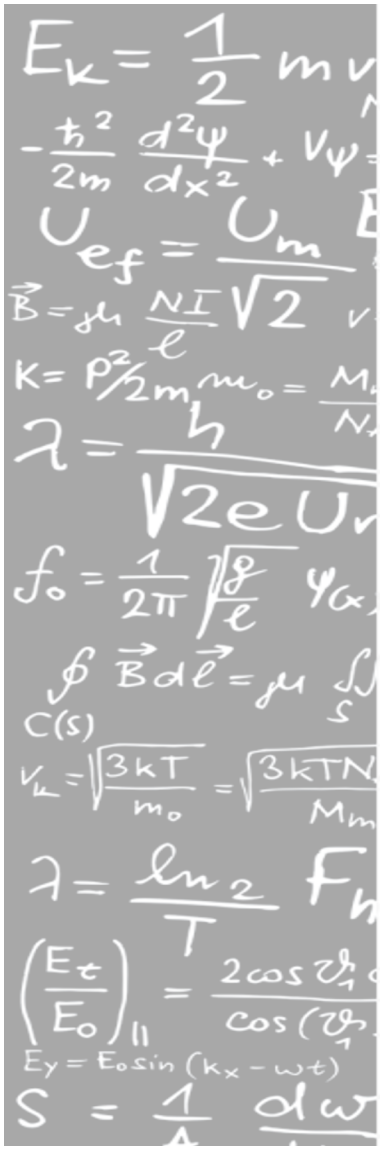
- Hartree-Fock-Skyrme (based Woods-Saxon Potential) (SHF-WS)
- Hartree-Fock-Skyrme (based Harmonic Oscillator Potential) (SHF-HO)
- Hartree-Fock-Bogolyubov-Skyrme (HFB-S)
- Hartree-Fock-Bogolyubov-Gogny (HFB-G)

methods. The obtained nucleon densities for ^{141}Pr have been compared with each other.



The Hartree-Fock (HF) codes are a useful tool to describe the ground states properties of the spherical nuclei. Spherical HF codes have been used for several years and are developed by deriving gradient iterations. In HF approach, equations are solved using an iteration, namely, harmonic oscillator or Wood-Saxon wave functions are recommended for wave function. The density $\rho(0)$ is generated based on these wave functions. Then, iterations are made between the intensity and $U(\rho)$ energy potential for φ wave function.

The Skyrme force is the effective force for HF calculations. In this way, quantities, which are very important and can be measured experimentally, such as core radii, density distributions and surface thickness can be calculated. HF codes are applied to the specified Skyrme force by fitting the least squares method. It is also fitted to ground states correlations and nucleus excitation studies.



Skyrme Forces

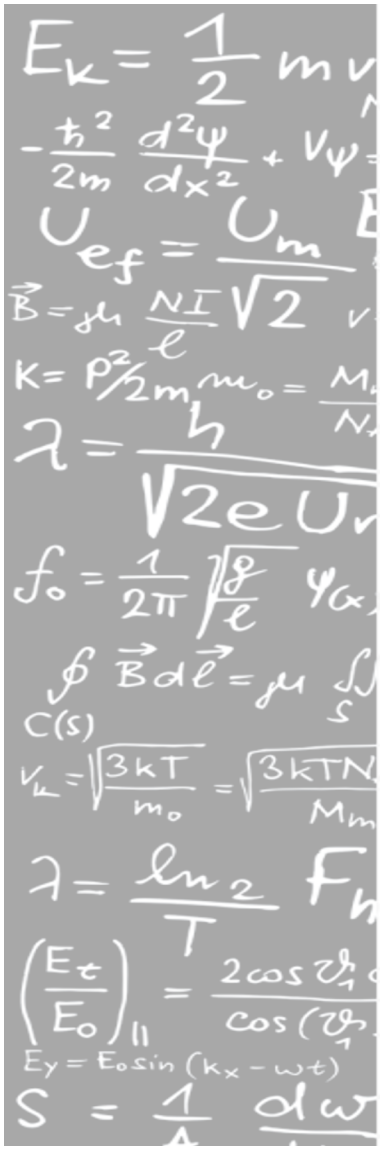
Skyrme forces are the best phenomenological force used to describe the ground states of nucleus. Skyrme interactions, including the interaction of three objects with two-body interaction is defined as;

$$\vec{V}_{\text{Skyrme}} = \sum_{i<j} \vec{V}(i,j) + \sum_{i<j<k} \vec{V}(i,j,k)$$

The first term defines two objects and the second term defines interaction of three bodies. Short-range two-body interaction is given as;

$$\vec{V}(i,j) = t_0(1+x_0P_x)\delta(\vec{r}_i-\vec{r}_j) + \frac{1}{2}t_1(1+x_1P_x)\{\vec{p}'^2_{12}\delta(\vec{r}_i-\vec{r}_j) + \delta(\vec{r}_i-\vec{r}_j)\vec{p}^2_{12}\}$$

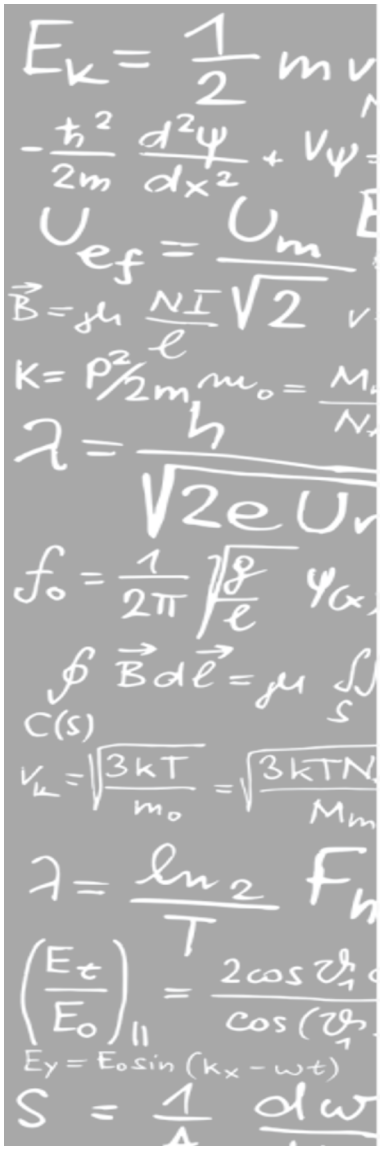
$$+ t_2(1+x_2P_x)\vec{p}'_{12}\delta(\vec{r}_i-\vec{r}_j)\vec{p}_{12} + it_4\vec{p}'_{12}\cdot\delta(\vec{r}_i-\vec{r}_j)(\vec{\sigma}_i+\vec{\sigma}_j)\times\vec{p}_{12}$$



Skyrme-HFB

Many properties of nuclei can be described in terms of a model comprised of independent particles moving in an average potential. The space dependence of this potential should closely follow the matter distribution from the Skyrme-HF method. Using this model, for independent particles, a single particle potential can be derived from the two-body interaction. This is done using a variational principle, with Slater determinants (of, for example, harmonic oscillator or Woods-Saxon wave functions) as trial wave functions.

The most general product wave functions in the Skyrme-HFB theory consist of independently moving quasi-particles. Such wave functions are determined using a variational principle and they take into account as many correlations as possible while remaining in a static single particle picture.



Gogny Forces

Another self-compatible field method is the Gogny model using Gogny interaction. Gogny force is the interaction of two non-divergent of particles and its parameters are arranged according to experimental data.

The Gogny force has been widely used in various mean-field calculations of nuclear structure. The parameters of he Gogny force were determined by mean-field calculations fitting to the experimental data of infinite nuclei and the properties of infinite nuclear matter. The effect of the three-body force is taken into account in both the Skyrme and Gogny forces through a density-dependent term, which is essential to describe various properties of nuclei and nuclear matter.

Nucleon Densities

The neutron and proton densities are given by:

$$\rho_q(\vec{r}) = \sum_{\beta \in q} w_\beta \psi_\beta(\vec{r})^+ \psi_\beta(\vec{r})$$

where q indicates neutron or proton density and w_β denotes the occupation probability of the state β.

For the SHF method, Ψ_β represents the single-particle wave function of the state β. Alternatively, for the HFB-S method, Ψ_β represents the quasiparticle wave function of the quasiparticle state β. Using either method, the rms radii of neutron and proton densities can be evaluated using upper Equation and the following formula:

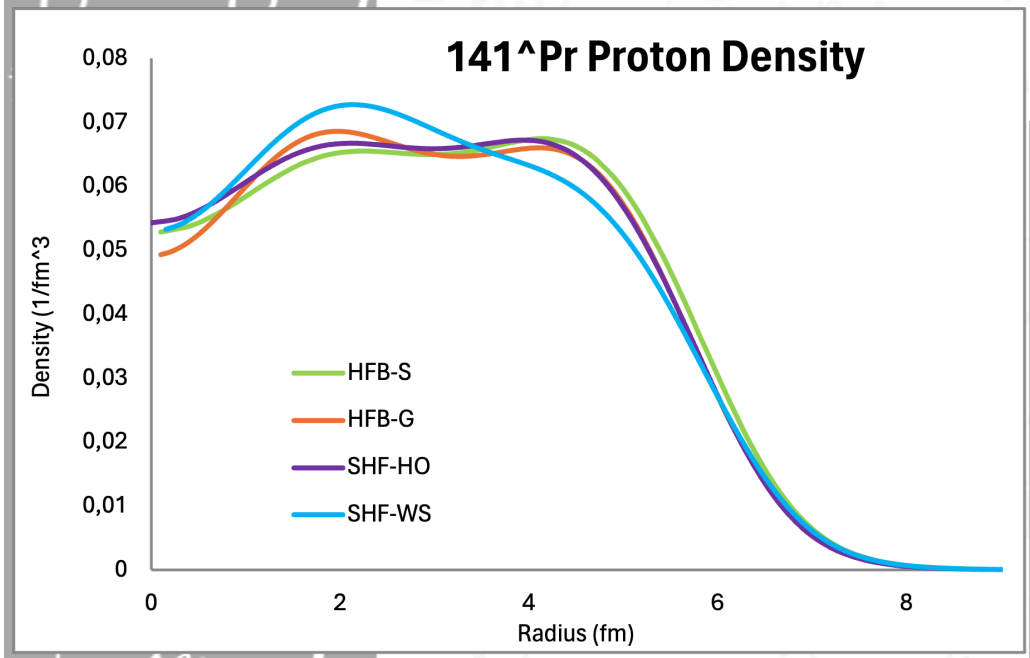
$$r_q = \left\langle r_q^2 \right\rangle^{1/2} = \left[\frac{\int r^2 \rho_q(r) dr}{\int \rho_q(r) dr} \right]^{1/2}$$

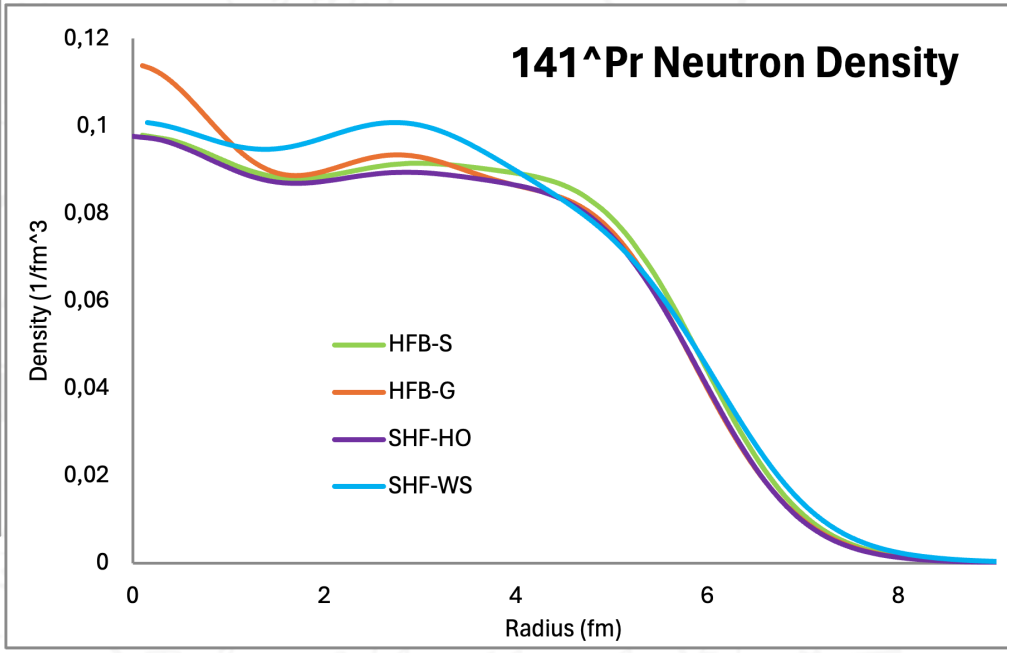
Skyrme Parameter Sets

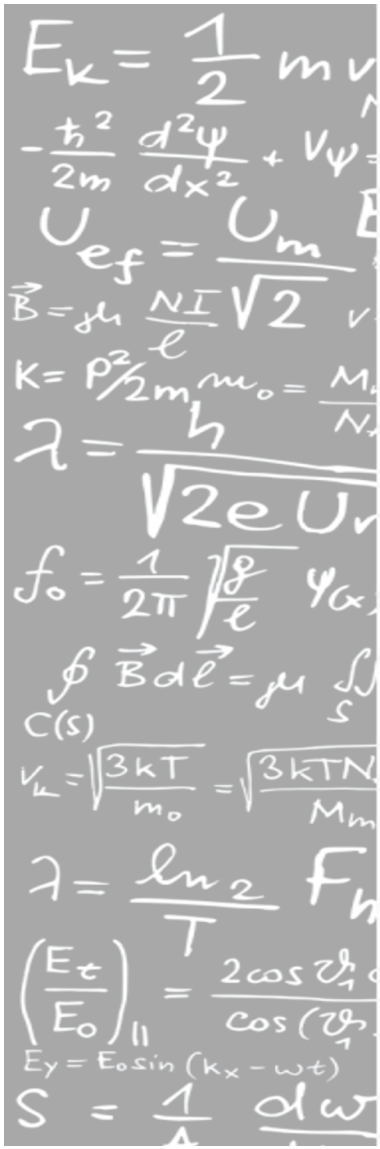
Skyrme	t_0 (Mev.fm^3)	t_1 (Mev.fm^5)	t_2 (Mev.fm^5)	t_3 (MeV.fm$^{3(s+1)}$)	x_0	x_1	x_2	x_3	b_4 (MeV.fm^5)	b_{4p} (MeV.fm^5)	W_0 (MeV.fm^5)	alpha
SKa	-1602.78	570.88	-67.7	8000.0	-0.02	0	0	-0.286	62.5	62.5	125.0	1/3
SGII	-2645.0	340.0	-41.9	15595.0	0.09	-0.0588	1.423	0.06044	52.5	52.5	105.0	1/6
SkM	-2645.0	385.0	-120.0	15595.0	0.09	0	0	0	65.0	65.0	130.0	1/6
SLy4	-2488.913	486.818	-546.395	13777.0	0.834	-0.3438	-1.0	1.354	61.5	61.5	123.0	1/6
SLy5	-2483.45	484.23	-556.69	13757.0	0.776	-0.317	-1.0	1.263	62.5	62.5	125.0	1/6
SLy6	-2479.5	462.18	-448.61	13673.0	0.825	-0.465	-1.0	1.355	61.0	61.0	122.0	1/6
SLy7	-2480.8	461.29	-433.93	13669.0	0.848	-0.492	-1.0	1.393	62.5	62.5	125.0	1/6
SLy8	-2481.41	480.78	-538.34	13731.0	0.8024	-0.3424	-1.0	1.3061	62.8	59.625	122.425	1/6
SLy9	-2511.13	510.6	-429.8	13716.0	0.7998	-0.6213	-1.0	1.3727	55.0	64.0	119.0	1/6
SLy10	-2506.77	430.98	-304.95	13826.41	1.0398	-0.6745	-1.0	1.6833	37.93	52.755	90.685	1/6

	Woods-Saxon	Harmonic Oscillator
R_{charge}	^{141}Pr	^{141}Pr
EXP	4.8919	4.8919
SkA	4.9031	5.0048
GS6	4.4951	4.5083
SkM*	4.8904	4.8912
SGII	4.9884	4.9033
SLy4	4.8991	4.9118
SLy5	4.8893	4.9018
SLy6	4.8878	4.9026
SLy7	4.8895	4.9049
SLy8	4.8904	4.9033
SLy9	4.9687	4.9836
SLy10	4.9134	4.9261

Proton and neutron density calculations by using HF code , HAFOMN and TALYS with chosen parameters







Single Folding Potential

Elastic scattering data can be analyzed using the HF code, in which the calculated values replace the phenomenological true potential. Double Folding Potential is used in the nucleus-nucleus interaction.

$$V_F(R) = N \iint \rho_1(r_1)\rho_2(r_2)V_{NN}(s)dr_1dr_2\,, \qquad (s = R + r_2 - r_1)$$

The terms ρ_1 and ρ_2 represent the nucleon densities of two colliding nuclei. R indicates the distance between the centers of mass of the nuclei. The relative vector (s) indicates the distance between the interacting nucleon pair, while N is the normalization factor. $V_{NN}(s)$ gives the effective nucleon-nucleon (NN) interaction.

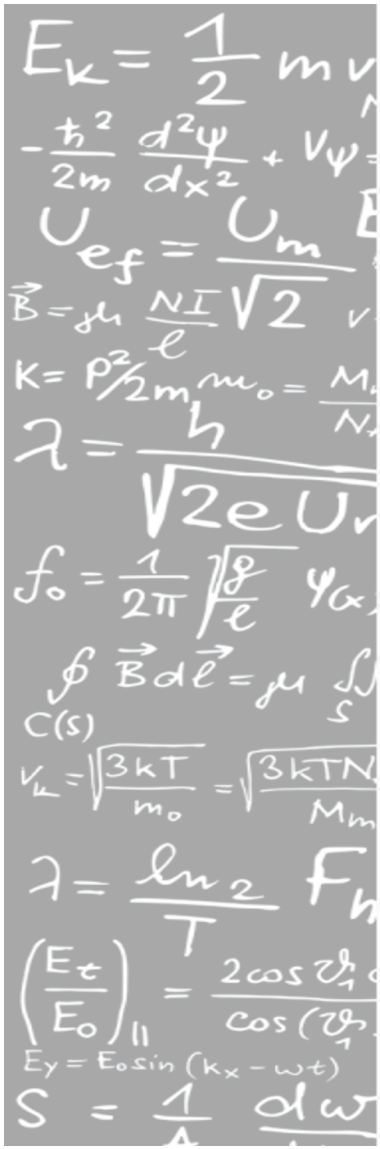
During nucleon-nucleus scattering, the displacement of two interacting nucleons is called "knock-on exchange". This event is included in the double folding integral as follows.

$$V'_{NN} = V_{NN} + J_{00}\,(E)\delta(s)$$

The V_{NN} component used in our calculations has the following form

$$V_{NN}(s) = \left[7999\,\frac{exp(-4s)}{4s} - 2134\,\frac{exp(-2.5s)}{2.5s}\right] MeV$$

and $J_{00} = -146$ MeV fm^3



The nucleon-nucleus reaction is called the standard single-fold optical model, Single Folding Potential, and is expressed as given below.

$$V_{SF}(R) = \int \rho(r)\, V_{NN}\left(|\vec{R} - \vec{r}|\right) d\vec{r}$$

Here,

$$V_{NN}(R - r_1) = \int dr_2 \rho_2(r_2) V(s), \qquad s = R + r_2 - r_1$$

The V interaction potential depends on the density because the folding integral is taken depending on the density of both interacting nuclei or nucleon. The nucleon-nucleus interacting potential includes effects that depend only on the density of a single nucleus. When the phenomenological potential mentioned in the single folding integral is included, the density effects of the other particle are neglected, while the V_{NN} potential is not directly related to the density of the nucleus.

Out[]= Veff[0.15, 1]

Out[]= DF11[0.15, 1]

In[]:= $\partial_{0.15`}$ 1934.5764676414826`

```
Do[
 VDDdf[r] = 1/(2 * π^2) * NIntegrate[ρtfit[k] * VDDfit[k] * SphericalBesselJ[0, k * r] * k^2,
    {k, kmin, kmax}];
 Print[" r = ", r "", "" , "  VDDdf[r]=", VDDdf[r]]
 , {r, rmin, rmax, rstep}]
```

General: 0.15` is not a valid variable.

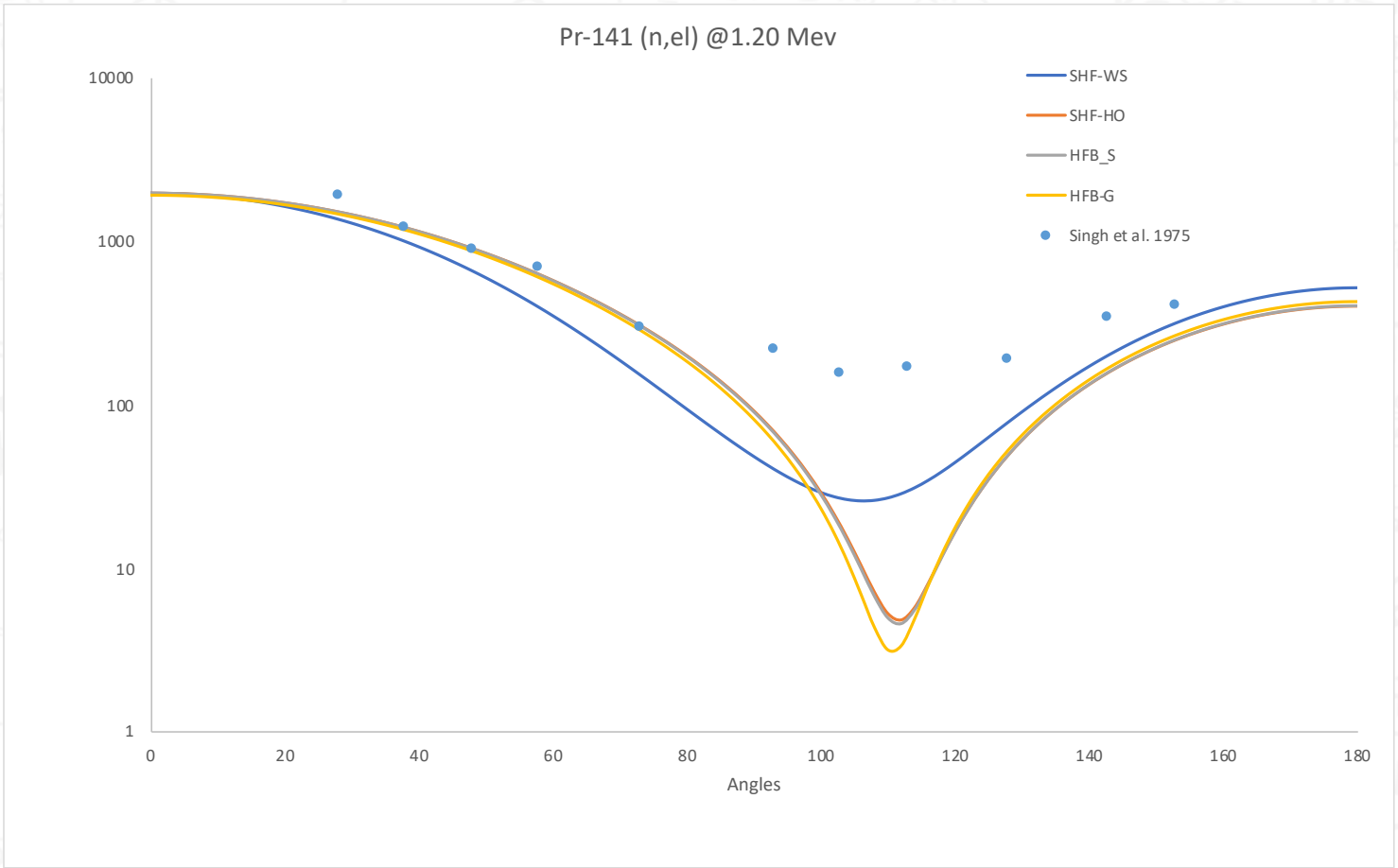
Out[]= $\partial_{0.15}$ 1934.58

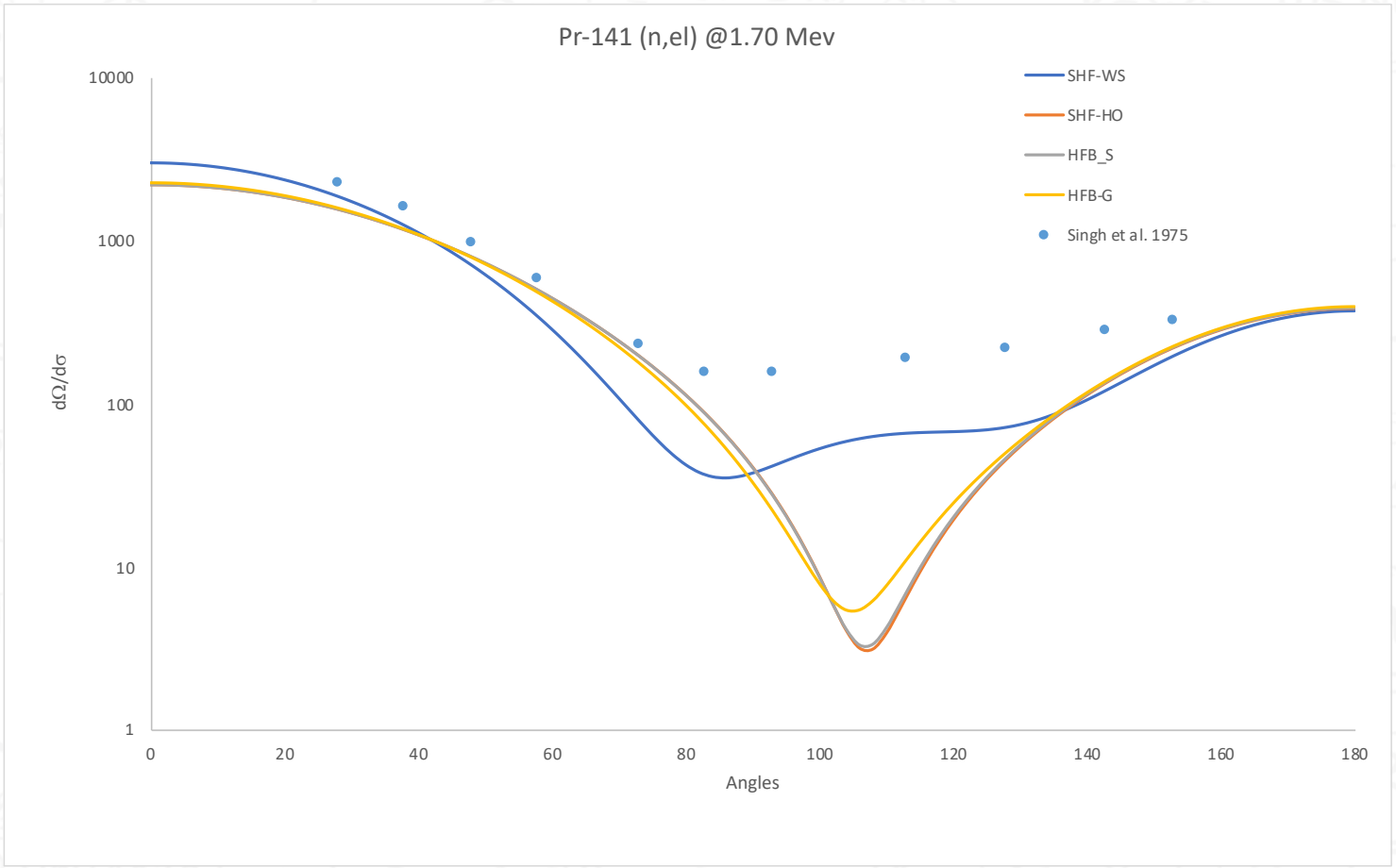
In[]:=

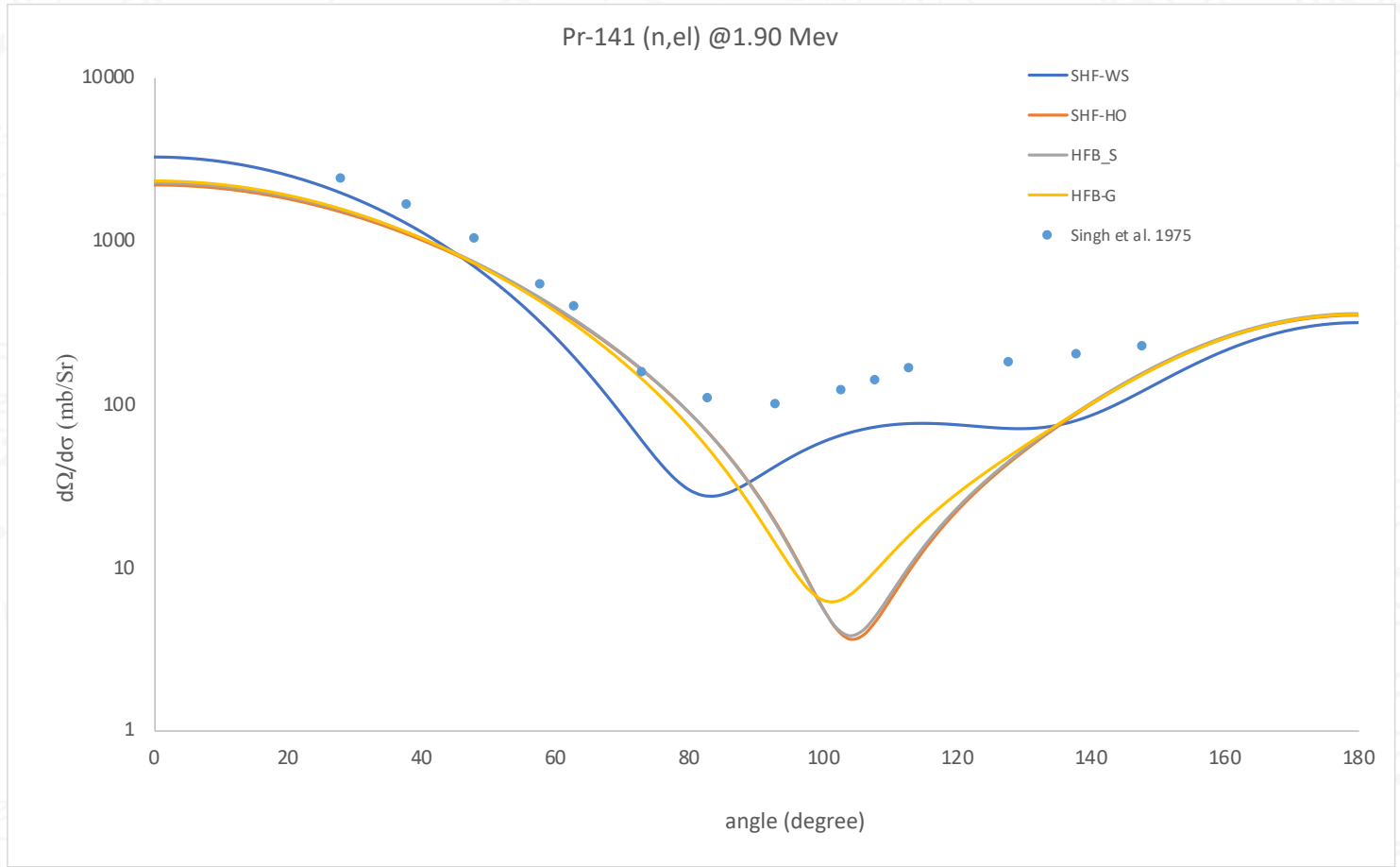
```
Print[" r = ", r "", "" , "  Veff[r,L]=", Veff[r, L], {r, rmin, rmax, rstep}]
```

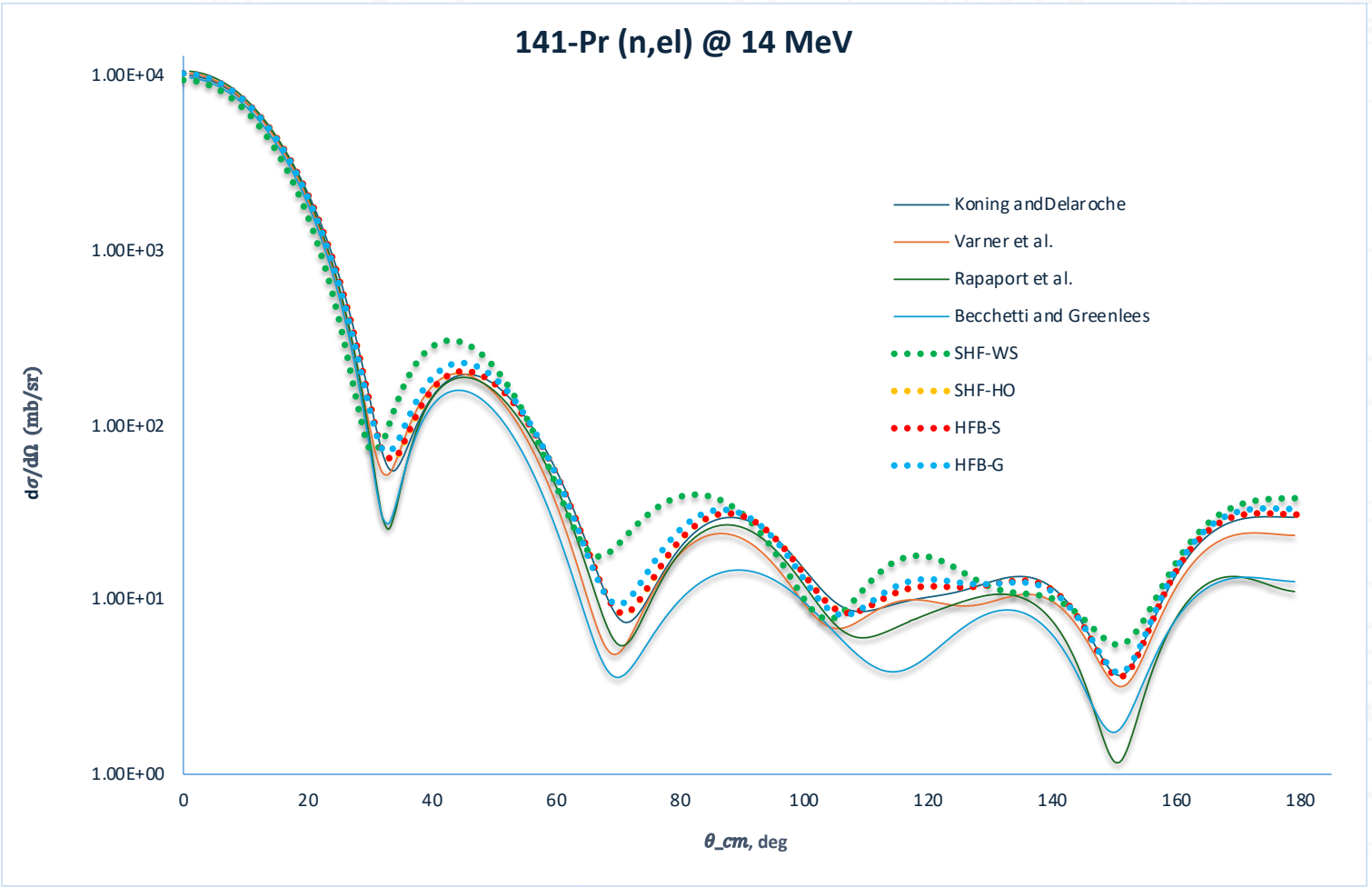
r = r Veff[r,L]= $\frac{21.0554\, L\, (1+L)}{r^2}$ − InterpolatingFunction[Domain: {{0., 30.}} Output: scalar][r] / $(1 + e^{0.769231 \times (-5.19818+r)})$ {r, 0, 30, 0.15}

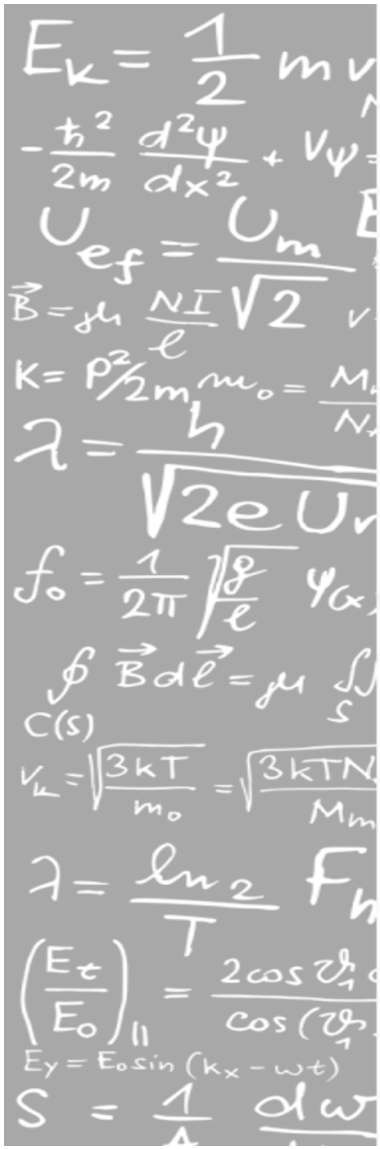
	Theoretical	Potential (MeV)	Potential (MeV)	This work
n+^{141}Pr 14 MeV	Koning and Delaroche	-44.964	-47.8521	SHF-WS
	Varner et al.	-46.577	-43.9269	SHF-HO
	Rapaport et al.	-46.301	-43.9754	HFB-S
	Becchetti and Greenlees	-47.905	-44.7173	HFB-G


Pr-141 (n,el) @1.20 Mev
10000
1000
100
10
1
0
20
40
60
80
100
120
140
160
180
Angles
SHF-WS
SHF-HO
HFB_S
HFB-G
Singh et al. 1975

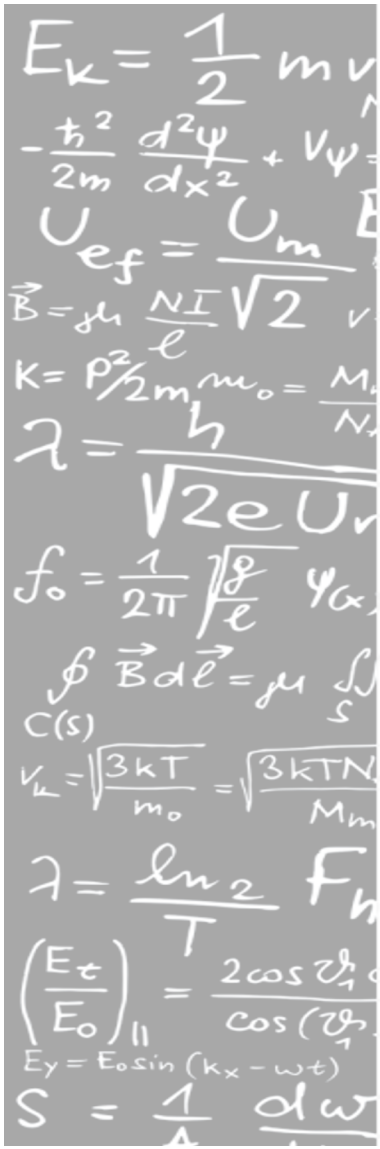

Pr-141 (n,el) @1.70 Mev
SHF-WS
SHF-HO
HFB_S
HFB-G
Singh et al. 1975
dΩ/dσ
10000
1000
100
10
1
0
20
40
60
80
100
120
140
160
180
Angles


Pr-141 (n,el) @1.90 Mev
10000
1000
100
10
1
dΩ/dσ (mb/Sr)
0
20
40
60
80
100
120
140
160
180
angle (degree)
SHF-WS
SHF-HO
HFB_S
HFB-G
Singh et al. 1975


141-Pr (n,el) @ 14 MeV
1.00E+04
1.00E+03
1.00E+02
1.00E+01
1.00E+00
dσ/dΩ (mb/sr)
0
20
40
60
80
100
120
140
160
180
θ_cm, deg
Koning andDelaroche
Varner et al.
Rapaport et al.
Becchetti and Greenlees
SHF-WS
SHF-HO
HFB-S
HFB-G



R^2		SHF-WS	SHF-HO	HFB-S	HFB-G
^{141}Pr(n,el) at 1.2 MeV	Singh et al. 1975	**0.982155**	0.959465	0.959609	0.961839
^{141}Pr(n,el) at 1.7 MeV		**0.997615**	0.980930	0.981322	0.985684
^{141}Pr(n,el) at 1.9 MeV		**0.993091**	0.985129	0.985675	0.990989
^{141}Pr(n,el) at 14 MeV	Koning and Delaroche	0.997146	0.999959	0.999952	0.999783



Thanks…